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MDCCCLXIX.

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THE Committee appointed by the *Royal Society* to direct the publication of the *Philosophical Transactions*, take this opportunity to acquaint the Public, that it fully appears, as well from the Council-books and Journals of the Society, as from repeated declarations which have been made in several former *Transactions*, that the printing of them was always, from time to time, the single act of the respective Secretaries till the Forty-seventh Volume; the Society, as a Body, never interesting themselves any further in their publication, than by occasionally recommending the revival of them to some of their Secretaries, when, from the particular circumstances of their affairs, the *Transactions* had happened for any length of time to be intermitted. And this seems principally to have been done with a view to satisfy the Public, that their usual meetings were then continued, for the improvement of knowledge, and benefit of mankind, the great ends of their first institution by the Royal Charters, and which they have ever since steadily pursued.

But the Society being of late years greatly enlarged, and their communications more numerous, it was thought advisable that a Committee of their members should be appointed, to reconsider the papers read before them, and select out of them such as they should judge most proper for publication in the future *Transactions*; which was accordingly done upon the 26th of March 1752. And the grounds of their choice are, and will continue to be, the importance and singularity of the subjects, or the advantageous manner of treating them; without pretending to answer for the certainty of the facts, or propriety of the reasonings, contained in the several papers so published, which must still rest on the credit or judgement of their respective authors.

It is likewise necessary on this occasion to remark, that it is an established rule of the Society, to which they will always adhere, never to give their opinion, as a Body, upon any subject, either of Nature or Art, that comes before them. And therefore the

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thanks, which are frequently proposed from the Chair, to be given to the authors of such papers as are read at their accustomed meetings, or to the persons through whose hands they received them, are to be considered in no other light than as a matter of civility, in return for the respect shown to the Society by those communications. The like also is to be said with regard to the several projects, inventions, and curiosities of various kinds, which are often exhibited to the Society; the authors whereof, or those who exhibit them, frequently take the liberty to report and even to certify in the public newspapers, that they have met with the highest applause and approbation. And therefore it is hoped that no regard will hereafter be paid to such reports and public notices; which in some instances have been too lightly credited, to the dishonour of the Society.

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Leipzig	Dr. C. Bruhns.
Lisbon	Senhor da Silveira.
Marburg	
Prague	K. Jelinek.
Stockholm	Professor H. Selander.
St. Petersburg	(Twelve copies for distribution to the Russian Mag. and Met. Obs.)
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PHILOSOPHICAL TRANSACTIONS.

I. *Researches on Solar Physics. Heliographical Positions and Areas of Sun-spots observed with the Kew Photoheliograph during the years* 1862 *and* 1863. *By* WARREN DE LA RUE, *Esq., Ph.D., F.R.S., F.R.A.S., Pres. C.S.,* BALFOUR STEWART, *Esq., LL.D., F.R.S., F.R.A.S., Superintendent of the Kew Observatory, and* BENJAMIN LOEWY, *Esq., F.R.A.S.*

Received March 31,—Read April 30, 1868.

1. WE have been hitherto engaged in preliminary researches devoted chiefly to the physical nature of the sun, but we now propose to give a first instalment of the measurements made with a view of making the solar photographs taken at Kew the basis of a new determination of the sun's astronomical elements. Accordingly the present paper contains the results of measurements of the Kew sun-pictures for 1862 and 1863, *i. e.* the heliographical longitudes and latitudes of all spots observed in these two years, together with a full description of the methods pursued in the reductions. The results of the succeeding years, and their final discussion with reference to the sun's elements, will be published hereafter.

In addition to those measurements which have reference to the heliographical position of each spot, we have also measured the area of each group on each occasion when it was observed; and we conceive that in thus giving the position and area of each group we give all the information regarding sun-spots that is capable of accurate numerical expression, at least in the present stage of our knowledge.

Measurements made with the view of determining the Heliographical Latitude and Longitude of Spots.

2. We need not again describe the measuring-apparatus and its various adjustments; for these have been already described by Mr. DE LA RUE in the Bakerian Lecture for 1862.

Suffice it to say that, in the photoheliograph by which the sun-pictures are taken, there is at the joint focus of the object-glass and eye-glass a system of two fixed wires,

one of which is perpendicular to the other; and consequently an impression of these wires is produced in every sun-picture which is obtained.

The nature of the measurements made will be better understood by supposing for the moment that this system of wires is in exact adjustment, and that their centre is also the centre of the sun-picture. Then by means of the measuring-instrument the following elements are measured:—

(1) The diameter of the sun in the picture taken (in inches).

(2) The distance of a spot from the sun's centre (in inches).

(3) The angle which the line joining the spot and the sun's centre makes with one of the wires.

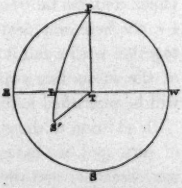
Sufficiency of Observational Data afforded by means of Measurements.

3. It will now be necessary to show that these observational data are sufficient for the determination of the heliographical position of the sun-spots. As will be seen from the following general considerations, two elements are required for this purpose, viz. the angular distance of a spot from the centre of the sun, and its angle of position with reference to a fixed line, at the time of the observation.

It is perhaps almost superfluous to remark that our object is to ascertain the heliographical latitude and longitude of spots—that is to say, the angular distance of a spot from the solar equator, and also from a certain meridian of solar longitude, the meridian chosen being that which passes through the vernal equinox, and the degrees being reckoned from west to east.

Let P E S W (fig. 1) represent the visible disk of the sun, of which circle T, the earth, is the pole, and let S′ be the place of a spot; also let W E denote the plane of the ecliptic, and let P and S be its poles.

Fig. 1.



In the spherical triangle P T S′ the angle T P S′ is obviously the difference of longitude of the spot S′ and the earth, while the arc P L S′ represents the north polar distance of the spot, and gives consequently the latitude. But in this triangle we have the necessary parts given for finding P S′ and T P S′;

for the side P T $=90°$,

the angle P T S′$=90°+$E T S′ is the angle of position of the spot S′,

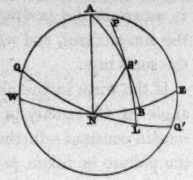
the side T S′ $=$the angular distance of the spot from the centre, which is given by the consideration, that

$$\text{the sine of the angular distance} = \frac{\text{measured distance T S′}}{\text{sun's semidiameter}}.$$

Hence we see that the observational data required for our purpose are, as far as the ecliptic is concerned, the measured length of the distance of a spot from the centre, and its angle of position, both of which data are given with the greatest precision by our measuring-instrument applied to the solar photographs.

To transform the ecliptical coordinates thus found into the heliographical, let P (fig. 2) be the true pole of the sun, A that of the ecliptic, which is represented by W E, while Q Q′ is the sun's equator, and N the node. Then, if S′ be a spot, the preceding considerations make known to us the sides S′ B and B N in the triangle S′ N B, while we really require S′ L and N L, which are the true heliographical coordinates. These, however, are easily deduced when we consider that in the triangle S′ N B

Fig. 2.



$$\cos N S' = \cos S' B \cos B N,$$
$$\cos S' N B = \cot N S' \tan B N,$$

and that in the triangle S′ N L the angle S′ N L = S′ N B + I, I being the inclination of the sun's equator and the plane of the ecliptic; therefore

$$\sin S' L = \sin N S' \sin S' N L,$$
$$\sin N L = \tan S' L \cot S' N L,$$

which gives the required coordinates.

4. The principles of reduction here explained, however, have been considerably modified, and we have throughout followed the elegant and convenient method given by Mr. Carrington in his volume of Sun-Observations. The method being now adapted to sun-observations by means of photography, we have thought it necessary to give in the following pages a detailed account of it, and here take the opportunity to express our gratitude to Mr. Carrington for the advice and instruction which he has kindly given to M. von Bose, to whom the first part of the reductions, extending over the year 1862, was entrusted.

We have also adopted for our calculations the values proposed by Mr. Carrington for the longitude of the sun's ascending node, and for the angle of inclination of the plane of the solar equator and that of the ecliptic (vide Mr. Carrington's Observations, page 244), viz.

$$I = 7° 15', \quad N = 73° 40' \text{ for } 1850{\cdot}0.$$

The auxiliary Tables necessary for the reductions, which are given in Mr. Carrington's volume, pages 20 to 26, were consequently recalculated; but as they can easily be deduced from those published by Mr. Carrington, by those who choose to adopt his elements for the calculation of the position of sun-spots, they are not given here.

Various Steps of the Process of Reduction.

5. We thus see that the measurements made are sufficient to determine, in the first place, the ecliptical latitude and longitude of a spot, and, in the second place, by knowing the longitude of the ascending node of the solar equator as well as the inclination of this to the plane of the ecliptic, to transform these ecliptical elements into others having reference to the solar equator instead of the ecliptic.

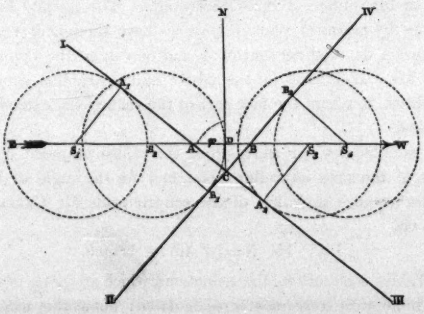
The first step in all this is to be able to trace on each solar photogram a line which

represents the projection of the plane of the ecliptic—or, what is equivalent to this, to ascertain the position-angle with reference to the ecliptic of the wires depicted on the sun-pictures, and which appear as bright lines untouched by the chemical action of the sun's rays.

If the wires in the photoheliograph could be so placed as to be always inclined at an angle of 45° exactly to a parallel of declination, if the declination of the sun could remain constant—further, if the wires could be made truly vertical to each other, and the picture be taken precisely in the centre of the photoheliograph, then obviously the wire I. (fig. 3) would make an angle of 45° with the true north point of the sun's disk, wire II. of 135°, and so on—the readings on the circle giving at once the precise angles of position of the wires and of any other line drawn to the centre, reckoning from north through east. These conditions, however, cannot be fulfilled; and the following steps are necessary for obtaining the corrections to be applied to the observed readings, in order to deduce the true position-angles.

6. To find the true inclination (ϕ) of the wire I. (fig. 3) to the line N D C (which is supposed to be drawn perpendicularly through the sun's path, and joining its centre and true north point), the following method, due to Mr. Carrington, has been adopted.

Fig. 3.



A plate of ground glass is inserted in the chamber provided in the heliograph for the reception of the prepared plates for the sun-pictures; and a bright image of the sun, and also of the black shadow which the wires throw upon it, in the form of well-defined lines, is seen on the plate. The sun is, of course, observed running along it from left to right, the true north limb being the upper one. In the figure the sun's centre is supposed to run along the line EW, passing north of the intersection-point of the wires, C. When the centre has arrived at a point S_1, such that the length of the perpendicular S_1A_1 is equal to the radius of the disk, a contact of the disk with the wire will take place, the instant of which is observed by a chronometer. There will be four such contacts, the times of which may be denoted by the letters A_1, B_1, A_2, B_2.

Neglecting the effects of hourly motion and of error of perpendicularity to each other of the wires, we have from the geometry of the figure, if we call R and δ respectively the sun's radius and north polar distance,

$$\frac{BC}{2AB}=\frac{S_1A_1}{S_1S_3}=\frac{R}{15\,.\sin\delta\,.\,(A_3-A_1)},$$

$$\frac{AC}{2\,.AB}=\frac{S_2B_1}{S_2S_4}=\frac{R}{15\,.\sin\delta\,.\,(B_3-B_1)},$$

whence

$$\frac{BC}{AC},\text{ or }\tan BAC=\cot\varphi=\frac{B_3-B_1}{A_3-A_1},\text{ or }\tan\varphi=\frac{A_3-A_1}{B_3-B_1}.$$

The observations for the determination of φ are repeated at intervals of a few weeks; and generally two sets of observations are taken, one on each side of the meridian. From the results and by interpolation the magnitude of φ is deduced for each sun-picture taken.

The following is an example of the observations:—

August 25th, 1864. Sun East of Meridian. Kew Mean Time.

	First Observation.			Second Observation.			Third Observation.		
	h	m	s	h	m	s	h	m	s
A_1	10	12	12·0	10	16	23·0	10	21	31·5
B_1		12	31·5		16	43·0		21	55·5
A_3		15	14·5		19	25·5		24	35·0
B_3		15	34·0		19	45·5		24	58·5
A_3-A_1			182·5			182·5			183·5
B_3-B_1			182·5			182·5			183·0

Mean $A_3-A_1=182{\cdot}83$ log $=2{\cdot}2620475$

$B_3-B_1=182{\cdot}67$ log $=2{\cdot}2616672$

$\varphi=45°\ 1'{\cdot}5$ log tan $=0{\cdot}0003803$

Sun West of Meridian.

	First Observation.			Second Observation.			Third Observation.			Fourth Observation.		
	h	m	s	h	m	s	h	m	s	h	m	s
A_1	2	58	19·0	3	2	17·5	3	6	16·5	3	10	22·0
B_1		58	31·5		2	34·0		6	32·5		10	36·5
A_3	3	1	20·5		5	19·0		9	19·0		13	24·5
B_3		1	32·5		5	35·0		9	34·0		13	38·0
A_3-A_1			181·5			181·5			182·5			182·5
B_3-B_1			181·0			181·0			181·5			181·5

Mean $A_3-A_1=182{\cdot}00$ log $=2{\cdot}2600714$

$B_3-B_1=181{\cdot}25$ log $=2{\cdot}2582780$

$\varphi=45°\ 7'{\cdot}1$ log tan $=0{\cdot}0017934$

7. It has been assumed in the preceding deduction that the line N D C, joining the sun's centre and north point, is perpendicular to the sun's path. This, however, is impossible on account of the change in the sun's declination. It is clear that such a line, drawn through the centre and perpendicular to the sun's path, will incline towards the west when the sun's north polar distance is increasing, and towards the east when it is decreasing. This deviation must therefore be applied as a correction.

If d be the sun's hourly increment of declination as given in the Nautical Almanack in seconds of arc, this correction will be (vide Mr. CARRINGTON'S Observations, page 11)

$$\iota = \frac{d}{15 \cdot \sin 8 \times 60 \times 60 \times \sin 1''}$$

with its proper sign.

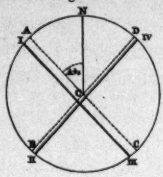
The following Table gives this correction in minutes, calculated for every fifth day of the year. The angle designated A + ι in our calculations represents the angle $\phi \pm \iota$.

Day.	ι.	Day.	ι.	Day.	ι.	Day.	ι.
	′		′		′		′
January 1.	+0·90	April 6.	+3·60	July 10.	−1·30	October 13.	−3·60
6.	+1·25	11.	+3·50	15.	−1·60	18.	−3·50
11.	+1·60	16.	+3·40	20.	−1·90	23.	−3·40
16.	+1·95	21.	+3·30	25.	−2·20	28.	−3·30
21.	+2·30	26.	+3·15	30.	−2·50	Nov. 2.	−3·10
26.	+2·55	May 1.	+3·00	August 4.	−2·70	7.	−2·90
31.	+2·80	6.	+2·80	9.	−2·90	12.	−2·70
February 5.	+3·05	11.	+2·60	14.	−3·05	17.	−2·50
10.	+3·30	16.	+2·35	19.	−3·20	22.	−2·20
15.	+3·40	21.	+2·10	24.	−3·35	27.	−1·90
20.	+3·50	26.	−1·80	29.	−3·50	Dec. 2.	−1·50
25.	+3·60	31.	+1·50	Sept. 3.	−3·55	7.	−1·10
March 2.	+3·70	June 5.	+1·15	8.	−3·60	12.	−0·70
7.	+3·75	10.	+0·80	13.	−3·65	17.	−0·30
12.	+3·80	15.	+0·45	18.	−3·70	22.	+0·10
17.	+3·80	20.	+0·10	23.	−3·70	27.	+0·50
22.	+3·80	25.	−0·25	28.	−3·70	January 1.	+0·90
27.	+3·75	30.	−0·60	October 3.	−3·70		
April 1.	+3·70	July 5.	−0·95	8.	−3·70		

8. The next operation is the rectification of the photograph by means of the wires of the heliograph depicted on it, and the determination of the error of perpendicularity of the wires. For this purpose it is necessary to determine the position-angles of the wires; for it is by means of these that we are enabled to find the angles of position of the lines joining the spots with the centre of the sun. This part of the measurement is very easily accomplished by rotating the larger divided circle of the measuring-instrument on its axis and bringing successively the ends of the wires, commencing with I., under the centre of the microscope, taking care that the image of the wire on the picture is exactly bisected by one of the wires of the microscope, while the other wire of the latter forms a tangent to the periphery of the sun at the middle of the indentation produced by the wire. Thus four readings are obtained for the position of the four wire-ends; and calling these readings in their proper order α, β, γ, δ, it will be easily seen from

fig. 4 that they give us the position-angles and the error of verticality with great precision. For let I., II., III., IV. represent the position of the wire-ends, and A, B, C, D the extremities of two lines, drawn parallel to the wires, but passing through the centre C of the sun-picture. If N represent the true north point of our picture, obviously the angle N C A is the angle $A+\iota$, which has been determined previously. But the position of the point A is given by the quantity

Fig. 4.



$$\frac{\alpha+\gamma}{2}-90^\circ=\alpha'=\text{reading corresponding to A.}$$

Similarly, the position of D by

$$\frac{\beta+\delta}{2}-90^\circ=\beta'=\text{reading corresponding to B;}$$

and obviously

$$\beta'-\alpha'=90^\circ,$$

if the imaginary lines through the centre, or the wires which are parallel to them, are perpendicular to each other. In general we shall find

$$\beta'-\alpha'=90^\circ\pm\theta;$$

and consequently the quantity $\mp\frac{\theta}{2}$ will have to be applied as a correction to $A+\iota$ on account of error of perpendicularity.

Finally, since the true reading of the point A, reckoning from the true north line, ought to be exactly

$$A+\iota\mp\frac{\theta}{2},$$

while it was found to be

$$\frac{\alpha+\gamma}{2}-90^\circ,$$

it is necessary to apply the quantity

$$\left(A+\iota\mp\frac{\theta}{2}\right)-\left(\frac{\alpha+\gamma}{2}-90^\circ\right)$$

with its proper sign, as a correction to every circle-reading, in order to deduce the true angle of position.

The following is an example of this part of the measurement of a picture, the numbers I., II., III., IV. corresponding to the readings α, β, γ, δ:—

May 26th, 1863.

I.	0·089	43°	20′·2	(α)	II.	0·096	134° 45′·3	(β)
III.	0·096	223	20·9	(γ)	IV.	0·091	312 19·6	(δ)
		2)266	41·1				2)447 4·9	
		133	20·5$_5$				223 32·4$_5$	

$$\frac{\alpha+\gamma}{2}-90^{\circ}= \quad 43^{\circ}\ 20'\cdot5_{s}=\alpha'$$

$$\frac{\beta+\delta}{2}-90^{\circ}=133\ 32'\cdot4_{s}=\beta'$$

$$\beta'-\alpha'= \quad 90\ 11'\cdot9$$

$$\theta= \quad +\ 11'\cdot9$$

$$\frac{\theta}{2}= \quad +\ 5'\cdot9_{s}$$

$$A+\iota=+45\ 1'\cdot8$$

$$-\frac{\theta}{2}=- \quad 5'\cdot9_{s}$$

$$-\left(\frac{\alpha+\gamma}{2}-90^{\circ}\right)=-43\ 20'\cdot5_{s}$$

Correction to readings $=+\ 1\ 35'\cdot3$

The readings on the circle for the wires I., II., III., IV. are, as seen in the above example, accompanied by the corresponding scale-readings of the sun's periphery at the wire-ends. They serve in the present instance chiefly as a control of the tangency of the microscope-wire, but will be of special use hereafter in an investigation on the sun's diameter, which we have in view for a future occasion.

9. The necessary corrections to the measurements being thus supplied, each group of spots as it appears on the photograph is then sketched, as represented in the accompanying form, which fills the next two pages, and is an exact representation of that used by us. The first page serves for entering the measurements necessary for determining the centre of the sun-picture, its radius, and the necessary corrections, derived by the preceding methods; while the second page contains the sketches of the groups, indicating by attached letters the different members of each group, the positions of which have been determined, and then gives for each of these members all necessary elements of calculation in five columns, viz. in

Column 1, the distance of a spot from the periphery, in terms of the scale.

Column 2, the distance of a spot from the centre, in terms of the scale (r).

Column 3, the distance of a spot from the centre, expressed as a fraction of the sun's radius $\left(\frac{r}{R}\right)$.

Column 4, the circle-reading corresponding to the position of the spot.

Column 5, the true angle of position, reckoned from north through east (east being left side of sun-picture)=P.

Even the smallest spots have in all cases been measured; and in larger spots, containing several nuclei, the position of each nucleus has been determined. The point chosen for measurement was always the geometrical centre of the figure of the spot or nucleus, as far as careful estimation could ascertain it. Proper care has also been taken to

bring the *centre* of the intersecting wires of the microscope to cover the point to be determined, so as to avoid any error arising from the thickness of the wires.

The following form, which gives the complete measurements and data for calculation of two pictures taken on the same day, hardly requires, after the preceding remarks, any further elucidation.

1862, August 25.

$10^h\ 37^m$

Vernier position 0°	0·093		Vernier position 90°	0·094	
	3·930	D=3·837		3·934	D=3·840
	C=2·012	R=1·918,		C=2·014	R=1·920

C=2·013

R=1·919

I. 0·099	38° 33′·5		II. 0·094	132° 17′·3	
III. 0·094	228 23·7		IV. 0·095	315 1·3	
	266 57·2			447 18·6	
	133 28·6, ∴ 43° 28′·6			223 39·3, ∴ 133° 39′·3	

$\theta = -10'{\cdot}7$

$A+\iota =$	$+45^\circ\ 35'{\cdot}5$
$\frac{1}{2}\theta =$	$-\quad 5{\cdot}3$
$\left(\frac{\alpha+\gamma}{2}-90^\circ\right)$ I. to III. =	$-43\ 28{\cdot}6$
	$+\ 2\ 1{\cdot}6$

$11^h\ 14^m$.

Vernier position 0°	0·095		Vernier position 90°	0·095	
	3·930	D=3·835		3·929	D=3·834
	C=2·013	R=1·917		C=2·012	R=1·917

C=2·012

R=1·917

I. 0·095	41° 51′·2		II. 0·102	132° 47′·2	
III. 0·095	224 53·3		IV. 0·097	314 14·7	
	266 44·5			447 1·9	
	133 22·2, ∴ 43° 22′·2			223 31·0, ∴ 133° 31′·0	

$\theta = -8'{\cdot}8$

$$A+\iota = +45^\circ\ 35'\cdot3$$
$$\tfrac{1}{2}\vartheta = -\quad\ 4'\cdot4$$
$$\left(\tfrac{\alpha+\gamma}{2}-90^\circ\right)\text{I. to III.} = -43'\ 22''\cdot2$$
$$+\ 2'\ 8''\cdot7$$

Fig. 5.

$10^h\ 37^m$.

	1.	2.	3.	4.	5.
P.	0·297	1·716	0·894	122° 7'·7	124° 9'·3
Q.	0·320	1·693	0·882	297 21·3	299 22·9
R.	1·764	0·249	0·130	353 54·0	355 55·6
S.	1·080	0·933	0·486	322 14·7	324 16·3
s.	1·129	0·884	0·461	330 37·5	332 39·1

$11^h\ 14^m$.

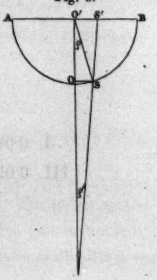
P.	0·308	1·704	0·889	121 56·5	124 5·2
Q.	0·319	1·693	0·883	297 15·3	299 24·0
R.	1·760	0·252	0·131	353 18·3	355 27·0
S.	1·077	0·935	0·488	322 5·8	324 14·5
s.	1·129	0·883	0·461	300 32·3	302 41·0

10. We pass now to the second part of the reduction of the measurements, viz. the calculation of the heliographical positions derived by knowing the observed distance of a spot from the centre of the sun and also its angle of position.

This part of the work may be conveniently divided into the following successive stages. As already mentioned, we have here entirely followed the method of Mr. CARRINGTON, who by introducing tabulated auxiliary values has condensed the two steps necessary for passing from the ecliptical longitude and latitude to the heliographical into one.

First, if r denote the measured distance of a spot from the centre, R the measured radius of the photogram, and (R) the tabular semi-diameter of the sun in minutes of arc; then, in order to express our measured distance in terms of the tabular radius, which required value we may call ϱ', we have the proportion

Fig. 6.



$$r : \varrho' :: R : (R)$$
$$\varrho' = \frac{r}{R}\cdot(R). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Now it will readily be seen from fig. 6 that O′S′ is larger than OS, or that the *measured* distance requires a correction to deduce from it the value $\frac{OS}{O'S}$ for finding the true *angular* distance of a spot from

the centre $=\varrho$. We have from the figure

$$\frac{r}{R}=\frac{O'S'}{O'B}=\frac{O'S'}{O'S}=\sin O'SS'=\sin(\varrho+\varrho'),$$

or

$$\varrho+\varrho'=\sin^{-1}\frac{r}{R};$$

hence

$$\varrho=\sin^{-1}\frac{r}{R}-\varrho'. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

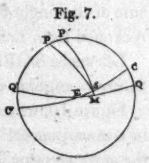
To facilitate the application of this correction the following Table has been calculated, which gives the logarithm of (R) for every tenth day of the year.

Values of log(R).

Jan. 1.	1·212	Mar. 22.	1·206	June 10.	1·198	Aug. 29.	1·201	Nov. 17.	1·210
11.	1·212	Apr. 1.	1·205	20.	1·198	Sept. 8.	1·202	27.	1·211
21.	1·212	11.	1·204	30.	1·198	18.	1·203	Dec. 7.	1·212
31.	1·211	21.	1·203	July 10.	1·198	28.	1·204	17.	1·212
Feb. 10.	1·211	May 1.	1·201	20.	1·198	Oct. 8.	1·206	27.	1·212
20.	1·210	11.	1·200	30.	1·199	18.	1·207		
Mar. 2.	1·209	21.	1·199	Aug. 9.	1·199	28.	1·208		
12.	1·207	31.	1·199	19.	1·200	Nov. 7.	1·209		

11. Next, let C'EC (fig. 7) represent the ecliptic, Q'EQ the celestial equator, and P, P' the poles of these circles. Then if S is the position of the sun, ES will be its longitude $=\odot$, and the angle SEM$=\varpi$, the inclination of the ecliptic.

Fig. 7.



Now, if the angle PSP'$=$G represents the inclination of two planes passing through the line joining the centres of the sun and earth, and the poles of the earth and ecliptic respectively, then in the triangle ESM, right-angled at M, we shall have

$$\cos ES=\cot SEM \cot ESM,$$

or

$$\cos\odot=\cot\varpi\tan G,$$

whence

$$\tan G=\tan\varpi\cos\odot. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

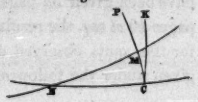
Similarly, if H be the inclination of two planes passing through the line joining the centres of the sun and earth and the poles of the sun and ecliptic respectively, we should obtain by a corresponding figure

$$\tan H=\tan I.\cos(\odot-N), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where I is the inclination of the sun's equator to the ecliptic, and N the longitude of the ascending node.

Further, let L be the heliographical longitude of the earth, and D its latitude, then it will easily be deduced from figure 8, in which NC is part of the ecliptic, NM the solar equator, N the ascending node, C the position of the earth, K the pole of the ecliptic, P the pole of the sun, that

Fig. 8.



$$L = N\,M,$$
$$D \text{ (negative)} = MC,$$

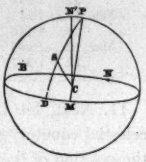
whence, since $M\,N\,C = I$, $N\,C = 180° + \odot - N$, $N\,M\,C = 90°$,

$$\tan L = \cos I \tan(\odot - N). \quad (5)$$
$$\sin D = \sin I \sin(\odot - N). \quad (6)$$

These four auxiliary angles, viz. G, H, L, D, have been calculated for every degree of the arguments, viz. $\odot$ for G, and $(\odot - N)$ for H, L, and D; and in these Tables, as mentioned in § 4, the elements I and N employed by us are those arrived at by Mr. CARRINGTON.

12. To pass now to the last step in the reduction, let P (fig. 9) be the sun's north pole, N′ the celestial pole, and N′C the meridian through the apparent centre of the sun; further, let N D M be the solar equator, N the position of the ascending node, and S that of a spot.

Fig. 9.



Then $N\,B\,D = l$ is the heliographical longitude of the spot reckoned along the solar equator from N. Also $NBDM = L$ = heliographic longitude of the earth.

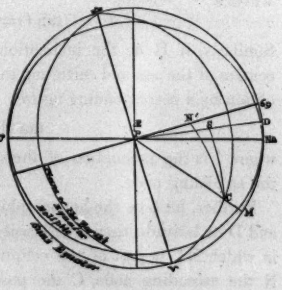
$D\,S = \lambda$ is the spot's heliographical latitude.

Now, in the triangle P S C the angle P C S is obviously the algebraical sum of the angle S C N′, or the angle of position of the spot S (=P), reckoned from north towards east, and the angle $N'\,C\,P = G + H$.

Let angle $S\,C\,P = S\,C\,N' + N'\,C\,P = P + G + H = \chi$, further we have $SC = \varrho$ = angular distance from centre, and $P\,C = 90° - D$.

Figure 10, which has been suggested to us by Sir JOHN HERSCHEL, places these facts in a more general and probably also clearer manner before the reader. In this figure the sun's equator is taken for a plane of projection, and the spherical triangles are referred to the centre of the sun. The letters are identical with those in fig. 9, while C in fig. 10, looked at from without by a spectator placed in the line of the earth's situation, corresponds to C in fig. 9, the centre of the visible disk.

Fig. 10.



The triangle P S C will give us, by well-known fundamental formulæ of spherical trigonometry,

$$\sin\lambda = \cos\varrho \sin D + \sin\varrho \cos D \cos\chi,$$
$$\sin(L - l) = \sin\chi \sin\varrho \sec\lambda,$$

from which the heliographical coordinates are at once obtained.

The following page exhibits the final reduction of all spots observed on August 25, 1862, the measurements of which, with their primary reductions, have been given on pages 10 & 11 above. On the pages 45 *et seq.* the results of the positional elements for the spots observed during 1862 and 1863 are given under the following heads:—

1. Year, month, and day.
2. Running number.
3. Mean time of sun-picture.
4. Number of group in the Kew Catalogue.
5. Distance from centre.
6. Angle of position.
7. Heliographical longitude from node.
8. True heliographical longitude.
9. Heliographical latitude.
10. Letter denoting the particular spot of the group.

The "heliographical longitude" is derived from the "longitude from node" in the following manner. Taking that great circle through the sun's poles, which at the epoch 1854·0 passed through the ascending node as first meridian, and assuming preliminarily 25·38 days as the sun's period of rotation, this prime meridian will obviously again coincide with the node at the periods

$$1854{\cdot}0+25^{d}{\cdot}38,\ 1854{\cdot}0+2(25^{d}{\cdot}38),\ 1854+n(25^{d}{\cdot}38).$$

A spot being on the sun at the time 1854·0, the calculated longitude of which was found to be 0°·0, would be on the prime meridian; but the calculated longitude of the same spot, a day afterwards, would be found to be $\frac{360^{\circ}}{25{\cdot}38}=14^{\circ}\,11'\,3''{\cdot}84$ (disregarding for our present purpose all modifying influences), this being its "longitude from node."

To reduce this longitude to the true heliographical, a small table is necessary, showing at a glance the civil dates of the times $1854{\cdot}0+25^{d}{\cdot}38+25^{d}{\cdot}38+\ldots\ldots$

For a sun-picture taken intermediately between two coincidences of the prime meridian with the nodal point, the time since the preceding coincidence is found by subtraction; and calling this time T, we have obviously

$$\text{longitude of prime meridian from node}=\frac{T\,.\,360^{\circ}}{25{\cdot}38}=l';$$

and if the longitude from node of a spot be $=l$, its angular distance from the prime meridian, reckoning always from west to east, will be

$$l-l',=\text{the "Heliographical Longitude."}$$

1862.	August 25 ($10^h\ 37^m$).	P.	August 25.	Q.	August 25.	R.	August 25.	S.	August 25.	s.	1862.
Log $\frac{r}{R}$.....	9·95145	124° 9′·3	9·94559	299° 22′·9	9·11313	355° 55′·6	9·68681	324° 16′·3	9·66338	332° 39′·1	P.
Log (R).....	1·201	−19 24·7	1·201	−19 24·7	1·201	−19 24·7	1·201	−19 24·7	1·201	−19 24·7	G+H.
Log ϱ'.....	1·152	104 44·6	1·147	279 58·2	0·314	336 30·9	0·888	304 51·6	0·864	313 14·4	χ.
Log cos ϱ ...	9·65446	63 24·6	9·67611	61 54·9	9·99635	7 27·3	9·94198	29 5·4	9·94868	27 25·8	$\varrho+\varrho'$.
Log sin D ...	+9·09111	14·2	+9·09111	14·0	+9·09111	2·1	+9·09111	7·7	+9·09111	7·3	ϱ'.
Log (1).....	+8·74557	63 10·4	+8·76722	61 40·9	+9·08746	7 25·2	+9·03309	28 57·7	+9·03979	27 18·5	ϱ.
Log cos χ ...	−9·40567	+9·98546	+9·23837	−9·99339	+9·96245	−9·60044	+9·75707	−9·91411	+9·83573	−9·86242	Log sin χ.
Log sin ϱ ...	9·95055	9·95055	9·94464	9·94464	9·11106	9·11106	9·68505	9·68505	9·66161	9·66161	Log sin ϱ.
Log cos D ...	9·99667	0·00634	9·99667	0·00977	9·99667	0·01287	9·99667	0·03436	9·99667	0·04247	Log sec λ.
Log (2).....	−9·35289	+9·94235	+9·17968	−9·94780	+9·07018	−8·72437	+9·43879	−9·63352	+9·49401	−9·56650	Log sin (L−l).
(1).....	+0·05566	257° 42′·3	+0·05851	257° 42′·3	+0·12231	257° 42′·3	+0·10792	257° 42′·3	+0·10960	257° 42′·3	L.
(2).....	−0·22537	+61° 7′·6	+0·15123	−62° 28′·0	+0·11754	−3° 2′·3	+0·27466	−25° 28′·3	+0·31189	−21° 37′·6	L−l.
Sum	−0·16971	196° 34′·7	+0·20974	320° 10′·3	+0·23985	260° 44′·6	+0·38258	283° 10′·6	+0·42149	279° 19′·9	l.
Log sin λ ...	−9·22971	− 9° 46′	+9·32168	+12° 6′	+9·37994	+ 13° 53′	+9·58272	+22° 30′	+9·62479	24° 56′	λ.

Measurement of the Areas of Sun-spots.

13. The second part of the results of our researches published in this paper, exhibits the amount of spotted surface for every day on which a solar photogram was taken, expressed as a fraction of the visible hemisphere. The information is given under two heads, viz. the area covered by the *penumbra*, *umbra*, and *whole* of the spots forming *one* group, and also the same values for all groups, taken together, which were visible on each day, thus enabling us to trace not only the behaviour, as regards increase or decrease, of every group, but also to express numerically the variations in the energy manifested by the sun's surface from time to time.

We had previously published the area-measurements of the whole of Mr. CARRINGTON's observations, and were induced to devote to his sketches the considerable labour which these measurements require, on account of the excellence and care with which they were executed. But if, as every one will probably admit, these area-measurements are a more scientific representation of the sun's activity than the mere numbering of the groups, it must also be conceded that solar photograms are a more trustworthy basis for such measurements than drawings, however carefully the sun's surface may be delineated.

We shall therefore continue these measurements from year to year, convinced that they will afford materials of very great scientific value.

14. The method adopted for measuring the areas is to place a plate of glass engraved with a series of small squares in contact with the spot to be measured, and to note independently how many squares and fractional parts of a square were occupied by the umbra, the penumbra, and the whole spot; if the number of squares occupied by the whole spot is equal to the sum of those occupied by the umbra and penumbra separately, then the observation is supposed to have been correctly made. If, in addition to the measurement, we know the angular distance of the spot from the visual centre, we have the elements for ascertaining the area occupied by the spot. This area we have expressed in millionths of the sun's hemispherical surface.

It ought perhaps to be noted that we could not place the engraved square absolutely in contact with the collodion side of the picture, for fear of destroying it; and it was therefore necessary to interpose the thickness of the glass plate betwixt the measuring-squares and the picture. To obviate an objection to the process which might arise from considering that the engraved lines were not precisely in contact with the object measured, an experiment was made which showed that this difference of distance did not occasion a perceptible error in the determination. The area of the square which forms the unit of measurement is $\frac{1}{10,000}$ of a square inch, the side of each square being $\frac{1}{100}$ inch.

To find the mean radius of the sun-pictures, we took for each month of the year two photograms distant from each other by about a fortnight, and found their respective radii to be of the following lengths, expressed in inches:—

January	1·958	May	1·904	September	1·920
	1·963		1·903		1·922
February	1·969	June	1·900	October	1·930
	1·959		1·897		1·943
March	1·956	July	1·900	November	1·949
	1·947		1·907		1·947
April	1·930	August	1·904	December	1·969
	1·927		1·911		1·967

The mean of these gave us 1·9326 as the mean radius, which we adopted in the reduction of the areas measured. The area of each square unit being ·0001 of a square inch, it follows that, if r be the radius of the disk in inches, the area of the visible hemispherical surface will be $2\pi r^2$, and in our pictures

$$=2\times\pi\times(1\cdot9326)^2=23\cdot46734;$$

therefore, at the *centre* of the visual disk, one of these small squares will represent a proportional area of the whole hemispherical surface, viz.

$$=\frac{\cdot0001}{23\cdot46734}=\frac{1}{234673\cdot4},$$

while at an angular distance α from the visual centre the proportion will be

$$\frac{\sec\alpha}{234673\cdot4}.$$

From this formula Table I. is calculated. It gives in the first column the angular distances, in the second the corresponding distances in terms of the radius as unity, and in the following nine columns the number of *millionths* of the sun's hemispherical surface covered in each position by 100, 200 &c. measured squares.

To obtain the required values for a number of squares between 10 and 99, and between 1 and 9, it is, of course, only necessary to cut off from the tabular values one or two decimals. An example will best illustrate the use of the Table.

Let the measurement of a group, situated at a distance from the centre of 0·483 (radius=1), show that the whole of it covered 963 squares; then, since 0·483 corresponds to an angular distance of 29° from the centre, we should find opposite to this distance

for the area of	900	squares		4383 millionths.
„ „	600	„	2922, ∴ for 60 . .	292·2 „
„ „	300	„	1461, ∴ for 3 . .	14·61 „
		consequently for 963 squares .		4690 millionths.

The use of a mean radius of the sun-pictures in the above formula will not sensibly affect the correctness of the yearly amount of spotted surface; for in our results the daily amount will be somewhat too small during the time of aphelion, and again slightly too great about the perihelion. But if for any special purposes the amount of spotted surface were to be determined with the greatest precision, the actual radius of each

picture would, of course, have to be employed in the reductions. The variation with the radius is as follows:—

With the smallest radius	= 1·897 in.,	the value of one square at the centre is			0·00000442,
„ mean	„ = 1·933	„	„	„	0·00000426,
„ greatest	„ = 1·969	„	„	„	0·00000411.

Distortion arising from optical deficiencies of the Photoheliograph.

15. The question how far the trustworthiness of solar photograms is limited by the effects of distortion, has caused us considerable anxiety. We have for several years past made various attempts to arrive at a satisfactory solution of this problem, and we have decided that our final discussion of the positional elements of sun-spots for the deduction of the sun's elements shall be preceded by an exhaustive investigation of the matter. At present we regard our researches and their results as only preliminary; but, on account of the importance of the subject for Celestial Photography generally, and in order to convince our readers that the subject is engaging our most earnest attention, we give in the following a short account of our latest steps in the matter.

16. It is obvious that if an image of some distant terrestrial object, which may be supposed to be of constant dimensions, is received through our photoheliograph on a prepared plate, we shall obtain a negative imprint of the object on a well-defined circular disk, which has been produced on the plate by the action of the whole diffused light which enters the object-glass. If the distant object be small, and placed in the axis of the instrument, it will also appear in the centre of the disk which has been produced, while if placed more or less out of the axis, it will also occupy a corresponding position on the photogram. Now in all positions its dimensions ought to be the same, for we supposed these dimensions to remain constant; hence it follows that if a relative change takes place in these dimensions, as determined by careful measurements of the same object in different positions, a distortion takes place in that part of the field where a change is observed.

This principle has been made use of in the following manner. The ornamental pinnacle of the Pagoda erected in the Kew Gardens, which is constructed of a number of rings of metal suspended in a horizontal position by vertical chains fastened to the top piece, was photographed by means of the photoheliograph in a series of twenty-five pictures, taken so as to bring its several parts into varying distances from the centre. The pictures were all taken within a few days, in which no considerable changes could possibly take place in the object itself, and any effect of expansion by changes in the temperature of the air would not sensibly affect the measurements. The distance of the Pagoda from the Observatory is 4398·24 feet*. The figure of the pinnacle, as it appears on the pictures, allows us to measure certain lines, produced photographically by its different

* We take this opportunity to express our great obligation to Colonel Sir Henry James of the Survey Office, who kindly presented us with a sketch of the relative positions of the Observatory and the Pagoda, from which the above numbers are taken.

parts, in directions which are perpendicular to one another. On Plate I., which accompanies this paper, all measurements of those lines which have the same direction are designated W W′, *i. e.* along a radius W W′, while those in a direction perpendicular to the former are designated "along the radius R R′."

All measurements were made with the greatest possible care by means of the Kathetometer of the Observatory, which reads to $\frac{1}{1000}$ of an inch.

17. Every line in the preceding figure being thus measured on the average about five times, so as to reduce errors as far as possible, the means of the measurements were taken; and of those lines for which a sufficient number of measurements existed, so as to trace their behaviour across the field with some degree of certainty, these measurements were represented in the form shown in Plate I. (A), where, however, only six lines are thus shown, although in reality for the behaviour in the direction W W′ ten distinct lines, and for that in the direction R R′ eight lines gave extensive data for our purpose, and therefore eighteen lines were finally employed for our mean results.

The mean radius of the disk produced by the whole light falling upon the plates was found to be 3·267 inches; and this length was subdivided into seven spaces, giving 0·267 inch to the central portion, and drawing concentric circles at distances of half an inch from each other; the central portion, having a radius of 0·267 inch, was considered as the centre; and, from the means of the measurements as above indicated, the behaviour of every line was determined at the centre and at each intersection of these concentric circles with the two radii W W′, R R′. These means are given in the following two Tables, which contain also the behaviour of each separate line, which we have called 1, 2, 3, &c., each of these numbers representing in fact a different object.

Behaviour of eighteen lines, photographed at different distances from the centre of the field in the Kew Photoheliograph.

A. Direction W W′.

Distance from centre, in inches.	Absolute length of each line in English inches at the corresponding distance.										
	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	Means.
+3·267	·1500	·1533	·1730	·1655	·1987	·2187	·2017	·1963	·2103	·2103	·1878
+3·000	·1494	·1526	·1708	·1647	·1984	·2156	·2009	·1964	·2100	·2100	·1869
+2·500	·1482	·1515	·1666	·1634	·1976	·2094	·1992	·1967	·2084	·2084	·1849
+2·000	·1469	·1508	·1627	·1623	·1969	·2027	·1985	·1971	·2071	·2071	·1831
+1·500	·1457	·1489	·1587	·1610	·1962	·1962	·1955	·1974	·2060	·2060	·1812
+1·000	·1446	·1479	·1547	·1598	·1955	·1896	·1938	·1978	·2046	·2046	·1793
+0·500	·1408	·1439	·1556	·1586	·1950	·1852	·1920	·1980	·2035	·2035	·1776
Centre.	·1370	·1400	·1586	·1575	·1952	·1859	·1955	·1980	·2023	·2017	·1772
−0·500	·1435	·1424	·1539	·1621	·1952	·1900	·1994	·1980	·2008	·1989	·1784
−1·000	·1446	·1447	·1529	·1623	·1943	·1894	·1936	·1900	·2050	·2058	·1783
−1·500	·1456	·1472	·1564	·1604	·1970	·1928	·1959	·1950	·2074	·2138	·1812
−2·000	·1467	·1495	·1598	·1594	·2000	·1942	·1980	·2010	·2097	·2139	·1832
−2·500	·1509	·1526	·1630	·1595	·2030	·1967	·2000	·2070	·2120	·2144	·1859
−3·000	·1581	·1566	·1644	·1664	·2060	·1998	·2020	·2128	·2145	·2145	·1895
−3·267	·1615	·1584	·1647	·1723	·2078	·2015	·2033	·2160	·2147	·2156	·1916

B. Direction R R′.

Distance from centre, in inches.	Absolute length of each line in English inches at the corresponding distance.								
	11.	12.	13.	14.	15.	16.	17.	18.	Means.
+2·500	·1800	·2008	·2216	·2410	·2630	·2890	·3020	·3250	·2528
+2·000	·1735	·1823	·2070	·2285	·2480	·2749	·2925	·3180	·2406
+1·500	·1651	·1838	·2026	·2264	·2440	·2688	·2868	·3104	·2348
+1·000	·1576	·1850	·2052	·2303	·2466	·2684	·2824	·3025	·2348
+0·500	·1660	·1866	·2078	·2249	·2465	·2633	·2774	·2990	·2339
Centre	·1672	·1812	·1998	·2194	·2395	·2584	·2767	·2976	·2300
−0·500	·1702	·1910	·2071	·2314	·2499	·2681	·2882	·3050	·2389
−1·000	·1695	·1922	·2114	·2333	·2520	·2729	·2928	·3112	·2419
−1·500	·1708	·1924	·2135	·2329	·2532	·2748	·2950	·3150	·2434
−2·000	·1760	·1946	·2142	·2351	·2595	·2771	·2970	·3177	·2464
−2·500	·1812	·1970	·2149	·2371	·2653	·2794	·2994	·3202	·2493

18. The mean results are graphically represented on Plate I. (B) in three curves, which have the distance from the centre for their abscissæ, while the mean length of the lines are the ordinates; both coordinates are given in inches. The curve W W′ shows the behaviour of all lines measured in one direction, the curve R R′ that of those measured in the direction at right angles to the former, and, finally, M M′ exhibits the mean of both. The irregularity of the curve R R′ is partly owing to the circumstance that the number of observations on which it is founded is considerably less than those at our disposal for the curve W W′; and it is our intention to repeat the whole investigation under more favourable circumstances. It is particularly our wish to place the determination on a more correct basis by erecting at a considerable distance from the photoheliograph a suitable scale of equal parts, which when repeatedly photographed in different positions, will in our opinion supply us with very correct constants of distortion. A similar contrivance would be useful in ascertaining the optical distortion of ordinary photographic lenses. In the meantime the following two facts may be regarded as established:—1st, that the image of any object photographically depicted is liable to a distortion, which varies at different distances from the centre of the field, and the amount of which may be determined for every instrument by methods similar to that employed by ourselves; 2nd, that in our case the image of an object is larger when formed near the edge of the field than at the centre, and that the amount of elongation of a unit of length at the centre increases with its distance from the centre. Taking the most trustworthy of our curves, W W′, as a preliminary result, we find the following values for the unit at the centre, when it passes to other positions in the field, the latter being given in fractions of the radius:—

Distance in parts of radius.	Corresponding length of central unit.
+1·000	1·06082
+0·918	1·05474
+0·765	1·04345
+0·612	1·03330
+0·459	1·02257
+0·306	1·01185

Distance in parts of radius.	Corresponding length of central unit.
+0·153	1·00226
±0·000	1·00000
−0·153	1·00677
−0·306	1·00621
−0·459	1·02257
−0·612	1·03386
−0·765	1·04910
−0·918	1·07041
−1·000	1·08126

19. The present state of this inquiry is not sufficiently far advanced to justify us in applying corrections to the calculated positions of sun-spots on account of the effect of distortion on the angular distances from the centre of the sun. The point will, however, have our careful attention when finally discussing the positions; and we hope to be able to give thoroughly satisfactory constants for the effect of displacement by our Photoheliograph.

With regard to the correctness of the calculated areas, it appears from the above that the side of our square unit, being ·01 of an inch at the centre, will in consequence of the elongation towards the limb be ·0106082 inch at the edge, and the areas measured there will obviously have to be diminished in the ratio of $(\cdot0106082)^2 : (\cdot01)^2$, and similarly at other distances, as seen in the following Table, which shows the numerical effect of the elongation on the values which we have assumed in our reduction for the square unit at different distances from the centre.

Distance from centre in terms of radius.	Proportion in which the areas ought to be diminished.	Assumed values for the square unit, without regard to elongation.	In millionths of sun's surface.	Corrected values, taking into account elongation.	In millionths of sun's surface.
+1·000	1·1125339 : 1	∞	∞	∞	∞
+0·918	1·1124764 : 1	·00001050	10	·00000944	9
+0·765	1·0887879 : 1	·00000663	7	·00000609	6
+0·612	1·0677089 : 1	·00000541	5	·00000507	5
+0·459	1·0454294 : 1	·00000478	5	·00000457	5
+0·306	1·0238404 : 1	·00000448	4	·00000437	4
+0·153	1·0045251 : 1	·00000431	4	·00000431	4
±0·000	1·0000000 : 1	·00000426	4	·00000426	4
−0·153	1·0135858 : 1	·00000431	4	·00000425	4
−0·306	1·0124586 : 1	·00000448	4	·00000442	4
−0·459	1·0454294 : 1	·00000478	5	·00000457	5
−0·612	1·0688665 : 1	·00000541	5	·00000506	5
−0·765	1·1006108 : 1	·00000663	7	·00000602	6
−0·918	1·1457776 : 1	·00001050	10	·00000916	9
−1·000	1·1691232 : 1	∞	∞	∞	∞

It will be seen from this Table that, our fractional values of the areas being expressed in millionths, the corrected values differ only from those employed by us in all spots which are very near the limb, and also in very large spots; in both these cases, however, the probable error of the measurements is not inconsiderable; and to introduce a correction on account of elongation was therefore considered by us a hardly necessary refinement in the calculations.

TABLE I.—Giving the areas in millionths of the Sun's visible hemisphere, corresponding to 100, 200, &c. measured unit-squares, for every degree, and also in terms of the radius=1.

One square at the centre of Sun=·00000426, the surface of the visible hemisphere being 1.

Distance from centre.		100.	200.	300.	400.	500.	600.	700.	800.	900.
In deg.	In terms of radius.									
0	0·000 to 0·009	426	852	1278	1704	2130	2556	2982	3408	3834
1	0·009 „ 0·026	426	852	1278	1704	2130	2556	2982	3408	3834
2	0·026 „ 0·044	426	852	1278	1704	2130	2556	2982	3408	3834
3	0·044 „ 0·061	427	854	1281	1708	2135	2562	2989	3416	3843
4	0·061 „ 0·078	427	854	1281	1708	2135	2562	2989	3416	3843
5	0·078 „ 0·096	428	856	1284	1712	2140	2568	2996	3424	3852
6	0·096 „ 0·113	428	856	1284	1712	2140	2568	2996	3424	3852
7	0·113 „ 0·130	429	858	1287	1716	2145	2574	3003	3432	3861
8	0·130 „ 0·148	430	860	1290	1720	2150	2580	3010	3440	3870
9	0·148 „ 0·165	431	862	1293	1724	2155	2586	3017	3448	3879
10	0·165 „ 0·182	433	866	1299	1732	2165	2598	3031	3464	3897
11	0·182 „ 0·199	434	868	1302	1736	2170	2604	3038	3472	3906
12	0·199 „ 0·216	436	872	1308	1744	2180	2616	3052	3488	3924
13	0·216 „ 0·233	437	874	1311	1748	2185	2622	3059	3496	3933
14	0·233 „ 0·250	439	878	1317	1756	2195	2634	3073	3512	3951
15	0·250 „ 0·267	441	882	1323	1764	2205	2646	3087	3528	3969
16	0·267 „ 0·284	443	886	1329	1772	2215	2658	3101	3544	3987
17	0·284 „ 0·301	446	892	1338	1784	2230	2676	3122	3568	4014
18	0·301 „ 0·317	448	896	1344	1792	2240	2688	3136	3584	4032
19	0·317 „ 0·334	451	902	1353	1804	2255	2706	3157	3608	4059
20	0·334 „ 0·350	453	906	1359	1812	2265	2718	3171	3624	4077
21	0·350 „ 0·367	456	912	1368	1824	2280	2736	3192	3648	4104
22	0·367 „ 0·383	460	920	1380	1840	2300	2760	3220	3680	4140
23	0·383 „ 0·399	463	926	1389	1852	2315	2778	3241	3704	4167
24	0·399 „ 0·415	466	932	1398	1864	2330	2796	3262	3728	4194
25	0·415 „ 0·431	470	940	1410	1880	2350	2820	3290	3760	4230
26	0·431 „ 0·446	474	948	1422	1896	2370	2844	3318	3792	4266
27	0·446 „ 0·462	478	956	1434	1912	2390	2868	3346	3824	4302
28	0·462 „ 0·477	483	966	1449	1932	2415	2898	3381	3864	4347
29	0·477 „ 0·492	487	974	1461	1948	2435	2922	3409	3896	4383
30	0·492 „ 0·508	492	984	1476	1968	2460	2952	3444	3936	4428
31	0·508 „ 0·522	497	994	1491	1988	2485	2982	3479	3976	4473
32	0·522 „ 0·537	502	1004	1506	2008	2510	3012	3514	4016	4518
33	0·537 „ 0·552	508	1016	1524	2032	2540	3048	3556	4064	4572
34	0·552 „ 0·566	514	1028	1542	2056	2570	3084	3598	4112	4626
35	0·566 „ 0·581	520	1040	1560	2080	2600	3120	3640	4160	4680
36	0·581 „ 0·595	527	1054	1581	2108	2635	3162	3689	4216	4743
37	0·595 „ 0·609	534	1068	1602	2136	2670	3204	3738	4272	4806
38	0·609 „ 0·623	541	1082	1623	2164	2705	3246	3787	4328	4869
39	0·623 „ 0·636	548	1096	1644	2192	2740	3288	3836	4384	4932
40	0·636 „ 0·649	556	1112	1668	2224	2780	3336	3892	4448	5004
41	0·649 „ 0·663	565	1130	1695	2260	2825	3390	3955	4520	5085
42	0·663 „ 0·676	573	1146	1719	2292	2865	3438	4011	4584	5157
43	0·676 „ 0·688	583	1166	1749	2332	2915	3498	4081	4664	5247
44	0·688 „ 0·701	592	1184	1776	2368	2960	3552	4144	4736	5328
45	0·701 „ 0·713	603	1206	1809	2412	3015	3618	4221	4824	5427
46	0·713 „ 0·725	613	1226	1839	2452	3065	3678	4291	4904	5517
47	0·725 „ 0·737	625	1250	1875	2500	3125	3750	4375	5000	5625
48	0·737 „ 0·749	637	1274	1911	2548	3185	3822	4459	5096	5733
49	0·749 „ 0·760	650	1300	1950	2600	3250	3900	4550	5200	5850
50	0·760 „ 0·772	663	1326	1989	2652	3315	3978	4641	5304	5967
51	0·772 „ 0·783	677	1354	2031	2708	3385	4062	4739	5416	6093

TABLE I. (continued).

Distance from centre.		100.	200.	300.	400.	500.	600.	700.	800.	900.
In deg.	In terms of radius.									
52	0·783 to 0·793	692	1384	2076	2768	3460	4152	4844	5536	6228
53	0·793 „ 0·804	708	1416	2124	2832	3540	4248	4956	5644	6372
54	0·804 „ 0·814	725	1450	2175	2900	3625	4350	5075	5800	6525
55	0·814 „ 0·824	743	1486	2229	2972	3715	4458	5201	5944	6687
56	0·824 „ 0·834	762	1524	2286	3048	3810	4572	5334	6096	6858
57	0·834 „ 0·843	782	1564	2346	3128	3910	4692	5474	6256	7038
58	0·843 „ 0·853	804	1608	2412	3216	4020	4824	5628	6432	7236
59	0·853 „ 0·862	827	1654	2481	3308	4135	4962	5789	6616	7443
60	0·862 „ 0·870	852	1704	2556	3408	4260	5112	5964	6816	7668
61	0·870 „ 0·879	879	1758	2637	3516	4395	5274	6153	7032	7911
62	0·879 „ 0·887	908	1816	2724	3632	4540	5448	6356	7264	8172
63	0·887 „ 0·895	939	1878	2817	3756	4695	5634	6573	7512	8451
64	0·895 „ 0·903	972	1944	2916	3888	4860	5832	6804	7776	8748
65	0·903 „ 0·910	1008	2016	3024	4032	5040	6048	7056	8064	9072
66	0·910 „ 0·917	1048	2096	3144	4192	5240	6288	7336	8384	9432
67	0·917 „ 0·924	1091	2182	3273	4364	5455	6546	7637	8728	9819
68	0·924 „ 0·930	1138	2276	3414	4452	5690	6828	7966	8904	10242
69	0·930 „ 0·937	1189	2378	3567	4756	5945	7134	8323	9512	10701
70	0·937 „ 0·943	1246	2492	3738	4984	6230	7476	8722	9968	11214
71	0·943 „ 0·948	1309	2618	3927	5236	6545	7854	9163	10472	11781
72	0·948 „ 0·954	1379	2758	4137	5516	6895	8274	9653	11032	12411
73	0·954 „ 0·959	1457	2914	4371	5828	7285	8742	10199	11656	13113
74	0·959 „ 0·964	1546	3092	4638	6184	7730	9276	10822	12368	13914
75	0·964 „ 0·968	1646	3292	4938	6584	8230	9876	11522	13168	14814
76	0·968 „ 0·972	1761	3522	5283	7144	8805	10566	12327	14288	15849
77	0·972 „ 0·976	1894	3788	5682	7576	9470	11364	13258	15152	17046
78	0·976 „ 0·980	2050	4100	6150	8200	10250	12300	14350	16400	18450
79	0·980 „ 0·983	2233	4466	6699	8932	11165	13398	15631	17864	20097
80	0·983 „ 0·986	2454	4908	7362	9816	12270	14724	17178	19632	22086
81	0·986 „ 0·989	2724	5448	8172	10896	13620	16344	19068	21792	24516
82	0·989 „ 0·991	3062	6124	9186	12248	15310	18372	21434	24496	27558
83	0·991 „ 0·994	3497	6994	10491	13988	17485	20982	24479	27976	31473
84	0·994 „ 0·995	4077	8154	12231	16308	20385	24462	28539	32616	36693
85	0·995 „ 0·997	4889	9778	14667	19556	24445	29334	34223	39112	44001
86	0·997 „ 0·998	6109	12218	18327	24436	30545	36654	42763	48872	54981
87	0·998 „ 0·999	8142	16284	24426	32568	40710	48852	56994	65136	73278
88	0·999 „ 0·999	12210	24420	36630	48840	61050	73260	85470	97680	109890
89	0·999 „ 1·000	24416	48832	73248	97664	122080	146496	170912	195328	219744

TABLE II.—Showing the areas of all Sun-spots observed at the Kew Observatory from February 7th, 1862 to December 31st, 1863.

The areas are expressed in millionths of the Sun's visible hemisphere.

Date.	Group.	Mean distance from centre, radius=1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1862.								
Feb. 7.	226	0·850	64	16	80			
	227	0·793	35	0	35			
	228	0·705	78	12	90			
	229	0·525	25	5	30			
	230	0·524	30	10	40			
	231	0·345	18	0	18			
	232	0·890	469	38	507			
	233	0·905	221	0	221	940	81	1021
8.	226	0·934	71	0	71			
	227	0·880	9	0	9			
	228	0·825	60	0	60			
	229	0·655	34	23	56			
	230	0·634	38	0	38			
	232	0·770	232	33	265			
	233	0·865	290	17	307	734	73	806
10.	229	0·911	147	31	178			
	230	0·810	72	29	101			
	232	0·485	513	100	613			
	233	0·586	208	82	290			
	234	0·949	345	55	400			
	235	0·775	15	12	27			
		0·775	34	20	54	1334	329	1663
16.	232	0·902	486	165	651			
	233	0·772	169	0	169			
	234	0·288	71	18	89			
	236	0·263	26	0	26			
	237	0·474	53	0	53			
	238	0·747	274	0	274			
	239	0·234	53	0	53	1132	183	1315
20.	234	0·902	282	29	311			
	239	0·857	45	21	66			
	240	0·910	178	42	220			
	241	0·304	18	0	18	523	92	615
21.	234	0·971	319	53	372			
	239	0·936	71	24	95			
	240	0·807	174	36	210			
	241	0·710	0	6	6			
	242	0·690	106	36	142	670	155	825
28.	240	0·686	129	41	170			
	242	0·852	330	297	627			
	243	0·652	175	119	294			
	244	0·252	159	0	159			
	245	0·345	154	68	222			
	247	0·730	119	94	212			
	248	0·947	131	65	196	1197	684	1880
Mar. 1.	240	0·822	119	52	171			
	242	0·940	324	349	673			
	243	0·537	234	61	295			
	244	0·062	94	34	128			
	245	0·160	181	82	263			
	247	0·619	86	27	114			
	248	0·870	88	44	132	1126	649	1776

TABLE II. (continued).

Date.	Group.	Mean distance from centre, radius = 1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1862.								
Mar. 3.	243	0·463	169	53	222			
	244	0·458	91	14	105			
	245	0·309	92	34	125			
	247	0·480	122	24	146			
	248	0·555	57	10	67			
	249	0·720	37	18	55	568	153	720
4.	243	0·541	229	61	290			
	244	0·627	74	25	99			
	245	0·489	34	34	68			
	247	0·534	15	10	25			
	248	0·408	23	19	42			
	249	0·554	41	15	56	416	164	580
5.	243	0·673	183	34	218			
	244	0·793	85	21	106			
	245	0·679	35	12	47			
	248	0·286	37	0	37			
	249	0·360	55	0	55			
	250	0·982	0	290	290	395	357	753
7.	243	0·967	296	82	379			
	248	0·230	17	0	17			
	250	0·706	277	48	326			
	251	0·941	125	0	125	715	130	847
19.	252	0·739	108	25	134			
	253	0·521	57	0	57			
	254	0·832	34	11	46	199	36	237
24.	252	0·297	62	18	80	62	18	80
27.	252	0·831	84	0	84	84	0	84
31.	255	0·705	175	30	205			
	256	0·921	251	76	327	426	106	532
April 3.	255	0·204	50	0	50			
		0·180	52	0	52			
	256	0·546	239	51	290			
	257	0·150	420	101	521			
	258	0·440	47	0	47			
	259	0·894	192	42	235	1000	194	1195
4.	255	0·456	201	14	215			
	256	0·403	224	74	298			
	257	0·175	390	138	528			
	258	0·278	115	27	142			
	259	0·777	162	27	190	1092	280	1373
12.	259	0·874	79	53	132			
	260	0·944	65	0	65	144	53	197
13.	259	0·960	93	62	155			
	260	0·774	20	14	34			
	261	0·985	466	0	466			
	262	0·443	62	0	62	641	76	717
14.	260	0·710	24	0	24			
	261	0·934	297	131	428			
	262	0·615	38	16	54			
	263	0·204	17	0	17	376	147	523
15.	261	0·777	291	115	406			
	263	0·369	5	0	5			
	264	0·869	26	8	34			
	265	0·960	402	0	402	724	123	847
17.	261	0·485	312	78	390			
	263	0·960	108	0	108			

TABLE II. (continued).

Date.	Group.	Mean distance from centre, radius = 1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1862.								
Apr. 17.	264	0·621	5	16	21			
	265	0·790	256	62	318	681	156	837
20.	261	0·195	282	52	334			
	265	0·285	111	27	138			
	266	0·510	50	45	95	443	124	567
21.	261	0·388	278	55	333			
	265	0·166	26	4	30			
	266	0·380	64	50	115			
	267	0·665	29	29	57	397	138	535
22.	261	0·595	283	48	331			
	265	0·248	22	18	40			
	266	0·209	57	17	74			
	267	0·460	38	0	38	400	83	483
23.	261	0·755	253	65	318			
	265	0·439	43	19	62			
	266	0·273	31	9	40			
	267	0·410	51	0	51			
	268	0·930	0	12	12	378	105	483
24.	261	0·892	244	66	310			
	265	0·665	52	0	52			
	266	0·458	57	0	57			
	267	0·185	113	35	148			
		0·220	78	44	122	544	145	689
25.	261	0·976	246	61	307			
	265	0·776	54	0	54			
	266	0·638	33	6	39			
	267	0·080	265	55	321			
	269	0·962	185	0	185	783	122	906
26.	267	0·310	327	45	372			
	269	0·850	249	96	346	576	141	718
27.	267	0·710	349	36	386			
	269	0·505	302	89	391			
	270	0·955	131	29	160			
	271	0·471	63	0	63	845	154	1000
28.	267	0·690	308	56	364			
	269	0·540	305	81	386			
	270	0·877	92	48	140			
	271	0·102	26	9	34	731	194	924
29.	267	0·850	145	24	169			
	269	0·385	315	97	414			
	270	0·754	32	32	65			
	271	0·160	52	43	95			
	272	0·873	40	31	70	584	227	813
30.	267	0·955	102	44	146			
	269	0·185	239	65	304			
	270	0·587	100	26	126			
	271	0·390	21	35	56			
	272	0·958	131	73	204			
	273	0·928	80	23	102	673	266	938
May 1.	269	0·164	685	194	879			
	270	0·409	84	37	121			
	271	0·575	36	10	47			
	273	0·965	181	49	230			
	274	0·984	98	270	368	1084	560	1645
2.	269	0·350	935	283	1217			
		0·314	94	40	134			

Table II. (continued).

Date.	Group.	Mean distance from the centre, radius = 1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1862.								
May 2.	270	0·311	85	45	130			
	273	0·698	41	12	53			
	274	0·915	136	21	157	1291	401	1691
5.	269	0·860	1174	314	1489			
	270	0·602	43	27	69			
	273	0·303	45	18	63			
	274	0·397	56	37	92			
	275	0·960	371	278	649	1689	674	2362
8.	273	0·673	46	23	69			
	274	0·682	17	35	52			
	275	0·562	308	103	411			
	276	0·654	90	45	136			
	277	0·941	87	62	149	548	268	817
9.	273	0·813	51	36	87			
	274	0·775	74	8	81			
	275	0·396	329	116	444			
	276	0·476	77	58	135			
	277	0·844	105	48	153	636	266	900
10.	273	0·930	59	36	95			
	274	0·910	42	0	42			
	275	0·262	357	101	459			
	276	0·264	40	22	62			
	277	0·683	76	35	111			
	278	0·928	50	40	89	624	234	858
12.	275	0·410	42	9	51			
	276	0·375	0	14	14			
	277	0·090	317	103	419			
	278	0·603	75	37	112			
	279	0·870	62	53	114			
	281	0·705	145	36	181	641	252	891
13.	275	0·615	5	5	11			
	276	0·525	75	0	75			
	277	0·238	263	88	351			
	278	0·243	44	26	70			
	279	0·662	130	56	186			
	281	0·524	60	0	60	577	175	753
16.	275	0·920	273	425	698			
	276	0·905	50	40	91			
	277	0·462	29	14	43			
	279	0·592	37	16	53			
	278	0·118	154	39	193			
	281	0·256	44	26	71			
	282	0·896	87	49	136			
	283	0·333	9	14	23	683	623	1306
17.	275	0·986	0	712	712			
	276	0·763	20	13	33			
	277	0·608	16	32	48			
	278	0·223	118	57	175			
	279	0·567	42	21	62			
	281	0·148	65	47	112			
	282	0·807	72	22	94			
	284	0·522	30	0	30	363	904	1266
18.	277	0·781	20	20	41			
	278	0·425	132	66	197			
	279	0·414	121	0	121			
	281	0·275	84	27	111			

TABLE II. (continued).

Date.	Group.	Mean distance from centre, radius=1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1862.								
May 18.	282	0·644	50	28	78	407	141	548
19.	278	0·607	160	53	214			
	279	0·210	65	48	113			
	281	0·460	14	10	24			
	282	0·462	77	19	97	316	130	448
22.	278	0·974	265	284	549			
	279	0·527	40	45	85			
	281	0·912	10	31	42			
	282	0·294	58	22	80			
	285	0·933	226	250	476	599	632	1232
24.	279	0·880	0	18	18			
	282	0·677	47	23	70			
	285	0·670	413	246	659			
	286	0·829	91	69	160			
	287	0·957	146	117	262			
	289	0·902	10	0	10	707	473	1179
26.	285	0·262	388	123	512			
	286	0·497	25	20	44			
	287	0·704	338	157	494			
	288	0·892	28	9	38			
	289	0·542	15	10	25	794	319	1113
29.	285	0·425	183	61	244			
	286	0·326	27	14	41			
	287	0·389	171	28	199			
	288	0·389	218	83	301			
	289	0·936	452	143	594	1051	329	1379
June 1.	286	0·943	75	12	87			
	287	0·654	62	17	79			
	288	0·340	353	91	444			
	289	0·488	326	73	399			
	290	0·445	52	38	90			
	291	0·572	26	5	31			
	292	0·755	474	110	585			
	293	0·957	772	248	1020	2140	594	2735
2.	287	0·775	88	47	135			
	288	0·525	311	95	407			
	289	0·303	331	76	408			
	290	0·617	189	76	265			
	291	0·370	18	0	18			
	292	0·625	575	131	707			
	293	0·875	624	255	879	2136	680	2819
3.	287	0·926	158	89	250			
	288	0·771	239	80	318			
	289	0·104	312	77	389			
	290	0·794	255	99	354			
	291	0·090	73	0	73			
	292	0·370	630	166	796			
		0·375	37	9	46			
	293	0·715	570	190	760			
	294	0·057	60	17	77	2334	727	3063
4.	287	0·971	106	106	211			
	288	0·863	264	60	324			
	289	0·204	183	61	244			
	290	0·891	169	122	291			
	291	0·155	43	4	47			
	292	0·260	547	119	666			

TABLE II. (continued).

Date.	Group.	Mean distance from centre, radius=1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1862.								
June 4.	293	0·595	780	211	991			
	294	0·187	52	13	65			
	295	0·850	24	16	40	2068	712	2879
7.	289	0·770	199	60	259			
	292	0·537	442	66	508			
	293	0·185	642	187	829			
	294	0·776	115	135	250			
	295	0·489	112	29	141			
	296	0·296	18	18	37	1528	495	2024
8.	289	0·902	224	58	282			
	292	0·700	574	124	699			
	293	0·328	681	203	884			
	294	0·911	105	147	251			
	295	0·332	185	45	230			
	297	0·321	68	36	104	1837	613	2450
9.	289	0·982	223	201	424			
	292	0·866	417	136	554			
	293	0·550	620	122	742			
	295	0·333	207	54	262			
	297	0·650	56	56	113	1523	569	2095
10.	292	0·942	299	100	399			
	293	0·694	574	154	728			
	295	0·445	218	43	261			
	297	0·712	115	42	157			
	298	0·293	22	27	49			
	299	0·539	25	15	41	1253	381	1635
11.	292	0·990	0	245	245			
	293	0·836	649	524	1173			
	295	0·642	161	50	211			
	297	0·884	200	18	218			
	298	0·472	58	29	87			
	299	0·349	5	14	18			
	300	0·367	2	2	4			
	301	0·970	176	229	404			
	302	0·438	57	14	71	1308	1125	2431
13.	295	0·911	31	31	63			
	298	0·812	7	22	29			
	300	0·240	9	4	13			
	301	0·831	831	229	1059			
	302	0·432	33	33	66			
	303	0·896	253	87	339	1164	406	1569
14.	295	0·979	20	41	61			
	300	0·424	14	0	14			
	301	0·725	1031	212	1244			
	302	0·633	22	22	44			
	303	0·779	332	68	399			
	304	0·980	205	123	328	1624	466	2090
15.	300	0·623	22	0	22			
	301	0·583	1302	285	1586			
	302	0·803	21	21	42			
	303	0·611	416	81	498			
	304	0·888	141	56	197			
	305	0·952	151	83	234	2053	526	2579
16.	301	0·463	1377	382	1758			
	303	0·430	240	85	324			
	304	0·748	147	57	204			

TABLE II. (continued).

Date.	Group.	Mean distance from centre, radius = 1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1862.								
June 16.	305	0·858	99	41	140	1863	565	2426
17.	301	0·405	1313	266	1580			
	303	0·263	181	48	229			
	304	0·588	174	63	237			
	305	0·736	37	31	69			
	306	0·811	0	7	7	1705	415	2122
18.	301	0·481	1389	292	1622			
	303	0·208	52	26	78			
	304	0·392	153	65	218			
	305	0·464	24	14	39			
	307	0·546	20	15	36			
	308	0·789	21	42	62	1599	454	2055
23.	301	0·967	362	132	494			
	303	0·773	34	14	47			
	304	0·645	33	44	78			
	305	0·329	18	9	27			
	309	0·525	50	30	80			
	310	0·659	203	62	265			
	311	0·721	123	43	165			
	311	0·638	22	22	44			
	312	0·720	31	18	49			
	313	0·664	11	23	34			
	314	0·705	314	90	404			
	315	0·795	255	113	368			
	316	0·871	369	70	439			
	317	0·927	80	45	125	1905	715	2619
24.	311	0·884	45	36	82			
	312	0·543	61	5	66			
	313	0·414	28	14	42			
	314	0·500	261	89	349			
	315	0·531	351	115	467			
	316	0·697	444	124	568			
	317	0·801	99	35	134	1289	418	1708
25.	304	0·892	28	9	38			
	305	0·741	64	25	89			
	309	0·878	35	18	53			
	310	0·938	274	75	349			
	311	0·986	147	123	270			
	312	0·353	18	9	27			
	313	0·243	26	18	44			
	314	0·263	242	75	317			
	315	0·404	391	93	485			
	316	0·526	582	136	718			
	317	0·665	132	46	178			
	318	0·909	101	71	172	2040	698	2740
26.	305	0·840	39	31	70			
	309	0·944	39	0	39			
	310	0·981	112	156	268			
	314	0·128	159	51	210			
	315	0·264	419	44	463			
	316	0·386	565	97	662			
	317	0·516	84	50	134			
	318	0·826	69	46	114	1486	475	1960
27.	305	0·961	46	77	124			
	314	0·192	161	52	213			
	315	0·115	510	60	570			

TABLE II. (continued).

Date.	Group.	Mean distance from centre, radius=1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1862.								
June 27.	316	0·148	572	95	666			
	317	0·302	31	13	45			
	318	0·637	100	22	122			
	319	0·872	26	44	70			
	320	0·979	102	164	266	1548	527	2076
28.	314	0·369	161	46	208			
	315	0·247	601	79	680			
	316	0·154	539	103	642			
	317	0·171	9	4	13			
	318	0·476	82	34	116			
	319	0·763	40	13	53			
	320	0·921	164	76	240	1596	355	1952
30.	314	0·735	100	50	150			
	315	0·622	671	108	779			
	316	0·523	532	110	642			
	318	0·156	65	52	116			
	320	0·668	241	74	315	1609	394	2002
July 1.	314	0·895	58	97	155			
	315	0·810	457	116	573			
	316	0·737	503	51	554			
	318	0·300	62	27	89			
	320	0·439	256	66	322	1336	357	1693
5.	318	0·927	45	34	80			
	320	0·462	333	87	420			
	321	0·147	30	34	64			
	322	0·972	106	194	299			
	323	0·431	14	14	28			
	324	0·736	50	19	69	578	382	960
6.	320	0·628	269	44	312			
	321	0·348	41	14	54			
	322	0·900	68	29	97			
	323	0·565	82	31	113			
	324	0·786	111	187	297	571	305	873
8.	320	0·914	210	178	388			
	321	0·841	78	86	164			
	322	0·612	81	27	108			
	323	0·912	73	63	136			
	324	0·939	0	87	87			
	325	0·900	214	39	253			
	326	0·859	165	74	239			
	327	0·760	45	0	45	866	554	1420
11.	322	0·307	152	107	260			
	325	0·443	204	81	284			
	326	0·301	85	49	133			
	327	0·323	32	32	63			
	328	0·922	65	55	120			
	329	0·585	84	26	111			
	331	0·977	76	56	131	698	406	1102
13.	322	0·599	182	27	208			
	325	0·258	123	48	172			
	326	0·178	48	17	65			
	327	0·381	60	0	60			
	329	0·344	27	23	50			
	330	0·680	64	29	93			
	331	0·793	111	35	145	615	179	793
14.	322	0·772	217	34	250			

TABLE II. (continued).

Date.	Group.	Mean distance from centre, radius=1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1862.								
July 14.	325	0·378	143	55	198			
	326	0·394	28	9	37			
	327	0·579	187	73	260			
	330	0·503	34	15	49			
	331	0·634	49	22	71	658	208	865
15.	322	0·900	136	39	175			
	326	0·585	21	11	32			
	327	0·735	431	87	519			
	325	0·544	132	56	188			
	330	0·330	18	18	36			
	331	0·443	47	28	76	785	239	1026
16.	322	0·980	89	223	313			
	325	0·707	96	42	139			
	326	0·712	121	24	145			
	327	0·871	431	149	580			
	330	0·217	26	13	39			
	331	0·227	65	22	87			
	332	0·308	22	13	36			
	333	0·365	32	0	32			
	334	0·397	19	9	28	901	495	1399
17.	325	0·844	72	24	96			
	326	0·871	132	132	263			
	327	0·962	448	170	618			
	330	0·300	0	18	18			
	331	0·064	26	0	26			
	332	0·250	9	4	13			
	333	0·272	31	4	35			
	334	0·628	77	93	170			
	335	0·362	18	41	59			
18.	325	0·940	75	25	100			
	326	0·959	102	29	131			
	330	0·465	10	10	19			
	331	0·238	48	18	66			
	332	0·377	18	23	41			
	333	0·320	77	41	117			
	334	0·790	270	103	373			
	335	0·543	249	81	330			
	336	0·965	33	132	165			
	337	0·925	0	397	397	882	859	1739
20.	330	0·738	178	51	229			
	331	0·657	40	11	51			
	332	0·649	68	119	186			
	334	0·989	0	163	163			
	335	0·854	107	132	240			
	336	0·747	70	32	102			
	337	0·655	260	79	339			
	338	0·945	209	0	209	932	587	1519
21.	330	0·868	111	77	187			
	331	0·811	0	29	29			
	332	0·791	55	42	97			
	335	0·947	0	79	79			
	336	0·592	258	32	290			
	337	0·496	207	69	276			
	338	0·982	0	134	134	631	462	1092
25.	336	0·331	81	49	131			
	337	0·452	163	43	206			

TABLE II. (continued).

Date.	Group.	Mean distance from centre, radius=1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1862.								
July 25.	340	0·722	809	172	981			
	341	0·524	40	0	40			
	342	0·823	728	238	966	1821	502	2324
26.	336	0·532	30	30	60			
	337	0·644	67	22	89			
	340	0·575	853	172	1024			
	341	0·360	14	5	18			
	342	0·693	959	142	1101	1923	371	2292
27.	336	0·710	48	36	84			
	337	0·801	71	14	85			
	340	0·386	574	139	713			
	342	0·525	707	110	818			
	343	0·923	284	44	327			
	345	0·969	352	143	495	2036	486	2522
28.	336	0·905	40	181	222			
	337	0·923	55	44	98			
	340	0·201	789	209	998			
	342	0·328	1037	234	1272			
	343	0·774	427	88	514			
	344	0·646	67	50	117			
	345	0·862	392	281	673			
	346	0·972	0	625	625	2807	1712	4519
29.	337	0·976	0	189	189			
	340	0·200	767	218	985			
	342	0·196	998	200	1198			
	343	0·626	422	88	510			
	344	0·775	54	61	115			
	345	0·756	448	253	702			
	346	0·898	243	204	447	2932	1213	4146
30.	340	0·378	819	230	1049			
	342	0·240	900	189	1089			
	343	0·462	328	150	478			
	344	0·893	0	28	28			
	345	0·603	507	182	689			
	346	0·771	292	80	371			
	347	0·949	14	28	41	2860	887	3745
31.	340	0·564	745	226	971			
	342	0·400	913	172	1086			
	343	0·326	230	86	316			
	345	0·445	664	86	749			
	346	0·614	130	54	184			
	347	0·850	16	16	32			
	348	0·985	368	172	540			
	349	0·571	83	31	114	3149	843	3992
Aug. 1.	340	0·721	999	202	1201			
	342	0·575	853	213	1066			
	343	0·319	266	140	406			
	345	0·344	381	127	507			
	346	0·453	234	81	315			
	347	0·714	6	18	24			
	348	0·927	57	102	158			
	349	0·730	25	19	44	2821	902	3721
2.	340	0·865	886	460	1346			
	342	0·760	822	172	994			
	343	0·444	337	104	441			
	345	0·362	342	132	474			

TABLE II. (continued).

Date.	Group.	Mean distance from centre, radius = 1.	Area of Penumbra.	Area of Umbra.	Area of whole spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1862.								
Aug. 2.	346	0·302	202	72	273			
	347	0·539	10	5	15			
	348	0·817	22	30	52			
	350	0·625	38	11	49			
	351	0·970	71	158	229			
	352	0·490	29	0	29	2759	1144	3902
3.	340	0·952	924	689	1613			
	342	0·884	518	245	763			
	343	0·592	390	95	485			
	345	0·481	224	49	273			
	346	0·284	205	71	276			
	347	0·376	5	9	14			
	348	0·668	29	17	46			
	350	0·453	100	19	119			
	351	0·901	97	58	156			
	352	0·364	23	27	50			
	353	0·936	131	95	226			
	354	0·969	0	282	282	2646	1656	4303
4.	340	0·995	0	1264	1264			
	342	0·969	282	370	652			
	343	0·774	305	68	372			
	345	0·639	128	44	172			
	346	0·408	126	60	186			
	348	0·428	28	19	47			
	350	0·307	85	18	103			
	351	0·785	90	35	125			
	353	0·825	320	91	411			
	354	0·892	206	94	300	1570	2063	3632
5.	343	0·895	347	103	451			
	345	0·776	81	61	142			
	346	0·575	203	62	265			
	348	0·382	18	5	23			
	350	0·295	18	31	49			
	351	0·650	40	0	40			
	353	0·679	291	87	379			
	354	0·767	232	106	338	1230	455	1687
6.	346	0·735	181	87	269			
	350	0·431	118	19	137			
	353	0·488	297	117	414			
	354	0·605	246	85	331			
	355	0·797	57	28	85	899	336	1236
7.	346	0·860	174	83	256			
	350	0·385	37	14	51			
	351	0·324	49	23	72			
	353	0·304	269	81	349			
	354	0·431	171	47	218			
	355	0·656	73	17	90	773	265	1036
10.	351	0·627	38	27	65			
	353	0·393	278	88	366			
	354	0·291	245	62	308			
	355	0·216	44	26	71			
	356	0·557	26	10	36	631	213	846
15.	357	0·669	137	34	172			
	358	0·934	0	119	119			
	359	0·662	11	11	23	148	164	314
18.	357	0·086	68	26	94			

TABLE II. (continued).

Date.	Group.	Mean distance from centre, radius = 1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1862.								
Aug. 18.	358	0·466	121	24	145			
	359	0·335	471	136	606			
	360	0·791	7	14	21	667	200	866
19.	357	0·118	77	26	103			
	358	0·271	111	22	133			
	359	0·391	463	60	523			
	360	0·675	46	34	80			
	361	0·965	477	148	624			
	362	0·824	15	15	30			
	363	0·994	0	163	163	1189	468	1656
20.	357	0·333	90	18	108			
	358	0·134	133	73	206			
	359	0·540	544	127	671			
	360	0·495	84	0	84			
	361	0·882	427	100	527			
	362	0·698	47	18	65			
	363	0·942	62	25	87	1387	361	1748
21.	357	0·533	85	20	105			
	358	0·210	87	35	122			
	359	0·688	592	100	692			
	360	0·368	74	41	115			
	361	0·759	318	117	435			
	362	0·538	30	46	76			
	363	0·845	72	16	88	1258	375	1633
22.	357	0·730	106	37	144			
	358	0·430	33	28	61			
	359	0·843	338	145	482			
	360	0·320	32	9	41			
	361	0·578	302	130	432			
	362	0·356	86	50	137			
	363	0·680	35	12	47	932	411	1344
23.	357	0·851	56	56	112			
	358	0·598	59	16	74			
	359	0·938	149	112	262			
	360	0·424	56	9	66			
	361	0·404	294	125	419			
	362	0·279	80	13	93			
	363	0·513	35	10	45	729	341	1071
24.	357	0·950	152	0	152			
	358	0·762	33	13	46			
	360	0·569	26	0	26			
	361	0·204	296	135	432			
	362	0·333	50	9	59			
	364	0·969	143	106	248	700	263	963
25.	358	0·882	9	9	18			
	360	0·686	12	12	23			
	361	0·130	396	129	525			
	362	0·486	49	10	58			
	364	0·894	150	66	216	616	226	840
26.	358	0·960	31	0	31			
	360	0·801	21	0	21			
	361	0·285	299	147	446			
	362	0·693	3	6	9			
	364	0·767	172	73	245			
	365	0·651	6	11	17	532	237	769
28.	361	0·669	327	132	458			

TABLE II. (continued).

Date.	Group.	Mean distance from centre, radius=1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1862.								
Aug. 28.	364	0·442	194	62	256			
	365	0·265	132	26	159			
	366	0·785	21	7	28	674	227	901
29.	361	0·825	366	152	518			
	364	0·301	202	67	269			
	365	0·160	112	34	146			
	367	0·969	0	143	143	680	396	1076
30.	361	0·922	338	109	447			
	364	0·302	188	54	242			
	365	0·271	62	22	84			
	367	0·901	87	19	107	675	204	880
Sept. 2.	364	0·777	190	47	237			
	365	0·824	30	0	30			
	367	0·632	44	22	66			
	368	0·673	120	52	172			
	369	0·854	149	33	182			
	370	0·420	23	9	33	556	163	720
3.	364	0·895	194	68	262			
	365	0·952	55	0	55			
	367	0·483	29	10	39			
	368	0·501	79	39	118			
	369	0·729	81	81	162	438	198	636
4.	364	0·963	170	124	294			
	367	0·277	27	13	40			
	368	0·371	78	41	120			
	369	0·610	314	87	400	589	265	854
5.	367	0·173	26	9	35			
	368	0·390	255	79	333			
	369	0·484	380	146	526	661	234	894
6.	367	0·236	31	18	48			
	368	0·423	202	70	273			
	369	0·355	488	128	615	721	216	936
9.	367	0·799	7	21	28			
	369	0·676	460	117	577			
	370	0·731	137	37	175			
	371	0·900	39	9	49	643	184	829
10.	367	0·924	11	11	23			
	369	0·861	314	223	539			
	370	0·578	364	146	510			
	371	0·771	13	0	13	702	380	1085
11.	369	0·931	83	48	130			
	370	0·462	507	126	633	590	174	763
12.	370	0·403	643	144	787	643	144	787
15.	370	0·697	370	163	533			
	372	0·983	294	123	416	664	286	949
16.	370	0·837	407	125	531			
	372	0·914	251	84	335			
	373	0·962	294	62	356			
	374	0·940	361	149	511	1313	420	1733
18.	370	0·980	313	0	313			
	372	0·966	658	197	856			
	373	0·390	157	42	199			
	374	0·394	241	93	333			
	375	0·940	50	50	100	1419	382	1801
19.	372	0·488	253	73	326			
	373	0·560	236	87	324			

TABLE II. (continued).

Date.	Group.	Mean distance from centre, radius=1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1862.								
Sept. 19.	374	0·530	392	125	517			
	375	0·872	97	35	132			
	376	0·160	26	13	39			
	377	0·977	697	0	697	1701	333	2035
20.	372	0·306	202	49	251			
	373	0·370	224	28	253			
	374	0·357	246	68	315			
	375	0·734	75	19	94			
	376	0·134	26	13	39			
	377	0·904	1270	524	1794	2043	701	2746
25.	372	0·782	162	115	277			
	373	0·825	114	46	160			
	376	0·972	417	0	417			
	377	0·390	2005	750	2755	2698	911	3609
30.	377	0·955	1690	612	2302			
	378	0·377	350	110	460			
	379	0·446	110	52	162	2150	774	2924
Oct. 1.	377	0·972	284	152	436			
	378	0·349	430	127	557			
	379	0·350	150	27	177	864	306	1170
3.	378	0·595	411	128	539			
	379	0·518	65	20	84			
	381	0·753	58	19	78	534	167	701
5.	378	0·872	228	352	580			
	379	0·811	14	14	29			
	382	0·658	51	28	79			
	383	0·774	20	27	47	313	421	735
8.	382	0·326	415	135	550			
	383	0·358	14	0	14			
	384	0·601	112	64	176			
	385	0·939	50	75	125	591	274	865
11.	382	0·773	602	135	738			
	384	0·980	67	67	134			
	385	0·542	25	15	41	694	217	913
16.	385	0·355	9	5	14			
	386	0·565	36	10	46			
	387	0·962	108	139	247	153	154	307
17.	386	0·362	22	14	36			
	387	0·863	111	51	162			
	388	0·925	68	34	102	201	99	300
21.	387	0·258	159	31	190			
	388	0·279	18	9	27			
	389	0·423	61	9	70	238	49	287
23.	387	0·418	381	33	414			
	388	0·187	17	0	17			
	389	0·379	28	5	32			
	390	0·932	296	71	368	722	109	831
24.	387	0·569	338	130	468			
	388	0·383	5	14	19			
	389	0·476	58	19	77			
	390	0·841	281	63	344			
	391	0·888	263	75	338	945	301	1246
26.	387	0·884	109	45	154			
	390	0·541	269	86	356			
	391	0·618	314	59	373			
	392	0·789	7	14	21			

TABLE II. (continued).

Date.	Group.	Mean distance from centre, radius = 1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1862.								
Oct. 26.	393	0·902	39	29	68	738	233	972
Nov. 11.	394	0·113	115	34	150			
	395	0·204	126	35	161			
	396	0·535	70	15	85			
	397	0·428	28	23	52			
	398	0·446	10	10	19	349	117	467
12.	394	0·310	215	40	255			
	395	0·419	230	56	287			
	396	0·687	35	23	58			
	397	0·251	22	0	22			
	399	0·616	32	11	43	534	130	665
13.	394	0·519	179	65	244			
	395	0·612	260	65	325			
	396	0·826	23	30	53			
	397	0·101	4	4	8			
	399	0·505	152	64	216			
	400	0·825	15	23	38	633	251	884
16.	394	0·941	50	199	249			
	395	0·970	106	53	158			
	396	0·919	55	11	65			
	399	0·300	491	111	602			
	400	0·351	0	14	14			
	401	0·694	195	59	254	897	447	1342
17.	499	0·472	338	58	396			
	400	0·273	9	0	9			
	401	0·516	194	54	248			
	402	0·903	50	60	111	591	172	764
23.	401	0·778	203	54	257			
	402	0·244	733	272	1005			
	403	0·315	18	22	40			
	404	0·444	9	24	33			
	405	0·465	29	24	53	992	396	1388
24.	401	0·901	224	49	272			
	402	0·442	915	171	1085			
	403	0·371	14	0	14			
	404	0·317	68	23	90			
	405	0·318	27	14	41	1248	257	1502
27.	402	0·893	601	103	704			
	404	0·542	5	10	15			
	405	0·489	5	10	15	611	123	734
Dec. 12.	406	0·844	169	56	225	169	56	225
25.	407	0·603	267	117	384			
	408	0·733	50	25	75			
	409	0·766	40	20	60			
	410	0·981	201	134	335			
	411	0·936	143	48	190	701	344	1044
30.	410	0·347	45	14	59			
	411	0·332	190	41	230			
	412	0·420	42	19	61			
	413	0·261	57	13	71			
	414	0·905	272	91	363	606	178	784
31.	410	0·504	128	20	148			
	411	0·499	182	54	236			
	412	0·403	42	23	65			
	413	0·220	210	79	289			
	414	0·776	420	318	738	982	494	1476

TABLE II. (continued).

Date.	Group.	Mean distance from centre, radius = 1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1863.								
Jan. 2.	410	0·700	0	6	6			
	411	0·749	136	84	221			
	412	0·640	11	0	11			
	413	0·530	449	153	602			
	414	0·466	807	309	1116	1403	552	1956
4.	411	0·948	152	55	207			
	413	0·840	446	101	547			
	414	0·256	1107	198	1305	1705	354	2059
7.	414	0·765	1061	324	1386			
	415	0·905	101	50	151	1162	374	1537
8.	414	0·894	958	319	1277			
	415	0·778	135	34	169	1093	353	1446
18.	416	0·279	443	93	536			
	417	0·492	49	25	74			
	418	0·902	301	68	369	793	186	979
20.	416	0·285	531	147	678			
	417	0·551	30	0	30			
	418	0·654	220	73	294			
	419	0·775	0	41	41	781	261	1043
25.	416	0·981	0	335	335			
	417	0·631	27	5	33			
	418	0·588	132	63	195			
	419	0·510	109	20	129			
	420	0·928	114	80	193			
	421	0·983	74	74	147			
	422	0·557	36	0	36	492	577	1068
28.	418	0·949	124	55	179			
	420	0·492	157	39	197			
	421	0·626	33	16	49			
	422	0·422	155	47	202			
	423	0·740	38	19	57	507	176	684
29.	420	0·324	185	63	248			
	421	0·417	61	14	75			
	422	0·632	153	38	192			
	423	0·574	42	26	68	441	141	583
Feb. 8.	424	0·163	121	22	142			
	425	0·392	65	19	83			
	426	0·907	0	20	20	186	61	245
9.	424	0·068	205	47	252			
	425	0·568	31	5	36	236	52	288
May 6.	437	0·503	49	15	64			
	438	0·752	26	6	32			
	439	0·263	18	9	26			
	440	0·561	36	26	62			
	441	0·827	183	46	229	312	102	413
7.	437	0·658	62	40	102			
	438	0·884	54	36	91			
	439	0·437	14	9	24			
	440	0·424	42	28	70			
	441	0·688	213	59	272	385	172	559
8.	437	0·518	30	30	60			
	438	0·960	15	108	124			
	439	0·599	32	21	53			
	440	0·275	75	27	102			
	441	0·516	194	40	234			
	442	0·884	9	18	27	355	244	600

TABLE II. (continued).

Date.	Group.	Mean distance from centre, radius=1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1863.								
May 13.	441	0·583	648	153	801			
	443	0·585	163	53	216			
	444	0·579	130	26	156	941	232	1173
14.	441	0·698	781	243	1024			
	443	0·408	149	61	210			
	444	0·469	68	39	106	998	343	1340
23.	441	0·768	7	27	33			
	445	0·969	0	71	71			
	446	0·987	136	27	163	143	125	267
24.	445	0·869	170	68	239			
	446	0·901	49	10	58	219	78	297
25.	445	0·739	338	102	440			
	446	0·809	109	36	145			
	447	0·882	172	27	200			
	448	0·914	63	31	94			
	449	0·591	42	11	53	724	207	932
26.	445	0·660	429	153	582			
	446	0·631	219	60	279			
	447	0·801	198	64	262			
	448	0·838	109	47	156			
	449	0·417	23	0	23	978	324	1302
27.	445	0·590	464	148	611			
	446	0·413	224	56	280			
	447	0·702	181	60	241			
	448	0·697	77	24	101			
	449	0·340	59	14	72	1005	302	1305
28.	445	0·411	442	126	568			
	446	0·189	256	39	295			
	447	0·547	132	36	168			
	448	0·505	59	20	79			
	449	0·270	35	9	44	924	230	1154
29.	445	0·216	454	87	542			
	446	0·208	187	35	222			
	447	0·340	77	27	104			
	448	0·337	68	18	86	786	167	954
June 1.	445	0·509	373	88	462			
	446	0·765	66	14	80			
	447	0·221	39	17	57			
	448	0·400	28	14	42			
	450	0·747	19	19	38			
	451	0·933	202	36	238	727	188	917
2.	445	0·683	437	93	530			
	456	0·893	19	9	28			
	447	0·434	71	14	85			
	448	0·593	16	16	32			
	450	0·577	125	10	135			
	451	0·838	227	47	273	895	189	1083
3.	445	0·836	407	109	516			
	447	0·602	59	11	69			
	450	0·382	179	55	235			
	451	0·672	155	52	206	800	227	1026
4.	445	0·939	374	212	586			
	447	0·765	40	7	46			
	450	0·203	279	70	349			
	451	0·492	207	44	251	900	333	1232
6.	450	0·374	193	64	258			

TABLE II. (continued).

Date.	Group.	Mean distance from centre, radius=1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1863.								
June 6.	451	0·092	98	26	124			
	452	0·323	9	45	54			
	453	0·947	92	52	144	392	187	580
8.	450	0·736	200	25	225			
	451	0·409	107	23	130			
	452	0·349	113	45	158			
	453	0·805	94	22	116	514	115	629
9.	450	0·875	88	35	123			
	451	0·608	96	32	128			
	452	0·513	109	30	139			
	453	0·599	59	21	80	352	118	470
10.	450	0·968	88	35	123			
	451	0·792	83	28	111			
	452	0·713	116	31	147			
	453	0·387	69	23	93	356	117	474
11.	451	0·911	63	52	115			
	452	0·850	72	24	96			
	453	0·198	43	17	60	178	93	271
13.	453	0·398	37	19	56			
	454	0·451	57	72	129	94	91	185
19.	455	0·646	11	11	22	11	11	22
20.	456	0·963	46	278	325	46	278	325
21.	456	0·891	244	85	329	244	85	329
23.	456	0·591	200	58	258			
	457	0·680	47	12	58	247	70	316
25.	456	0·146	129	39	168			
	457	0·299	36	9	45			
	458	0·741	38	13	51	203	61	264
26.	456	0·112	124	39	163			
	457	0·249	18	0	18			
	458	0·864	51	26	77	193	65	258
27.	456	0·343	109	41	149			
	457	0·288	36	9	45	145	50	194
29.	456	0·715	135	25	159			
	457	0·742	185	64	248	320	89	407
30.	456	0·866	128	43	170			
	457	0·835	31	47	78	159	90	248
July 1.	456	0·946	144	39	182			
	459	0·868	0	43	43			
	460	0·880	27	0	27	171	82	252
2.	456	0·985	49	0	49			
	459	0·690	47	24	71			
	460	0·729	19	12	31	115	36	151
3.	459	0·493	10	30	39			
	460	0·567	10	21	31	20	51	70
4.	459	0·219	4	13	17	4	13	17
5.	459	0·095	9	43	51			
	461	0·654	73	40	113			
	462	0·994	408	1345	1753	490	1425	1917
6.	459	0·162	9	52	60			
	461	0·789	173	62	235			
	462	0·895	282	78	360			
	463	0·944	222	118	340	686	310	995
7.	459	0·263	9	4	13			
	461	0·915	157	63	220			
	462	0·771	265	73	338			

TABLE II. (continued).

Date.	Group.	Mean distance from centre, radius=1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1863.								
July 7.	463	0·844	249	64	313	680	204	884
9.	459	0·658	6	56	62			
	462	0·428	249	61	310			
	463	0·546	142	81	223			
	464	0·638	11	17	28	408	215	623
10.	459	0·855	214	116	331			
	462	0·240	171	61	233			
	463	0·357	182	46	228			
	464	0·431	137	38	175	704	261	967
12.	462	0·303	170	67	237			
	463	0·249	167	48	215			
	464	0·306	121	72	193	458	187	645
14.	462	0·677	245	82	326			
	463	0·608	224	75	299			
	464	0·664	126	34	160			
	465	0·874	202	79	281	797	270	1066
15.	462	0·807	167	58	225			
	463	0·747	127	46	172			
	464	0·799	71	28	99			
	465	0·767	252	60	312	617	192	808
17.	463	0·956	102	73	175			
	464	0·981	22	134	156			
	465	0·510	204	69	273	320	276	604
18.	465	0·411	186	51	238			
	466	0·286	31	9	40	217	60	278
19.	465	0·417	174	56	230			
	466	0·474	39	10	48	213	66	278
24.	465	0·888	197	47	244	197	47	244
25.	465	0·931	143	71	214	143	71	214
26.	467	0·989	0	153	153	0	153	153
28.	467	0·835	188	70	258	188	70	258
29.	467	0·693	160	53	213			
	468	0·913	73	0	73	233	53	286
30.	467	0·509	189	60	248			
	468	0·790	173	41	214	362	101	462
31.	467	0·305	242	31	273			
	468	0·634	120	55	175	362	86	448
Aug. 1.	467	0·143	228	52	279			
	468	0·450	129	29	158	357	81	437
3.	467	0·365	155	36	191			
	468	0·190	43	17	60			
	469	0·916	126	42	168	324	95	419
4.	467	0·535	135	45	181			
	468	0·576	47	16	62			
	469	0·823	201	59	260	383	120	503
5.	467	0·734	181	44	225			
	468	0·761	46	26	73			
	469	0·646	284	50	334			
	470	0·769	46	0	46	557	120	678
7.	467	0·960	185	93	278			
	468	0·904	212	71	282			
	469	0·257	203	31	234			
	470	0·393	28	9	37	628	204	831
8.	468	0·952	41	14	55			
	469	0·058	90	26	115			
	470	0·202	83	17	100	214	57	270

TABLE II. (continued).

Date.	Group.	Mean distance from centre, radius=1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1863.								
Aug. 10.	469	0·386	60	14	74			
	470	0·248	22	9	31			
	471	0·741	146	19	166			
	472	0·936	107	36	143			
	473	0·954	248	44	291	583	122	705
11.	469	0·583	37	16	53			
	470	0·471	14	5	19			
	471	0·569	130	47	177			
	472	0·845	201	64	265			
	473	0·912	251	84	335	633	216	849
12.	471	0·376	101	32	133			
	472	0·687	163	52	216			
	473	0·803	227	85	311			
	474	0·815	37	22	59	528	191	719
14.	470	0·954	335	0	335			
	471	0·221	17	9	26			
	472	0·476	43	19	63			
	473	0·449	177	48	225	572	76	649
15.	471	0·409	14	9	23			
	472	0·425	75	9	85			
	473	0·241	197	53	250			
	474	0·975	114	57	170	400	128	528
17.	473	0·222	109	31	140			
	474	0·724	110	55	165			
	475	0·593	47	16	63	266	102	368
18.	473	0·476	106	39	145			
	474	0·209	148	100	248			
	475	0·484	29	15	44			
	476*a*	0·451	43	24	67			
	476*b*	0·549	66	20	86	392	198	590
19.	473	0·668	52	29	80			
	474	0·290	214	89	303			
	475	0·413	56	28	84			
	476	0·287	152	45	196	474	191	663
22.	473	0·817	30	22	52			
	475	0·483	39	10	49	69	32	101
24.	475	0·683	23	12	35	23	12	35
29.	476	0·950	55	14	69	55	14	69
Sept. 7.	477	0·927	23	23	45	23	23	45
10.	477	0·495	59	29	88			
	478	0·049	21	9	30			
	479	0·122	56	17	73	136	55	191
11.	477	0·274	62	22	84			
	478	0·240	18	0	18			
	479	0·349	41	14	54	121	36	156
16.	477	0·553	0	21	21	0	21	21
22.	480	0·936	131	71	202	131	71	202
23.	480	0·844	193	48	241	193	48	241
24.	480	0·689	243	71	314	243	71	314
25.	480	0·541	330	102	432	330	102	432
26.	480	0·357	292	132	424	292	132	424
28.	480	0·192	295	74	369			
	481	0·927	11	45	57			
	482	0·551	10	0	10	316	119	436
29.	480	0·363	233	32	264			
	481	0·827	411	99	510			

TABLE II. (continued).

Date.	Group.	Mean distance from centre, radius = 1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1863.								
Sept. 29.	482	0·749	45	19	65	689	150	839
30.	480	0·534	176	40	216			
	481	0·665	521	160	682			
	482	0·892	113	28	141	810	228	1039
Oct. 1.	480	0·690	219	30	249			
	481	0·512	691	144	835			
	482	0·995	196	98	293	1106	272	1377
5.	481	0·472	478	72	551			
	483	0·835	117	39	156	595	111	707
6.	481	0·648	406	111	517			
	483	0·686	117	41	157	523	152	674
13.	483	0·829	137	46	183			
	484	0·378	41	18	60	178	64	243
14.	483	0·913	73	105	178			
	484	0·482	53	19	73	126	124	251
16.	485	0·931	119	48	166			
	486	0·973	19	19	38	138	67	204
17.	485	0·821	141	59	201	141	59	201
22.	487	0·339	109	27	136			
	488	0·238	26	13	39	135	40	175
23.	487	0·509	60	30	89			
	488	0·549	36	25	61	96	55	150
31.	489	0·817	193	45	238			
	490	0·178	160	35	195			
	491	0·229	131	35	166			
	492	0·961	46	62	108	530	177	707
Nov. 1.	489	0·922	164	65	229			
	490	0·247	176	66	241			
	491	0·148	108	43	151			
	492	0·867	34	43	77	482	217	698
2.	489	0·982	134	89	223			
	490	0·444	123	47	171			
	491	0·301	125	22	148	382	158	542
6.	491	0·974	76	284	360	76	284	360
12.	493	0·943	92	52	144			
	495	0·972	265	114	379	357	166	523
13.	493	0·864	128	34	162			
	494	0·916	63	0	63			
	495	0·905	292	151	443	483	185	668
19.	493	0·466	121	82	203			
	495	0·305	237	58	295			
	496	0·615	92	22	114			
	497	0·869	17	0	17	467	162	629
20.	493	0·634	99	33	132			
	495	0·479	166	102	268			
	496	0·431	76	28	104			
	498	0·462	14	34	48			
	499	0·901	29	0	29	384	197	581
21.	493	0·753	91	19	110			
	495	0·660	215	40	254			
	496	0·214	65	22	87			
	498	0·267	22	13	36	393	94	487
22.	493	0·826	15	30	46			
	495	0·805	210	66	275			
	496	0·089	86	21	107			
	498	0·130	17	30	47	328	147	475

TABLE II. (continued).

Date.	Group.	Mean distance from centre, radius=1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1863.								
Nov. 23.	493	0·933	59	0	59			
	495	0·921	174	76	251			
	496	0·301	54	13	67			
	498	0·290	9	18	27	296	107	404
28.	500	0·530	141	35	176	141	35	176
Dec. 3.	501	0·880	245	64	309			
	502	0·698	83	30	112			
	503	0·570	104	21	125	432	115	546
4.	501	0·962	155	93	248			
	502	0·829	84	23	107			
	503	0·721	92	24	116	331	140	471
10.	504	0·678	6	12	17			
	505	0·588	279	74	353			
	506	0·712	241	30	271			
	507	0·930	24	48	71	550	164	712
12.	504	0·899	19	0	19			
	505	0·112	385	107	491			
	506	0·501	89	39	128			
	507	0·627	38	16	55			
	508	0·050	9	13	21	540	175	714
14.	505	0·339	313	86	399			
	506	0·234	26	18	44			
	507	0·342	32	9	41			
	508	0·344	18	14	32			
	509	0·429	19	9	28	408	136	544
16.	505	0·600	240	80	320			
	506	0·309	22	0	22	262	80	342
18.	505	0·953	233	193	427			
	510	0·708	585	157	741	818	350	1168
19.	510	0·614	866	276	1141	866	276	1141
22.	510	0·284	945	263	1209			
	511	0·777	61	41	102			
	512	0·646	189	72	261			
	513	0·889	197	47	244	1392	423	1816
23.	510	0·402	783	270	1053			
	511	0·611	49	11	59			
	512	0·545	361	102	462			
	513	0·633	77	16	93			
	514	0·330	41	9	49	1311	408	1719
24.	510	0·613	741	200	941			
	511	0·501	20	10	30			
	512	0·324	478	113	591			
	513	0·501	69	30	98			
	514	0·477	136	29	165			
	515	0·900	68	49	117	1512	431	1942
25.	510	0·775	602	122	724			
	511	0·370	78	14	92			
	512	0·126	292	99	390			
	513	0·401	65	19	84			
	514	0·685	151	23	175			
	515	0·786	159	28	187	1347	305	1652
30.	512	0·916	409	105	514			
	513	0·720	147	31	178			
	515	0·323	171	32	203			
	516	0·231	52	13	65	779	181	960

Remark.—No spots were observed on the sun's disk on the following days:—1863, August 28, 31; September 1, 3, 17, 21; November 9.

TABLE III.—Showing the Heliographical Elements of all Sun-spots observed at the Kew Observatory from February 7th, 1862 to December 31st, 1863.

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1862.									
Feb. 7.	1	37·507	232	0·860	51° 23·0	9° 13·2	150° 24′	+16° 33′	P.
	2		232	0·890	54 31·5	4 55·2	146 6	+14 57	Q.
	3		233	0·950	55 51·2	355 33·8	136 45	+15 58	R.
	4		228	0·655	288 52·7	98 33·5	239 45	+15 54	s.
	5		228	0·705	285 56·8	103. 8·4	244 19	+16 4	S.
	6		227	0·793	289 50·3	108 47·5	249 59	+22 11	T.
	7		226	0·850	283 13·5	116 50·7	258 2	+19 40	U.
	8		231	0·345	321 42·2	72 15·2	213 26	+11 55	V.
	9		230	0·524	318 53·7	78 36·4	219 47	+21 37	W.
	10		229	0·525	306 3·7	84 30·1	225 41	+17 52	X.
	11	37·625	232	0·843	51 5·5	11 15·5	150 46	+16 9	P.
	12		232	0·872	54 10·9	7 10·8	146 41	+14 38	Q.
	13		233	0·936	55 37·4	358 11·5	137 42	+15 36	R.
	14		228	0·669	287 48·2	100 2·6	239 32	+15 51	s.
	15		228	0·720	285 9·4	104 39·0	244 8	+16 8	S.
	16		227	0·805	288 59·0	110 22·1	249 52	+22 8	T.
	17		226	0·862	282 43·4	118 24·6	257 55	+19 46	U.
	18		230	0·534	315 27·9	80 42·9	220 13	+21 15	W.
	19		229	0·537	303 39·4	86 8·7	225 39	+17 35	X.
8.	20	38·448	232	0·754	45 18·6	22 2·5	149 53	+16 57	P.
	21		232	0·779	49 10·1	18 42·3	146 32	+15 8	Q.
	22		233	0·865	51 37·5	9 25·2	137 15	+16 13	R.
	23		226	0·934	278 59·8	129 54·8	257 45	+19 49	S.
	24		228	0·825	279 4·5	116 20·5	244 11	+15 44	T.
	25		229	0·655	291 51·8	98 19·4	226 9	+17 43	U.
	26		230	0·634	294 24·1	95 58·2	223 48	+18 8	V.
	27	38·490	232	0·744	45 13·0	22 51·9	150 6	+16 39	P.
	28		232	0·772	48 55·1	19 23·9	146 38	+15 4	Q.
	29		233	0·860	51 36·3	10 0·9	137 15	+16 3	R.
	30		226	0·938	278 40·6	130 36·4	257 50	+19 41	S.
	31		228	0·830	278 41·8	117 0·1	244 14	+15 37	T.
	32		229	0·663	291 7·8	99 9·4	226 23	+17 38	U.
	33		230	0·642	293 41·6	96 46·4	223 11	+18 7	V.
10.	34	40·521	232	0·481	18 12·8	50 56·6	149 23	+16 56	P.
	35		232	0·488	24 46·3	48 4·6	146 31	+15 18	Q.
	36		233	0·586	33 14·5	39 49·8	138 16	+16 30	R.
	37		233	0·600	35 20·7	38 14·1	136 40	+16 6	s.
	38		234	0·986	51 42·5	350 29·6	88 56	+20 34	S.
	39		234	0·949	64 12·9	356 2·8	94 29	+ 7 4	T.
	40		235	0·671	253 43·7	109 28·8	207 55	− 5 10	V.
	41		235	0·700	254 32·6	111 43·0	210 10	− 4 25	v.
	42		229	0·911	278 4·9	128 32·5	226 59	+18 40	W.
	43		230	0·829	277 58·2	118 55·5	217 22	+15 32	X.
	44		230	0·786	281 14·5	113 54·4	212 20	+16 29	x.
	45		230	0·775	273 5·7	115 10·1	213 36	+10 11	Y.
	46	40·534	232	0·480	18 8·3	51 0·6	149 15	+16 55	P.
	47		232	0·487	24 41·4	48 10·4	146 24	+15 16	Q.
	48		233	0·585	33 18·8	39 51·7	138 6	+16 26	R.
	49		233	0·599	35 16·4	38 19·5	136 34	+16 6	r.
	50		234	0·988	51 47·6	349 37·2	87 51	+20 39	S.
	51		234	0·951	64 12·4	356 42·7	94 57	+ 7 8	T.
	52		235a	0·550	221 16·6	97 20·1	195 34	−23 3	U.
	53		235a	0·582	220 59·9	99 28·2	197 42	−24 7	u.
	54		235	0·672	253 42·2	109 34·3	207 48	− 5 10	V.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1862.									
Feb. 10.	55	40·534	235	0·703	254° 28·4	111° 56·0	210° 10	− 4° 27	v.
	56		229	0·911	277 55·6	128 31·2	226 45	+18 31	W.
	57		230	0·831	277 41·6	119 11·1	217 25	+15 23	X.
	58		230	0·785	280 51·1	113 58·4	212 12	+16 11	x.
	59		230	0·777	272 45·8	115 27·7	215 15	+10 0	Y.
	60	40·562	232	0·477	17 28·1	51 25·7	149 17	+16 56	P.
	61		232	0·482	24 2·9	48 38·6	146 30	+15 15	Q.
	62		233	0·580	33 0·3	40 19·1	138 10	+16 19	R.
	63		233	0·596	35 1·1	38 39·1	136 30	+16 4	r.
	64		234	0·986	51 42·2	350 25·9	88 17	+20 35	S.
	65		234	0·946	64 12·1	357 40·1	95 31	+ 6 58	T.
	66		235*a*	0·554	221 31·4	97 44·4	195 35	−23 3	U.
	67		235*a*	0·588	221 15·3	100 2·1	197 53	−24 10	u.
	68		235	0·679	253 38·1	110 5·5	207 57	− 5 10	V.
	69		235	0·709	254 22·3	112 26·1	210 17	− 4 28	v.
	70		229	0·913	277 52·9	128 54·3	226 45	+18 35	W.
	71		230	0·834	277 28·6	119 40·1	217 31	+15 21	X.
	72		230	0·790	280 40·8	114 29·1	212 20	+16 14	x.
	73		230	0·732	272 39·8	115 56·4	213 47	+10 4	Y.
16.	74	46·552	238	0·747	45 30·1	29 56·4	42 49	+14 22	P.
	75		238	0·680	44 6·4	35 27·7	48 21	+13 0	p.
	76		236	0·234	211 34·5	84 1·7	96 55	−15 31	Q.
	77		232	0·902	272 18·9	134 17·6	147 11	+14 59	R.
	78		232	0·772	277 0·1	119 23·1	132 16	+14 16	r.
	79		234	0·288	306 43·4	82 59·6	95 53	+ 6 42	S.
	80		234	0·263	326 23·4	77 27·9	90 21	+ 7 39	s.
	81		239	0·474	341 34·7	73 35·5	86 29	+21 15	T.
	82	46·572	238	0·738	45 41·8	30 35·5	43 12	+13 54	P.
	83		238	0·668	42 36·8	36 48·6	49 25	+13 27	p.
	84		236	0·240	212 42·4	84 30·9	97 7	−15 30	Q.
	85		236	0·197	201 9·7	81 0·0	93 36	−15 38	q.
	86		232	0·905	271 56·6	134 49·0	147 25	+14 47	R.
	87		232	0·776	276 29·9	119 57·0	132 33	+14 3	r.
	88		234	0·291	305 54·4	83 17·7	95 54	+ 6 41	S.
	89		234	0·269	325 30·1	77 48·7	90 25	+ 7 56	s.
	90		239	0·475	340 56·8	73 55·9	86 32	+21 29	T.
20.	91	50·594	240	0·344	356 1·7	72 50·5	28 24	+ 9 55	P.
	92		242	0·910	60 14·6	13 30·5	329 4	+ 6 23	Q.
	93		234	0·902	262 20·6	140 13·6	95 47	+ 7 19	R.
	94		239	0·857	281 4·1	129 53·6	85 27	+21 27	S.
	95		239*a*	0·288	325 53·4	81 45·4	37 18	+ 9 2	T.
21.	96	51·479	240	0·807	56 56·9	26 26·3	329 26	+ 6 26	P.
	97		242	0·630	78 12·6	39 9·9	342 10	−10 26	Q.
	98		242	0·664	76 22·1	36 37·6	339 38	− 9 17	q.
	99		242	0·620	74 23·6	39 56·9	342 57	− 8 4	q.
	100		234	0·971	259 38·3	153 8·7	96 9	+ 7 10	R.
	101		234	0·783	258 40·4	123 16·9	66 17	+ 1 25	r.
	102		234	0·750	260 20·9	125 53·7	68 54	+ 2 39	s.
	103		239	0·936	276 31·3	142 37·7	85 38	+21 22	S.
	104	51·500	240	0·811	57 3·5	26 5·4	328 47	+ 6 26	P.
	105		242	0·636	78 5·0	38 45·2	341 27	−10 22	Q.
	106		242	0·670	76 25·5	36 10·3	338 52	− 9 19	q.
	107		242	0·626	74 28·5	39 28·2	342 10	− 8 6	q.
	108		234	0·969	259 48·3	152 48·2	95 30	+ 7 17	R.
	109		234	0·776	258 42·5	128 26·0	71 8	+ 1 54	r.
	110		234	0·744	260 20·8	125 25·3	68 7	+ 2 33	s.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1862.									
Feb. 21.	111	51·500	239	0·932	276° 39·6	141° 58·5	84 40	+21° 19′	S.
28.	112	58·625	243	0·652	27 58·2	54 11·1	255 49	+18 46	P.
	113		247	0·730	31 59·8	47 7·7	248 46	+19 59	p.
	114		248	0·947	56 12·7	16 17·0	217 55	+ 9 2	Q.
	115		244	0·251	56 12·7	71 29·0	273 7	− 3 59	R.
	116		245	0·343	63 26·7	65 24·8	267 3	− 5 6	r.
	117		242	0·855	240 50·8	146 51·8	348 30	−10 11	S.
	118		242	0·815	238 23·9	140 42·8	342 21	−12 19	s.
	119		240	0·683	265 57·8	126 49·9	328 28	+ 6 33	T.
Mar. 1.	120	59·531	244	0·061	0 59·5	85 12·3	273 59	− 4 0	P.
	121		245	0·159	56 36·8	77 13·0	266 0	− 5 15	p.
	122		243	0·537	14 41·1	66 54·8	255 42	+18 41	Q.
	123		247	0·618	21 46·5	59 39·6	248 27	+20 5	q.
	124		248	0·870	53 32·4	28 24·2	217 11	+ 8 53	R.
	125		242	0·947	240 19·0	158 27·0	347 14	− 9 44	S.
	126		242	0·918	237 52·7	153 49·2	342 36	−12 17	s.
	127		240	0·822	261 33·4	139 55·7	328 43	+ 6 40	T.
3.	128	61·646	248	0·555	40 3·0	59 1·5	217 49	+ 8 33	P.
	129		249	0·715	83 47·7	43 9·4	201 56	−16 34	Q.
	130		249	0·738	79 56·0	40 54·3	199 41	−13 56	q.
	131		249	0·456	255 2·8	115 30·8	274 18	− 3 7	R.
	132		244	0·395	251 3·0	111 54·5	270 42	− 5 19	ρ.
	133		245	0·309	252 5·3	106 40·2	265 27	− 5 32	r.
	134		245	0·992	255 23·2	170 21·4	329 8	+ 6 41	S.
	135		243	0·461	321 9·7	96 37·4	255 24	+18 57	T.
	136		247	0·466	337 28·3	88 48·9	247 36	+20 23	t.
	137		247	0·496	334 35·2	90 20·3	249 7	+22 20	τ.
	138	61·659	248	0·554	40 1·0	59 4·8	217 41	+ 8 33	P.
	139		249	0·712	83 48·0	43 23·0	201 59	−16 33	Q.
	140		249	0·737	79 51·6	41 0·0	199 36	−13 53	q.
	141		249	0·458	255 0·0	115 41·0	274 17	− 3 6	R.
	142		244	0·397	251 5·8	112 2·1	270 38	− 5 18	ρ.
	143		245	0·313	251 53·3	106 55·1	265 31	− 5 34	r.
	144		245	0·992	255 24·8	170 1·7	328 38	+ 6 40	S.
	145		243	0·463	320 42·8	96 52·2	255 28	+18 58	T.
	146		247	0·466	337 7·1	88 58·5	247 35	+20 24	t.
	147		247	0·496	334 9·1	90 34·7	249 11	+22 17	τ.
4.	148	62·507	248	0·408	25 46·4	71 41·0	218 15	+ 8 47	P.
	149		249	0·554	85 46·4	56 31·0	203 5	−16 7	Q.
	150		249	0·633	83 1·6	50 28·7	197 3	−15 26	q.
	151		244	0·627	251 39·2	128 9·8	274 34	− 3 2	R.
	152		245	0·577	249 10·1	124 48·5	271 23	− 4 56	ρ.
	153		245	0·489	248 52·4	118 51·3	265 25	− 5 38	r.
	154		243	0·541	301 52·5	108 57·3	255 31	+19 11	S.
	155		247	0·534	313 22·7	103 5·9	249 40	+22 1	s.
	156	62·521	248	0·403	25 34·8	71 58·6	218 22	+ 8 39	P.
	157		249	0·550	86 28·6	56 53·8	203 17	−16 26	Q.
	158		249	0·632	83 23·9	50 36·5	197 0	−15 39	q.
	159		244	0·629	251 13·8	128 23·3	274 46	− 3 17	R.
	160		245	0·579	248 51·3	124 56·7	271 20	− 5 6	ρ.
	161		245	0·491	248 32·8	119 1·7	265 25	− 5 47	r.
	162		243	0·540	301 26·9	109 7·0	255 30	+18 59	S.
	163		247	0·534	312 52·3	103 22·3	249 45	+21 54	s.
	164	62·584	248	0·390	24 19·2	72 37·1	218 6	+ 8 48	P.
	165		249	0·540	86 22·0	57 39·6	203 9	−16 14	Q.
	166		249	0·624	83 25·7	51 18·8	196 48	−15 36	q.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Heliographical Longitude.	Heliographical Latitude.	Spot.
1862.					° ′	° ′	° ′	° ′	
Mar. 4.	167	62·584	244	0·637	251 26·8	128 58·4	274 27	− 3 4	R.
	168		245	0·588	249 5·7	125 36·6	271 6	− 4 54	ρ.
	169		245	0·503	248 44·7	119 49·8	265 19	− 5 32	r.
	170		243	0·552	300 32·6	110 6·9	255 36	+19 19	S.
	171		247	0·542	311 54·4	104 11·5	249 40	+22 9	s.
5.	172	63·548	250	0·982	50 28·0	14 21·2	146 9	+14 48	P.
	173		249	0·344	95 17·5	72 18·9	204 7	−16 10	Q.
	174		249	0·390	84 40·8	68 13·5	200 2	−13 25	q.
	175		248	0·793	248 53·5	142 58·2	274 46	− 3 6	R.
	176		248	0·679	246 15·8	133 31·1	265 19	− 5 58	r.
	177		243	0·673	287 6·9	123 40·5	255 29	+19 27	S.
	178		243	0·649	295 22·2	118 26·6	250 15	+22 27	s
	179		244	0·286	352 9·0	86 21·7	218 10	+ 8 45	T.
7.	180	65·479	250	0·706	36 59·6	53 22·7	157 48	+14 54	P.
	181		251	0·941	43 31·1	27 47·6	132 13	+17 24	Q.
	182		248	0·239	213 13·6	104 24·7	208 50	−14 39	R.
	183		248	0·223	202 38·0	102 6·5	206 32	−16 2	r.
	184		243	0·967	269 0·6	163 54·4	268 19	+19 16	S.
	185	65·500	250	0·702	36 38·9	53 48·8	157 54	+14 58	P.
	186		251	0·941	43 21·0	26 43·8	130 51	+19 7	Q.
	187		248	0·249	213 34·0	104 58·9	209 6	−14 53	R.
	188		248	0·232	202 36·2	102 31·0	206 38	−16 23	r.
	189		243	0·969	268 54·7	164 19·1	268 26	+19 16	S.
12.	190	70·639	250*a*	0·483	0 16·2	85 34·9	116 48	+18 57	P.
	191		250*b*	0·523	43 4·8	68 52·3	100 5	+ 5 20	Q.
	192		250*b*	0·499	42 31·0	70 30·5	101 44	+ 4 56	q.
	193		251*a*	0·915	233 5·7	164 43·2	195 56	−14 28	R.
	194		251*a*	0·898	229 44·3	162 23·3	193 36	−17 32	r.
	195		251*b*	0·441	303 11·6	111 55·8	143 9	+14 47	S.
19.	196	77·570	253	0·499	39 53·3	77 45·4	10 40	+ 5 48	P.
	197		253	0·862	40 28·3	49 44·2	342 39	+16 44	Q.
	198		252	0·739	70 54·4	56 48·4	349 43	− 9 18	R.
	199		255	0·521	96 29·3	76 17·7	9 13	−22 3	S.
	200		255	0·504	93 45·5	76 48·5	9 44	−20 20	s.
	201		254	0·984	266 44·0	180 46·9	113 42	+20 6	T.
	202		254	0·832	275 19·8	154 22·9	87 18	+20 33	U.
	203	77·584	253	0·494	39 57·4	78 2·2	10 45	+ 5 38	P.
	204		253	0·461	40 25·1	49 53·9	342 37	+16 44	Q.
	205		252	0·734	71 4·6	57 14·3	349 57	− 9 26	R.
	206		255	0·519	96 58·4	76 33·5	9 17	−22 14	S.
	207		254	0·988	266 35·4	182 6·8	114 50	+20 12	T.
	208		254	0·837	275 1·2	155 0·6	87 44	+20 32	U.
24.	209	82·510	252	0·297	236 9·1	126 51·1	349 41	− 8 53	R.
	210	82·520	252*a*	0·983	43 51·8	33 30·2	256 12	+18 24	P.
	211		252*b*	0·938	237 51·4	179 50·2	42 32	− 8 15	Q.
	212		252	0·300	235 54·1	127 1·7	349 44	− 8 59	R.
27.	213	85·528	252	0·831	237 51·9	169 5·0	349 7	− 8 44	P.
31.	214	89·521	256	0·921	45 36·4	52 29·4	175 52	+13 47	P.
	215		255	0·702	75 8·0	71 58·7	195 22	−12 39	Q.
	216		255	0·705	73 36·7	71 40·0	195 3	−11 36	q.
	217		255	0·760	76 1·7	66 58·4	190 21	−13 36	q.
	218		255	0·483	77 41·0	88 2·4	211 25	−12 25	R.
	219		255	0·465	80 2·2	89 27·6	212 51	−13 17	r.
	220	89·624	256	0·916	45 34·2	53 15·6	175 12	+13 40	P.
	221		255	0·698	75 5·5	72 24·3	194 20	−12 36	Q.
	222		255	0·700	73 37·0	72 8·2	194 4	−11 35	q.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1862.									
Mar. 31.	223	89·624	255	0·758	75° 56′·7	67° 17′·7	189° 14′	−13° 31′	q.
	224		255	0·493	74 31·4	87 18·2	209 44	−10 58	p.
	225		255	0·480	77 46·5	88 20·3	210 16	−12 25	R.
	226		255	0·459	80 21·0	89 58·5	211 55	+15 52	r.
Apr. 3.	227	92·468	256	0·546	26 2·0	93 11·1	174 46	+13 41	P.
	228		259	0·894	42 10·5	59 48·4	141 23	+15 52	Q.
	229		258	0·440	101 25·0	97 40·2	179 15	−21 30	R.
	230		258	0·406	104 38·0	100 25·3	182 0	−21 23	r.
	231		257	0·109	122 0·5	116 12·4	197 47	−11 36	S.
	232		257	0·173	98 2·8	111 10·6	192 46	−11 51	s.
	233		257	0·230	98 37·3	108 24·5	190 0	−13 45	σ.
	234		255	0·204	250 6·8	131 10·3	212 45	− 4 52	T.
	235		255	0·150	260 6·5	127 47·4	209 22	− 3 48	t.
	236		255	0·092	254 29·2	124 42·8	206 18	− 5 18	τ.
	237	92·514	256	0·540	25 48·6	93 36·4	174 32	+13 34	P.
	238		259	0·888	42 3·5	60 38·1	141 34	+15 46	Q.
	239		258	0·436	102 12·3	98 8·8	179 5	−21 39	R.
	240		258	0·404	105 33·1	100 48·7	181 45	−21 36	r.
	241		257	0·105	124 46·3	116 36·7	197 33	−11 34	S.
	242		257	0·164	99 8·3	111 46·0	192 42	−11 43	s.
	243		257	0·220	100 26·1	109 11·1	190 7	−13 45	σ.
	244		255	0·215	250 54·6	131 51·8	212 48	− 4 36	T.
	245		255	0·158	259 11·5	128 17·1	209 13	− 3 49	t.
	246		255	0·099	253 26·5	125 7·9	206 4	− 5 19	τ.
4.	247	93·490	256	0·403	7 22·0	107 15·0	174 40	+13 23	P.
	248		259	0·777	37 35·7	74 12·9	141 18	+15 32	Q.
	249		258	0·278	136 31·3	115 32·6	182 38	−21 33	R.
	250		258	0·317	123 7·1	110 35·2	177 40	−21 59	r.
	251		257	0·138	165 24·5	122 12·3	189 17	−14 0	S.
	252		257	0·135	192 54·5	125 33·0	192 38	−12 13	s.
	253		257	0·203	217 28·5	131 10·1	198 15	−11 14	σ.
	254		255	0·456	244 11·0	147 41·5	214 47	− 5 16	T.
	255		255	0·400	249 7·3	143 53·8	210 59	− 3 31	t.
	256		255	0·317	245 15·7	139 0·0	206 5	− 5 23	τ.
12.	257	101·458	260	0·944	65 26·7	57 39·0	11 43	− 3 36	P.
	258		263	0·215	227 5·4	140 20·9	94 25	− 9 4	Q.
	259		263	0·201	224 21·7	139 23·8	93 28	− 9 22	q.
	260		259	0·874	264 22·6	185 59·9	140 4	+15 1	R.
	261		262	0·288	344 1·4	125 21·6	79 26	+10 40	S.
	262		262	0·356	5 55·7	117 12·6	71 17	+11 51	s.
13.	263	102·500	261	0·774	76 8·2	78 4·2	17 59	+13 11	P.
	264		261	0·732	73 28·6	82 17	21 34	−10 59	p.
	265		261	0·985	72 11·4	48 46	348 3	− 9 20	Q.
	266		263	0·443	234 58·0	155 33	94 50	− 8 52	R.
	267		263	0·361	230 22·5	150 12	89 29	−10 0	r.
	268		259	0·960	260 58·7	200 37	139 54	+14 50	S.
	269		262	0·344	305 27·0	138 55	78 12	+12 0	T.
	270		262	0·297	327 7·9	131 22	70 39	+11 28	U.
14.	271	103·393	261	0·934	71 36·3	60 51	347 28	− 9 20	Q.
	272		261	0·710	62 50·2	85 9	11 46	− 3 8	q.
	273		261	0·682	48 17·7	89 0	15 37	+ 6 16	q.
	274		263	0·204	150 6·5	129 31	56 8	−14 46	R.
	275		262	0·615	237 22·8	168 13	94 50	− 8 16	S.
	276		262	0·601	236 33·0	167 13	93 50	− 8 44	S'.
	277		262	0·534	233 39·2	162 25	89 2	−10 3	σ.
	278		262	0·374	296 13·2	143 39	70 16	+11 46	T.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1862.									
Apr. 14.	279	103·406	264	0·971	35° 46′·4	58° 56′	345° 22′	+25° 19′	P.
	280		261	0·929	71 26·1	61 44	348 10	− 9 12	Q.
	281		261	0·705	62 33·6	85 32	11 58	− 3 7	q.
	282		262	0·615	237 7·1	168 13	94 39	− 8 25	S.
	283		262	0·602	236 11·3	167 16	93 42	− 8 56	S′.
15.	284	104·743	264	0·869	30 52·2	78 11	345 40	+24 59	P.
	285		264	0·914	41 28·3	69 6	336 35	+17 51	Q.
	286		265	0·971	77 14·5	54 44	322 13	−14 23	R.
	287		265	0·940	75 58·3	61 2	328 31	−13 16	r.
	288		261	0·777	71 55·5	80 25	347 57	− 9 43	S.
	289		263	0·369	207 57·5	149 49	57 18	−17 36	T.
17.	290	106·458	264	0·621	16 12·6	106 29	349 38	+22 32	P.
	291		261	0·485	73 55·1	104 28	347 37	− 9 27	Q.
	292		265	0·814	77 45·7	78 49	321 58	−14 19	R.
	293		265	0·747	77 8·9	85 4	328 13	−13 22	r.
	294		265	0·747	70 56·4	84 50	327 59	− 8 45	ϱ.
	295		265	0·858	77 30·6	74 7	317 16	−14 21	r′.
	296		263	0·579	223 19·6	167 23	50 32	−16 6	S.
	297		263	0·960	235 21·4	207 27	90 36	− 9 42	T.
	298	106·479	264	0·914	15 14·7	107 5	349 56	+22 24	P.
	299		261	0·476	74 1·4	105 2	347 53	− 9 26	Q.
	300		265	0·809	77 43·9	79 20	322 11	−14 15	R.
	301		265	0·741	77 5·9	85 40	328 31	−13 17	r.
	302		265	0·741	70 49·9	85 25	328 16	− 8 39	ϱ.
	303		265	0·852	77 30·4	74 46	317 37	−14 19	r′.
	304		263	0·584	223 40·4	167 52	50 43	−15 59	S.
	305		263	0·961	235 30·9	207 37	90 28	− 9 32	T.
20.	306	109·488	266	0·585	45 47·2	102 26	302 36	+ 6 26	P.
	307		266	0·521	41 14·9	107 27	307 37	+ 7 16	p.
	308		265	0·287	98 40·7	122 11	322 21	−14 11	Q.
	309		265	0·351	94 12·7	117 57	318 7	−14 52	q.
	310		265	0·274	88 0·3	121 28	321 38	−11 12	q′.
	311		261	0·197	219 59·1	146 42	346 52	− 9 34	R.
	312		261	0·193	225 11·5	146 49	346 59	− 8 32	R′.
21.	313	110·400	266	0·344	27 56·0	121 0	308 14	+ 6 57	P.
	314		266	0·409	37 23·5	115 42	302 56	+ 6 1	p.
	315		267	0·678	76 29·9	94 41	281 55	−11 53	Q.
	316		267	0·645	77 0·9	97 18	284 32	−11 57	q.
	317		265	0·166	141 4·5	134 53	322 7	−14 13	R.
	318		265	0·183	132 36·2	133 8	320 22	−14 40	r.
	319		261	0·386	230 56·3	159 24	346 38	− 9 41	S.
	320		261	0·388	234 32·2	159 46	347 0	− 8 20	S′.
22.	321	111·428	266	0·209	345 28·5	135 49	308 28	+ 6 55	P.
	322		266	0·225	8 7·0	130 56	303 35	+ 5 55	p.
	323		267	0·441	82 43·1	112 51	285 30	−12 20	Q.
	324		267	0·499	81 4·6	108 55	281 34	−12 27	q.
	325		265	0·248	203 52·9	149 17	321 56	−14 1	R.
	326		261	0·599	236 57·7	174 49	347 28	− 8 20	S.
	327		261	0·590	234 45·7	174 7	346 46	− 9 36	S′.
	328		261	0·622	233 20·5	176 22	349 1	−10 40	s.
23.	329	112·400	268	0·930	70 42·9	70 28	229 20	− 7 30	P.
	330		267	0·380	74 7·9	116 56	275 48	− 8 2	Q.
	331		267	0·439	73 34·1	113 10	272 2	− 8 14	q.
	332		265	0·426	221 27·1	162 48	321 40	−13 57	R.
	333		265	0·593	228 38·4	174 45	333 37	−13 11	r.
	334		265	0·589	230 48·3	174 40	333 32	−11 53	ϱ.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1862.									
Apr. 23.	335	112·400	261	0·752	235° 59′·6	187° 50′	346° 42′	− 9° 35′	S.
	336		261	0·760	237 25·6	188 33	347 25	− 8 29	S′.
	337		266	0·273	291 59·6	149 42	308 34	+ 6 51	T.
	338	112·431	268	0·929	70 46·6	70 38	229 3	− 7 34	P.
	339		267	0·376	74 22·4	160 58	319 23	− 8 6	Q.
	340		267	0·436	73 44·8	113 26	271 51	− 8 17	q.
	341		265	0·431	221 46·6	163 8	321 33	−13 54	R.
	342		265	0·599	228 39·6	175 12	333 37	−13 15	r.
	343		265	0·597	230 54·4	175 19	333 44	−11 54	ρ.
	344		261	0·757	235 52·3	188 16	346 41	− 9 39	S.
	345		261	0·764	237 25·1	188 57	347 22	− 8 30	S′.
	346		266	0·277	291 18·6	150 3	308 28	+ 6 56	T.
	347	112·488	268	0·921	70 46·9	71 55	229 32	− 7 36	P.
	348		267	0·362	75 1·1	118 11	275 48	− 8 12	Q.
	349		267	0·422	74 37·3	114 26	272 3	− 8 33	q.
	350		265	0·444	222 31·6	164 7	231 44	−13 51	R.
	351		265	0·609	228 48·1	176 2	333 39	−13 17	r.
	352		265	0·605	231 11·1	175 57	333 34	−11 49	ρ.
	353		261	0·767	236 4·3	189 16	346 33	− 9 32	S.
	354		261	0·776	237 26·6	190 2	347 39	− 8 30	S′.
	355		266	0·285	288 43·6	150 57	308 54	+ 6 44	T.
24.	356	113·442	267	0·149	84 51·4	132 0	276 1	− 7 34	P.
	357		267	0·229	82 14·9	127 23	271 24	− 8 30	p.
	358		261	0·888	236 25·9	202 52	346 53	− 9 26	R.
	359		261	0·895	237 30·6	203 39	347 40	− 8 28	R′.
	360		266	0·458	269 39·4	164 40	308 41	+ 6 52	S.
	361	113·642	267	0·105	92 47·5	134 55	276 10	− 7 28	P.
	362		267	0·188	87 29·8	130 13	271 28	− 8 45	p.
	363		267*a*	0·634	228 49·3	178 57	320 12	−13 35	Q.
	364		267*a*	0·680	228 40·2	182 29	333 44	−14 14	q.
	365		261	0·910	236 7·0	205 53	347 8	− 9 43	R.
	366		261	0·915	237 6·2	206 36	347 51	− 8 48	R′.
	367		261	0·903	232 46·2	205 1	346 16	−12 44	r.
	368		266	0·492	267 11·5	167 25	308 40	+ 6 42	S.
25.	369	114·441	269	0·962	75 16·4	66 40	196 35	−11 20	P.
	370		269*a*	0·740	68 15·1	93 21	223 16	− 5 39	Q.
	371		267	0·099	218 38·9	146 8	276 3	− 7 2	R.
	372		267	0·074	163 59·6	141 44	271 39	− 8 45	r.
	373		265	0·776	232 26·1	191 45	321 40	−12 27	S.
	374		265	0·764	230 52·1	190 31	320 26	−13 33	s.
	375		261	0·976	236 49·8	218 46	348 41	− 8 45	T.
	376		261*a*	0·884	236 12·6	203 16	333 11	− 9 43	U.
	377		266	0·635	261 31·1	178 52	308 47	+ 6 57	V.
	378	114·458	269	0·961	75 28·7	66 47	196 27	−11 32	P.
	379		269*a*	0·735	68 39·6	93 46	223 26	− 5 57	Q.
	380		267	0·103	218 22·2	146 22	276 2	− 7 10	R.
	381		267	0·079	165 29·1	141 54	271 34	− 9 0	r.
	382		265	0·776	232 4·4	191 47	321 27	−12 44	S.
	383		265	0·765	230 39·9	190 41	320 21	−13 43	s.
	384		261	0·972	236 31·9	217 44	347 24	− 9 6	T.
	385		261*a*	0·883	233 55·0	203 12	332 52	− 9 53	U.
	386		266	0·637	258 53·8	178 54	308 34	+ 6 36	V.
26.	387	115·554	269	0·867	75 57·5	82 0	196 7	−11 46	Q.
	388		269	0·809	77 51·0	88 16	202 23	−13 3	q.
	389		267	0·340	237 26·7	161 53	276 0	− 6 44	R.
	390		267	0·276	226 53·7	157 29	271 36	− 9 12	r.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1862.					° ′	° ′	° ′	° ′	
Apr. 26.	391	11·646	270	0·992	51 12·8	61 9	173 59	+12 58	P.
	392		269	0·855	76 5·0	83 24	196 14	−11 49	Q.
	393		269	0·795	78 9·0	89 46	202 36	−13 10	q.
	394		267	0·363	237 35·0	163 24	276 14	− 6 49	R.
	395		267	0·299	227 21·5	158 55	271 45	− 9 26	r.
27.	396	11·476	270	0·955	50 2·3	72 7	173 10	+12 56	P.
	397		269	0·743	76 57·1	95 16	196 19	−11 43	Q.
	398		269	0·667	79 36·8	101 43	202 46	−12 54	q.
	399		271	0·481	90 46·8	116 21	217 24	−15 54	R.
	400		271	0·441	89 35·9	118 37	219 40	−14 30	r.
	401		267	0·535	239 1·6	175 16	276 19	− 6 55	S.
	402		267	0·468	232 31·6	170 30	271 33	− 9 42	s.
	403	11·538	270	0·950	49 51·7	73 7	173 17	+12 58	P.
	404		269	0·732	77 0·9	96 18	196 28	−11 41	Q.
	405		269	0·655	79 49·2	102 41	202 51	−12 54	q.
	406		271	0·478	92 12·2	116 46	216 56	−16 30	R.
	407		271	0·428	90 30·4	119 38	219 48	−14 34	r.
	408		267	0·549	239 14·9	176 16	276 26	− 6 51	S.
	409		267	0·481	232 42·4	171 26	271 36	− 9 45	s.
	410	11·556	270	0·947	49 58·4	73 41	173 36	+12 47	P.
	411		269	0·730	77 14·2	96 30	196 25	−11 40	Q.
	412		269	0·655	80 16·9	102 45	202 40	−13 12	q.
	413		271	0·476	92 35·7	117 5	217 0	−16 35	R.
	414		271	0·425	90 39·2	119 52	219 47	−14 33	r.
	415		267	0·553	239 11·2	109 35	209 30	− 6 54	S.
	416		267	0·485	232 50·7	171 43	271 28	− 9 43	s.
28.	417	11·425	270	0·877	47 49·4	85 0	172 35	+12 59	P.
	418		269	0·588	78 54·0	108 28	196 3	−11 27	Q.
	419		269	0·498	84 22·2	115 9	202 44	−13 8	q.
	420		271	0·324	105 0·9	128 58	216 33	−16 4	r.
	421		271	0·278	106 51·7	131 36	219 11	−14 46	R.
	422		271	0·083	104 10·5	140 13	227 48	− 7 15	S.
	423		267	0·707	240 11·7	188 53	276 28	− 6 35	T.
	424		267	0·648	235 7·2	184 5	271 40	− 9 47	t.
	425	11·482	270	0·874	47 47·5	85 19	172 6	+12 58	P.
	426		269	0·583	79 42·2	108 51	195 38	−11 29	Q.
	427		269	0·495	84 51·0	115 30	202 17	−13 16	q.
	428		271	0·273	108 12·2	132 8	218 55	−14 51	R.
	429		271	0·127	96 5·2	137 43	224 30	− 7 57	S.
	430		271	0·079	107 19·7	140 36	227 23	− 7 18	s.
	431		267	0·711	240 8·5	189 17	276 4	− 6 38	T.
	432		267	0·651	235 2·3	184 25	271 12	− 9 52	t.
29.	433	11·411	273	0·990	48 52·8	64 58	138 34	+15 38	P.
	434		270	0·754	44 20·8	98 56	172 32	+12 46	Q.
	435		269	0·401	85 2·8	122 18	195 54	−11 36	R.
	436		269	0·310	97 13·5	129 14	202 50	−13 25	r.
	437		271	0·127	212 2·1	151 2	224 38	− 8 9	S.
	438		271	0·196	229 40·3	155 47	229 23	− 7 8	s.
	439		267	0·850	240 18·6	203 9	276 45	− 6 31	T.
	440		267	0·808	235 12·1	198 43	272 19	−10 41	t.
	441		272	0·873	258 51·8	204 7	277 43	+ 9 35	U.
	442		272	0·864	259 56·6	202 55	276 31	+10 19	u.
	443	11·484	273	0·986	48 48·5	66 36	137 10	+15 32	P.
	444		270	0·740	43 56·5	100 15	172 49	+12 42	Q.
	445		269	0·386	85 59·0	123 25	195 59	−11 39	R.
	446		269	0·296	98 28·0	130 16	202 50	−13 18	r.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1862.					° ′	° ′	° ′	° ′	
Apr. 29.	447	118·484	271	0·141	216 1·5	152 4	224 38	− 8 6	S.
	448		271	0·200	227 19·5	156 0	228 34	− 7 39	s.
	449		267	0·858	240 11·0	204 6	276 40	− 6 37	T.
	450		267	0·817	235 8·2	199 42	272 16	−10 47	t.
	451		272	0·873	259 42·2	204 1	276 35	+10 18	u.
	452		272	0·853	259 46·8	201 43	274 17	+ 9 55	u′.
	453		272	0·869	257 32·0	203 56	276 30	+ 8 22	u″.
30.	454	119·482	273	0·928	47 12·8	80 10	138 35	+15 20	P.
	455		270	0·587	37 3·6	114 7	172 32	+12 46	Q.
	456		269	0·200	107 14·1	137 11	195 36	−11 39	R.
	457		269	0·236	112 47·1	136 27	194 52	−13 58	ρ.
	458		269	0·164	147 37·9	144 36	203 1	−13 23	r.
	459		269	0·122	160 21·1	146 31	204 56	−11 3	r′.
	460		271	0·450	238 21·6	172 30	230 55	− 6 52	S.
	461		271	0·355	233 57·3	166 26	224 51	− 8 58	s.
	462		267	0·955	239 55·6	218 48	277 13	− 6 36	T.
	463		272	0·945	259 7·6	215 19	273 44	+11 22	U.
	464		272	0·958	257 11·9	217 56	276 21	+ 9 53	V.
	465		272	0·975	255 17·6	222 2	280 27	+ 8 31	W.
	466	119·521	273	0·926	46 54·1	80 39	138 31	+15 33	P.
	467		270	0·585	36 36·9	114 24	172 16	+12 56	Q.
	468		269	0·196	108 10·8	137 31	195 23	−11 38	R.
	469		269	0·231	113 21·3	136 48	194 40	−13 50	ρ.
	470		269	0·161	149 58·6	145 3	202 55	−13 15	r.
	471		269	0·123	161 25·6	146 42	204 34	−11 3	r′.
	472		269	0·150	179 45·4	149 33	207 25	−11 54	r″.
	473		269	0·174	166 20·9	147 52	205 44	−13 51	r‴.
	474		271	0·455	238 37·3	172 53	230 45	− 6 47	S.
	475		267	0·954	239 53·6	218 33	276 25	− 6 39	T.
	476		272	0·946	259 8·1	215 33	273 25	+11 24	U.
	477		272	0·958	257 15·6	217 58	275 50	+ 9 56	V.
	478		272	0·974	255 11·4	221 49	279 41	+ 8 22	W.
May 1.	479	120·598	270	0·409	21 42·3	129 34	172 9	+12 40	P.
	480		273	0·813	44 34·6	95 38	138 13	+14 36	Q.
	481		273	0·858	41 29	91 39	134 14	+18 24	q.
	482		274	0·965	47 40	74 28	117 3	+16 16	R.
	483		274	0·982	46 25	70 0	112 35	+18 8	r.
	484		269	0·198	176 44	151 12	193 47	−14 35	S.
	485		269	0·164	191 20	152 34	195 9	−11 35	T.
	486		269	0·230	189 59	154 40	197 15	−14 51	U.
	487		269	0·221	203 1	156 33	199 8	−12 30	V.
	488		269	0·289	209 17	160 48	203 23	−13 43	v.
	489		271	0·575	238 3	181 56	224 31	− 7 40	W.
	490		271	0·651	239 39	187 31	230 6	− 6 59	X.
	491	120·632	270	0·405	20 57	130 0	172 6	+12 43	P.
	492		273	0·808	44 8	96 17	138 23	+14 48	Q.
	493		273	0·853	41 18	92 21	134 27	+18 23	q.
	494		274	0·958	47 28	75 51	117 57	+16 16	R.
	495		274	0·977	46 15	71 45	113 51	+18 4	r.
	496		269	0·198	179 28	151 44	193 50	−14 20	S.
	497		269	0·172	193 29	153 11	195 17	−11 43	T.
	498		269	0·234	191 43	155 10	197 16	−14 47	U.
	499		269	0·229	205 11	157 15	199 21	−12 26	V.
	500		269	0·295	210 35	161 20	203 26	−13 35	v.
	501		271	0·579	238 4	182 17	224 23	− 7 41	W.
	502		271	0·657	239 38	188 0	230 6	− 7 1	X.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1862.					° ′	° ′	° ′	° ′	
May 2.	503	121·434	273	0·698	40 6	107 28	138 11	+14 43	P.
	504		273	0·750	36 52	104 14	135 57	+18 31	p.
	505		274	0·935	44 22	81 33	112 16	+18 31	Q.
	506		274	0·895	45 34	87 8	117 51	+16 14	q.
	507		269	0·320	210 52	163 25	194 8	−14 16	R.
	508		269	0·314	220 26	164 33	195 16	−11 27	r.
	509		269	0·362	214 48	166 18	197 1	−14 34	ę.
	510		269	0·292	223 39	169 28	200 11	−12 8	S.
	511		269	0·443	223 46	172 39	203 22	−13 7	s.
	512		270	0·311	356 55	154 21	185 4	+12 56	T.
	513		270	0·275	348 23	144 22	175 5	+11 37	t.
4.	514	123·482	274	0·382	14 3	135 55	137 36	+13 57	P.
	515		274	0·494	14 52	130 51	132 32	+19 10	p.
	516		274	0·569	20 38	124 46	126 27	+20 38	p′.
	517		274	0·718	35 21	109 21	111 2	+18 51	Q.
	518		269	0·668	228 40	190 41	192 22	−14 25	R.
	519		269	0·693	233 17	193 9	194 50	−11 38	r.
	520		269	0·776	233 34	200 21	202 2	−12 11	S.
	521		269	0·739	231 57	196 51	198 32	−13 2	s.
	522		270	0·436	286 44	169 36	171 17	+12 54	T.
	523		270	0·408	282 54	169 9	170 50	+10 33	t.
5.	524	124·496	275	0·954	53 36	79 38	66 55	+11 11	P.
	525		275	0·984	58 28	71 43	59 0	+ 7 16	p.
	526		269	0·820	231 14	204 19	191 36	−14 32	Q.
	527		269	0·835	234 38	207 12	194 29	−11 52	q.
	528		269	0·903	235 0	215 14	202 31	−11 56	R.
	529		270	0·602	274 48	183 41	170 58	+13 31	S.
	530		270	0·578	274 3	182 1	169 18	+12 55	s.
	531		274	0·303	338 57	150 1	137 18	+13 59	T.
	532		274	0·397	346 48	146 29	133 46	+19 20	U.
	533		274	0·487	3 18	137 8	124 25	+22 12	v.
	534		274	0·583	25 46	123 2	110 19	+19 9	U.
	535	124·530	275	0·947	53 37	80 56	67 2	+11 1	P.
	536		275	0·983	58 22	72 10	58 16	+ 7 22	p.
	537		269	0·835	231 20	206 58	193 4	−14 37	Q.
	538		269	0·852	234 39	209 4	195 10	−11 59	q.
	539		269	0·912	234 53	216 38	202 44	−12 6	R.
	540		270	0·619	273 33	185 13	171 19	+13 20	S.
	541		274	0·300	334 34	151 28	137 34	+13 49	T.
	542		274	0·395	343 26	147 59	134 5	+19 26	t.
	543		274	0·570	24 47	124 13	110 19	+19 4	U.
8.	544	127·492	275	0·562	41 9	122 51	67 39	+10 46	P.
	545		276	0·654	51 20	114 4	58 52	+10 26	p.
	546		277	0·941	83 50	83 34	28 22	−16 45	Q.
	547		277*a*	0·578	86 56	119 43	64 31	−13 54	R.
	548		273	0·673	272 34	192 22	137 10	+14 12	S.
	549		273	0·682	280 33	190 41	135 29	+19 23	s.
	550		274	0·643	284 35	186 32	131 20	+20 12	σ.
9.	551	128·474	275	0·396	27 33	136 35	67 27	+11 41	P.
	552		276	0·466	36 11	130 45	61 37	+11 4	p.
	553		278	0·476	44 35	128 25	59 17	+ 7 48	Q.
	554		277	0·844	84 58	97 46	28 38	−16 26	R.
	555		274	0·813	267 41	206 18	137 10	+14 22	S.
	556		274	0·820	273 48	205 35	136 27	+19 22	s.
	557		274	0·775	277 28	200 12	131 4	+20 35	σ.
10.	558	129·614	275	0·262	349 53	152 28	67 10	+11 48	P.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1862.					° ′	° ′	° ′	° ′	
May 10.	559	129·614	276	0·264	22 45	144 52	59 34	+ 7 46	p.
	560		278	0·988	77 13	74 27	349 9	− 9 51	Q^1.
	561		278	0·980	72 53	77 8	351 50	− 5 40	Q^2.
	562		278	0·810	72 51	101 40	16 22	− 5 55	Q^4.
	563		278	0·923	79 18	88 27	3 9	−11 51	Q^5.
	564		277	0·683	88 48	114 15	28 57	−16 26	R.
	565		273	0·930	264 57	222 14	136 56	+14 49	S.
	566	129·640	275	0·265	348 12	152 54	67 14	+12 1	P.
	567		276	0·264	21 15	145 12	59 32	+ 8 1	p.
	568		278	0·985	77 3	75 25	349 45	− 9 42	Q_1.
	569		278	0·979	72 36	77 36	351 56	− 5 24	Q_3.
	570		278	0·952	70 12	83 44	358 4	− 3 18	Q_3.
	571		278	0·805	72 27	102 15	16 35	− 5 36	Q_4.
	572		278	0·916	78 58	89 27	3 47	−11 31	Q_5.
	573		277	0·678	88 33	114 37	28 57	−16 11	R.
	574		273	0·931	265 5	222 20	136 40	+14 56	S.
13.	575	132·511	282	0·524	10 7	141 3	14 39	+23 49	P.
	576		279	0·662	80 29	117 34	351 10	− 9 56	Q.
	577		278	0·726	80 53	112 28	346 4	−10 49	q.
	578		281	0·211	86 25	146 52	20 28	− 6 20	R.
	579		281	0·268	83 20	143 25	17 1	− 6 31	r.
	580		277	0·238	175 51	155 24	29 0	−16 1	S.
	581		275	0·615	272 52	193 21	66 57	+12 30	T.
	582		276	0·525	271 54	187 37	61 13	+ 9 41	t.
	583	132·526	282	0·524	9 30	141 21	14 44	+24 0	P.
	584		279	0·664	80 19	117 26	350 49	− 9 50	Q.
	585		278	0·724	80 48	113 37	347 0	−10 45	q.
	586		278	0·878	86 16	97 52	331 15	−16 53	q^1.
	587		281	0·202	86 39	147 23	20 46	− 6 14	R.
	588		281	0·263	84 29	143 48	17 11	− 6 44	r.
	589		277	0·234	146 19	155 34	28 57	−15 51	S.
	590		275	0·620	272 37	193 45	67 8	+12 29	T.
16.	591	135·683	282	0·896	59 43	98 40	287 16	+ 7 32	P.
	592		279	0·647	90 51	123 2	311 38	−15 30	Q.
	593		279	0·552	91 11	129 47	318 23	−13 48	q.
	594		283	0·333	112 49	147 3	335 39	−15 27	r_1.
	595		281	0·256	103 10	149 5	337 41	−10 25	r_2.
	596		281	0·212	112 12	152 27	341 3	−10 33	r_3.
	597		278	0·118	171 33	162 57	351 33	− 8 54	R.
	598		277	0·462	239 39	188 41	17 17	− 6 29	S.
	599		277	0·649	227 28	200 7	28 43	−15 45	s.
	600		275	0·940	262 5	230 25	59 1	+11 9	T.
	601		275	0·912	260 56	226 16	54 52	+ 9 35	t.
	602		275	0·924	263 37	227 42	56 18	+12 16	r.
	603		275	0·981	262 51	239 2	67 38	+12 46	U.
17.	604	136·420	282	0·807	58 28	109 23	287 32	+ 7 33	P.
	605		279	0·606	82 26	125 34	303 43	− 9 32	Q.
	606		279	0·567	82 56	128 23	306 32	− 9 23	q.
	607		279	0·522	95 57	133 20	311 29	−15 20	q^1.
	608		281	0·159	140 23	159 11	337 20	−10 49	r.
	609		281	0·444	172 8	164 2	342 11	−10 15	r.
	610		278	0·223	217 56	173 15	351 24	− 8 52	R.
	611		277	0·608	241 14	199 20	17 29	− 6 48	S.
	612		277	0·763	230 28	210 36	28 45	−15 53	s.
	613		275	0·986	261 15	241 47	59 56	+11 10	T.
18.	614	137·442	282	0·644	55 26	124 24	288 4	+ 7 32	P.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1862.									
May 18.	615	137·442	279	0·414	86° 42′	139° 39′	303° 19′	− 8° 48′	Q.
	616		279	0·344	92 25	144 30	308 10	− 9 33	q.
	617		281	0·233	209 8	173 31	337 11	−10 47	ρ.
	618		281	0·315	223 8	179 46	343 26	−10 8	r.
	619		278	0·425	233 18	187 27	351 7	− 8 50	R.
	620		277	0·781	242 45	214 20	18 0	− 6 50	S.
	621		277	0·891	233 4	225 25	29 5	−15 51	s.
19.	622	138·434	282	0·462	48 44	138 31	288 6	+ 8 16	P.
	623		279	0·232	104 19	152 58	302 33	− 9 25	Q.
	624		279	0·167	113 35	157 8	306 43	− 8 32	q.
	625		281	0·421	255 57	187 8	336 43	−11 43	ρ.
	626		281	0·505	233 3	193 25	343 0	−10 14	r.
	627		278	0·607	238 20	200 58	350 33	− 8 42	R.
22.	628	141·427	285	0·933	85 22	98 31	205 39	−13 55	P.
	629		285	0·932	87 15	98 49	205 57	−15 39	p.
	630		285	0·988	88 16	86 11	193 19	−17 15	π.
	631		279	0·745	236 6	214 16	321 24	−12 11	Q.
	632		279	0·608	237 54	203 40	310 48	− 9 14	r.
	633		279	0·561	236 47	200 18	307 26	− 9 18	ρ.
	634		279	0·527	236 56	197 59	305 7	− 8 46	R.
	635		278	0·974	242 12	243 46	350 54	− 9 0	S.
	636		281	0·938	241 48	236 29	343 37	− 9 15	T.
	637		281	0·912	239 4	232 21	339 29	−11 36	t.
	638		282	0·294	285 28	181 6	288 14	+ 7 55	U.
	639	141·445	285	0·933	85 25	98 34	205 27	−13 57	P.
	640		285	0·932	87 16	98 52	205 45	−15 40	p.
	641		285	0·990	88 14	85 27	192 20	−17 13	π.
	642		279	0·747	236 31	214 27	321 20	−11 54	Q.
	643		279	0·611	237 53	203 55	310 48	− 9 18	r.
	644		279	0·564	236 51	200 30	307 23	− 9 18	ρ.
	645		279	0·530	236 53	198 12	305 5	− 8 50	R.
	646		278	0·973	242 13	243 36	350 29	− 9 0	S.
	647		281	0·939	242 2	236 39	343 32	− 9 2	T.
	648		281	0·914	239 17	232 44	339 37	−11 26	t.
	649		282	0·297	286 22	181 7	288 0	+ 8 14	U.
24.	650	143·492	287	0·957	48 52	97 59	175 50	+21 20	P.
	651		287	0·941	49 6	99 39	177 30	+22 8	p.
	652		286	0·829	91 44	116 20	194 11	−17 14	Q.
	653		286	0·779	90 52	119 17	197 8	−15 39	q.
	654		285	0·670	89 54	128 19	206 10	−13 4	R.
	655		285	0·661	93 15	129 35	207 26	−15 3	r.
	656		285	0·576	91 54	135 27	213 18	−12 35	ρ.
	657		282	0·677	266 12	210 18	288 9	+ 8 39	S.
26.	658	145·694	287	0·704	40 6	131 37	178 13	+21 12	P.
	659		287	0·778	41 3	125 5	171 41	+23 5	p.
	660		288	0·892	60 49	108 53	155 29	+ 9 51	Q.
	661		289	0·542	52 46	140 4	186 40	+ 9 31	R.
	662		289	0·497	44 11	144 34	191 10	+12 33	r.
	663		289	0·533	36 37	144 19	190 55	+17 6	ρ.
	664		285	0·497	106 27	145 39	192 15	−17 4	S.
	665		285	0·335	122 24	158 8	204 44	−15 54	t.
	666		285	0·262	124 54	161 39	208 15	−13 3	T.
	667		285	0·215	140 5	166 16	212 52	−12 33	U.
	668		285	0·190	152 6	169 4	215 40	−11 53	V.
29.	669	148·456	287	0·339	351 41	170 16	177 42	+21 44	P.
	670		287	0·436	4 23	163 59	171 25	+23 8	p.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1862.					° ′	° ′	° ′	° ′	
May 29.	671	148·456	288	0·389	56 27	151 52	159 18	+ 5 45	Q.
	672		288	0·490	52 8	146 16	153 42	+ 9 30	q.
	673		289	0·936	79 16	104 38	112 4	− 5 42	R.
	674		286	0·326	193 52	183 35	191 1	−17 2	S.
	675		285	0·419	228 40	196 23	203 49	−10 49	T.
	676		285	0·519	230 13	202 48	210 14	−12 29	t.
June 1.	677	151·485	293	0·960	65 59	103 14	67 42	+ 8 7	P.
	678		293	0·954	62 18	104 48	69 16	+11 33	p.
	679		292	0·755	61 0	128 34	93 2	+ 9 54	Q.
	680		292	0·784	60 46	126 2	90 30	+10 29	q.
	681		289	0·488	86 25	148 0	112 28	− 6 6	R.
	682		291	0·572	82 53	142 6	106 34	− 5 5	r.
	683		286	0·943	242 26	246 35	211 3	−11 35	S.
	684		288	0·340	270 5	195 39	160 7	+ 4 46	T.
	685		290	0·282	292 28	189 32	154 0	+ 9 30	t.
	686		287	0·445	283 30	199 55	164 23	+11 58	U.
	687		287	0·411	288 21	197 0	161 28	+12 44	u.
	688		287	0·654	289 2	211 49	176 17	+21 17	V.
2.	689	152·382	293	0·882	65 3	116 9	67 54	+ 8 32	P.
	690		293	0·872	61 33	117 42	69 27	+11 27	p.
	691		293	0·940	63 10	108 7	59 52	+10 54	π.
	692		292	0·608	57 36	141 28	93 13	+10 7	Q.
	693		292	0·645	57 42	138 47	90 32	+10 43	q.
	694		289	0·303	94 11	160 46	112 31	− 6 2	R.
	695		288	0·525	264 33	208 35	160 20	+ 4 44	S.
	696		288	0·448	277 10	202 9	153 54	+ 9 25	s.
	697		290	0·617	275 49	213 25	165 10	+12 33	T.
	698		290	0·563	278 59	209 9	160 54	+12 57	t.
	699		287	0·775	283 4	224 21	176 6	+21 9	U.
3.	700	153·734	293	0·352	43 59	161 1	93 35	+10 23	P.
	701		293	0·398	46 25	158 1	90 35	+10 57	p.
	702		292	0 699	62 50	135 15	67 49	+ 8 38	Q.
	703		292	0·737	62 31	132 13	64 47	+ 9 21	q.
	704		292	0·786	60 41	128 6	60 40	+11 25	q'.
	705		289	0·104	177 10	179 55	112 29	− 6 0	R.
	706		294	0·057	283 37	181 34	114 8	+ 1 22	r.
	707		294	0·058	12 51	177 10	109 44	+ 2 48	ρ.
	708		288	0·771	262 0	228 44	161 18	+ 4 55	S.
	709		290	0·782	272 44	228 36	161 10	+13 17	s.
	710		290	0·801	270 1	230 46	163 20	+11 29	σ.
	711		287	0·926	278 58	244 27	177 1	+21 35	T.
4.	712	154·438	293	0·577	59 54	145 19	67 54	+ 8 57	P.
	713		293	0·608	57 11	143 35	66 10	+11 2	p.
	714		293	0·662	59 59	139 13	61 48	+10 14	π.
	715		292	0·235	23 36	170 59	93 34	+10 33	Q.
	716		292	0·279	30 37	167 51	90 26	+11 15	q.
	717		289	0·204	227 4	189 42	112 17	− 5 41	R.
	718		294	0·230	263 2	192 32	115 7	+ 1 36	r.
	719		294	0·144	276 10	187 5	109 40	+ 2 47	ρ.
	720		288	0·863	261 27	238 42	161 17	+ 4 53	T.
	721		290	0·891	268 36	241 28	164 3	+11 23	U.
	722		287	0·971	277 56	254 6	176 41	+21 29	V.
	723	154·666	293	0·536	58 59	148 26	67 47	+ 8 51	P.
	724		293	0·572	56 23	146 26	65 47	+10 53	p.
	725		293	0·568	61 18	146 1	65 22	+ 8 6	p'.
	726		293	0·625	59 15	142 14	61 35	+10 11	π.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1862.					° ′	° ′	° ′	° ′	
June 4.	727	154·666	292	0·206	13 51	173 57	93 18	+10 24	Q.
	728		292	0·245	23 30	170 52	90 13	+11 4	q.
	729		289	0·250	232 5	192 49	112 10	− 5 48	R.
	730		294	0·285	260 47	196 1	115 22	+ 1 22	r.
	731		294	0·190	269 19	190 12	109 33	+ 2 29	s.
	732		288	0·886	261 26	241 38	160 59	+ 4 56	T.
	733		290	0·912	268 24	244 34	163 55	+11 25	U.
	734		287	0·975	277 55	255 22	174 43	+21 29	V.
7.	735	157·417	296	0·943	87 7	112 15	352 35	− 9 31	P.
	736		295	0·641	63 7	143 17	23 37	+ 8 59	Q.
	737		295	0·489	114 28	158 24	38 44	−17 0	R.
	738		294*a*	0·666	180 4	184 23	64 43	− 8 59	S.
	739		294*a*	0·152	173 31	183 13	63 33	− 8 23	s.
	740		289	0·770	249 5	232 4	112 24	− 5 49	T.
	741		294	0·823	257 59	237 25	117 45	+ 1 2	t.
	742		294	0·748	259 53	230 24	110 44	+ 2 24	τ.
	743		292	0·537	277 4	212 59	93 19	+10 52	U.
	744		292	0·491	281 26	209 14	89 34	+11 58	u.
	745		293	0·175	318 22	187 2	67 22	+ 9 5	V.
	746		293	0·195	333 45	184 47	65 7	+11 10	W.
	747		293	0·148	331 40	184 27	64 47	+ 8 28	X.
	748	157·447	296	0·941	87 7	112 35	352 29	− 9 29	P
	749		295	0·508	72 20	151 53	31 47	+ 2 34	Q'.
	750		295	0·480	114 56	159 3	38 57	−16 52	R.
	751		295	0·428	121 33	163 43	43 37	−17 10	r.
	752		294*a*	0·166	182 27	184 48	64 42	− 8 54	S.
	753		294*a*	0·150	175 54	183 35	63 29	− 8 11	s.
	754		289	0·772	249 4	232 14	112 8	− 5 51	T.
	755		294	0·825	258 6	237 36	117 30	+ 1 8	t.
	756		294	0·751	259 49	230 39	110 33	+ 2 21	τ.
	757		292	0·541	277 8	213 13	93 7	+10 58	U.
	758		292	0·497	281 32	209 39	89 33	+12 10	u.
	759		293	0·178	316 1	187 32	67 26	+ 9 3	V.
	760		293	0·195	332 17	185 6	65 0	+11 7	W.
	761		293	0·151	329 54	184 47	64 41	+ 8 32	X.
8.	762	158·447	296	0·489	58 7	155 22	21 5	+ 9 35	P.
	763		296	0·440	58 37	158 22	24 5	+ 8 27	p.
	764		296	0·842	88 54	126 37	352 20	− 9 29	Q.
	765		295	0·342	137 16	172 55	38 38	−16 44	R.
	766		295	0·310	152 47	178 32	44 15	−16 58	r.
	767		295	0·321	230 15	199 52	65 35	− 7 59	S.
	768		289	0·902	250 33	247 10	112 53	− 5 55	T.
	769		294	0·939	258 10	252 46	118 29	+ 0 55	t.
	770		294	0·880	259 58	244 38	110 21	+ 2 31	τ.
	771		292	0·716	272 19	227 48	93 31	+10 56	U.
	772		292	0·675	274 57	224 8	89 51	+12 4	u.
	773		293	0·353	283 2	201 53	67 36	+ 9 9	V.
	774		293	0·336	292 6	199 26	65 9	+11 23	W.
	775		293	0·306	285 25	198 56	64 39	+ 8 38	X.
	776	158·466	296	0·482	58 3	155 49	21 16	+ 9 30	P.
	777		296	0·436	58 17	158 41	24 8	+ 8 31	p.
	778		296	0·837	89 24	127 16	352 43	− 9 49	Q
	779		295	0·341	138 13	173 16	38 43	−16 50	R.
	780		295	0·311	153 49	178 51	44 18	−17 8	r.
	781		295	0·326	230 26	200 10	65 37	− 8 3	S.
	782		295	0·359	228 37	201 44	67 11	− 9 30	s.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Heliographical Longitude.	Heliographical Latitude.	Spot.
1862.					° ′	° ′	° ′	° ′	
June 8.	783	158·466	295	0·308	222 59	198 4	63 36	− 9 34	σ.
	784		289	0·902	250 24	247 12	112 39	− 6 4	T.
	785		294	0·938	258 16	252 38	118 5	+ 1 1	t.
	786		294	0·884	259 51	245 4	110 31	+ 2 25	τ.
	787		292	0·901	239 23	243 11	108 38	− 8 52	δ.
	788		292	0·719	272 17	228 0	93 27	+10 56	U.
	789		292	0·677	274 51	224 21	89 48	+12 1	u.
	790		293	0·356	282 40	202 9	67 36	+ 9 7	V.
	791		293	0·339	291 40	199 43	65 10	+11 22	W.
	792		293	0·309	284 44	199 15	64 42	+ 8 32	X.
9.	793	159·594	296	0·220	35 2	174 54	24 21	+ 9 5	P.
	794		296	0·272	42 49	171 18	20 45	+ 9 27	p.
	795		296	0·678	93 0	142 48	352 15	− 9 49	Q.
	796		295	0·308	182 32	188 57	38 24	−16 43	R.
	797		295	0·358	198 59	195 24	44 51	−17 12	r.
	798		297	0·518	237 35	213 41	63 8	− 9 48	S.
	799		297	0·604	240 19	219 53	69 20	− 9 58	s.
	800		289	0·982	251 22	263 3	112 30	− 6 12	T.
	801		292	0·871	269 38	244 9	93 36	+10 32	U.
	802		292	0·845	271 47	241 1	90 28	+12 3	u.
	803		292	0·863	275 45	242 31	91 58	+15 41	u′.
	804		293	0·575	272 58	218 17	67 44	+ 9 4	V.
	805		293	0·547	277 54	215 45	65 12	+11 16	W.
	806		293	0·526	272 56	215 2	64 29	+ 8 20	X.
	807		293	0·675	280 15	224 23	73 50	+15 18	Y.
	808		293	0·656	282 9	222 34	72 1	+16 4	Z.
10.	809	160·431	299	0·837	44 11	133 18	330 52	+28 11	P.
	810		299a	0·539	97 7	154 3	351 37	− 9 30	Q.
	811		295	0·320	173 16	186 47	24 21	−17 51	r.
	812		295	0·293	196 1	193 10	30 44	−14 19	R.
	813		295	0·389	208 49	200 19	37 53	−16 27	S.
	814		295	0·471	217 52	207 3	44 37	−17 3	T.
	815		297	0·737	244 20	231 24	68 58	− 9 39	U.
	816		297	0·658	242 31	224 53	62 27	− 9 41	u.
	817		293	0·711	270 12	229 37	67 11	+ 8 58	V.
	818		293	0·685	274 11	227 6	64 40	+11 22	W.
	819		293	0·667	269 59	227 12	64 46	+ 8 20	X.
	820		293	0·569	272 27	218 48	56 22	+ 8 35	Y.
	821		293	0·529	273 57	215 57	53 31	+ 8 49	υ.
	822		292	0·942	269 16	254 59	92 33	+10 41	Z.
11.	823	161·410	301	0·970	100 43	111 44	295 25	−21 14	P.
	824		299	0·349	108 26	168 12	351 53	− 9 15	Q.
	825		298	0·438	220 8	206 44	30 25	−15 2	r.
	826		298	0·472	224 15	209 40	33 21	−14 40	R.
	827		295	0·546	225 13	214 18	37 59	−16 43	s.
	828		295	0·642	229 19	221 50	45 31	−17 35	S.
	829		302	0·430	264 22	211 13	34 54	+ 3 10	T.
	830		302	0·387	264 37	208 33	32 14	+ 3 2	t.
	831		297	0·884	247 14	247 19	71 0	− 9 36	U.
	832		297	0·816	246 11	239 41	63 22	− 9 35	u.
	833		293	0·857	268 39	244 25	68 6	+ 9 1	V.
	834		293	0·835	272 6	240 51	64 32	+ 8 18	W.
	835		293	0·821	268 8	240 39	64 20	+ 8 17	X.
	836		293	0·766	270 35	235 17	58 58	+ 9 40	Y.
	837		293	0·697	275 20	228 50	52 31	+12 8	Z.
13.	838	163·482	303	0·910	90 37	123 21	277 39	− 9 39	P.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1862.									
June 13.	839	163·482	303	0·875	89° 57′	127° 40′	281° 58′	− 8° 41′	p.
	840		301	0·862	108 27	132 32	286 50	−24 4	Q.
	841		301	0·800	108 24	139 5	293 23	−22 4	q.
	842		301	0·773	106 48	141 9	295 27	−20 5	q'.
	843		300	0·240	214 51	197 52	352 10	− 8 40	R.
	844		302	0·812	240 19	240 17	34 35	−14 46	S.
	845		302	0·911	239 31	251 41	45 59	−17 35	s.
	846		295	0·991	267 14	270 7	64 25	+ 7 53	T.
	847		298	0·406	276 14	210 55	5 13	+ 7 38	U.
	848		298	0·469	269 51	215 26	9 44	+ 5 44	u.
14.	849	164·454	304	0·980	89 25	110 52	251 23	− 9 9	P.
	850		303	0·798	93 0	137 4	277 35	− 9 45	Q.
	851		303	0·761	93 16	140 28	280 59	− 9 24	q.
	852		301	0·764	113 45	144 56	285 27	−24 20	R.
	853		301	0·668	115 10	153 6	293 37	−21 44	r.
	854		301	0·629	113 55	155 25	295 56	−19 38	ε.
	855		302	0·424	236 41	212 0	352 31	− 8 32	S.
	856		302	0·999	241 52	265 50	46 21	−17 19	T.
	857		298	0·601	272 45	255 3	5 34	+ 8 36	U.
	858		298	0·668	267 42	230 23	10 54	+ 6 4	u.
15.	859	165·490	304	0·888	90 37	127 59	253 48	− 8 34	P.
	860		305	0·952	90 25	118 26	244 15	− 9 14	p.
	861		303	0·632	98 4	152 21	278 10	−10 6	Q.
	862		303	0·585	98 43	155 46	281 35	− 9 35	q.
	863		301	0·631	123 38	159 41	285 30	−24 30	R.
	864		301	0·528	128 7	167 31	293 20	−21 46	r.
	865		301	0·491	128 18	169 29	295 18	−20 8	ε.
	866		300	0·623	244 5	229 59	355 48	− 8 59	S.
	867		302	0·831	266 38	245 49	11 38	+ 5 57	T.
	868		302	0·776	271 18	240 12	6 1	+ 9 19	t.
16.	869	166·507	306	0·278	8 33	185 45	297 8	+16 38	P.
	870		305	0·748	94 2	143 34	254 57	− 8 57	Q.
	871		304	0·858	92 7	132 39	244 2	− 9 1	q.
	872		303	0·448	107 7	214 45	326 8	−10 11	R.
	873		303	0·398	109 48	170 14	281 37	− 9 50	r.
	874		303	0·399	101 33	168 50	280 13	− 6 52	ε.
	875		301	0·513	138 6	173 15	284 38	−24 11	S.
	876		301	0·425	148 59	181 6	292 29	−21 47	T.
	877		301	0·382	152 43	183 37	295 0	−19 52	U.
	878		301	0·945	266 51	261 28	12 51	+ 6 14	V.
17.	879	167·417	306	0·736	94 47	145 30	243 59	− 8 57	p.
	880		305	0·588	98 52	157 16	255 45	− 9 4	P.
	881		304	0·321	124 14	177 54	276 23	−11 12	q.
	882		303	0·282	127 31	180 20	278 49	−10 17	Q.
	883		303	0·248	135 5	183 12	281 41	−10 3	q'.
	884		301	0·444	158 44	185 43	284 12	−24 6	R.
	885		301	0·403	170 1	191 12	289 41	−22 10	S.
	886		301	0·379	183 46	196 44	295 13	−20 9	T.
18.	887	168·424	307*a*	0·789	92 45	141 30	225 42	− 7 47	p.
	888		307*a*	0·817	95 23	139 6	223 18	−10 17	ε.
	889		307	0·546	87 44	159 51	244 3	− 2 1	P.
	890		305	0·566	100 4	159 51	244 3	− 8 58	q.
	891		304	0·464	105 26	167 18	251 30	− 9 20	q'.
	892		304	0·392	108 12	171 59	256 11	− 8 35	Q.
	893		303	0·221	168 24	191 55	276 7	−11 4	ε.
	894		303	0·208	179 58	194 24	278 36	−10 14	r.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1862.									
June 18.	895	168·424	303	0·218	193° 29′	197° 21′	281° 35′	−10° 2′	R.
	896		301	0·486	184 1	199 20	283 32	−26 37	S.
	897		301	0·451	184 4	198 45	282 57	−24 25	S′.
	898		301	0·459	194 8	205 25	289 37	−22 34	T.
	899		301	0·476	209 50	210 53	295 5	−20 21	U.
23.	900	173·470	317	0·927	70 27	129 52	142 29	+13 11	p.
	901		316	0·871	74 47	137 3	149 40	+ 8 55	P.
	902		315	0·795	72 57	145 10	157 47	+ 9 55	P_1.
	903		314	0·705	77 12	152 48	165 25	+ 6 11	P_2.
	904		314	0·612	71 54	160 17	172 54	+ 8 58	π.
	905		313	0·664	97 10	156 56	169 33	− 7 8	Q.
	906		313	0·720	102 44	153 26	166 3	−11 55	Q_1.
	907		313	0·761	98 18	149 15	161 52	− 9 30	q.
	908		304	0·341	232 56	214 30	227 7	− 7 57	R.
	909		305	0·268	218 56	208 28	221 5	− 8 44	r
	910		301	0·967	234 45	269 15	281 52	−27 19	S.
	911		303	0·773	253 5	247 5	259 42	− 6 50	T.
	912		303	0·645	247 17	236 0	248 37	− 8 50	t.
	913		311	0·659	260 48	238 23	251 0	− 0 19	U.
	914		311	0·525	257 45	228 44	241 21	− 1 18	u.
	915		309	0·388	304 44	215 8	227 45	+16 44	V.
25.	916	175·718	317	0·692	65 30	157 8	137 52	+15 1	p.
	917		317	0·640	65 28	161 13	141 57	+14 7	π.
	918		316	0·526	70 43	168 30	149 14	+ 9 26	P.
	919		315	0·404	66 56	176 42	157 26	+ 9 21	P_1.
	920		314	0·275	67 57	184 14	164 58	+ 6 55	P_2.
	921		314	0·173	42 10	192 10	172 54	+ 9 2	P_3.
	922		313	0·903	93 24	135 42	116 26	− 6 39	Q.
	923		318	0·243	124 22	188 42	169 26	− 6 29	R.
	924		318	0·353	127 49	184 21	165 5	−11 30	r.
	925		305	0·741	251 50	246 9	226 53	− 7 54	S.
	926		311	0·906	251 19	263 13	243 57	−11 8	t.
	927		311	0·986	257 5	279 18	260 2	− 7 12	T.
	928		310	0·938	262 55	269 0	249 44	− 0 56	U.
	929		310	0·878	262 20	260 39	241 28	− 1 0	u.
26.	930	176·422	317	0·576	61 49	167 6	137 51	+15 15	p.
	931		317	0·516	60 58	171 15	142 0	+14 22	π.
	932		316	0·386	66 25	178 24	149 19	+ 9 26	P.
	933		315	0·264	58 26	186 27	157 12	+ 9 14	P_1.
	934		314	0·131	47 34	194 12	164 57	+ 7 5	P_2.
	935		313	0·118	336 25	202 24	173 9	+ 8 54	P_3.
	936		318	0·826	94 43	145 24	116 9	− 6 26	Q.
	937		309	0·159	166 47	198 53	169 38	− 6 29	R.
	938		309	0·266	152 0	194 7	164 52	−11 34	r.
	939		305	0·840	253 47	256 11	227 1	− 8 6	S.
	940		310	0·981	263 32	278 35	249 20	− 1 4	T.
	941		310	0·944	263 10	270 33	241 18	− 1 1	t.
27.	942	177·650	320	0·979	75 57	123 2	76 22	+10 5	P.
	943		319	0·872	73 10	140 59	94 19	+12 13	p.
	944		318	0·637	98 50	162 56	116 16	− 6 13	Q.
	945		317	0·375	54 1	182 17	135 37	+13 49	r.
	946		317	0·302	40 59	188 36	141 56	+14 48	ρ.
	947		316	0·148	33 56	196 5	149 25	+ 9 17	R.
	948		315	0·115	337 48	203 25	156 45	+ 8 55	R_1.
	949		314	0·192	290 13	211 29	164 49	+ 7 10	R_2.
	950		314	0·334	286 4	219 48	173 8	+ 9 10	R_3.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Heliographical Longitude.	Heliographical Latitude.	Spot.
1882.									
June 27.	951	177·650	305	0·299	209° 46′	211° 11′	164° 31′	−11° 40′	S.
	952		310	0·961	255 58	274 21	227 41	− 8 35	T.
28.	953	178·537	320	0·921	76 5	135 13	75 58	+10 18	P.
	954		319	0·763	71 59	153 8	93 53	+12 30	p.
	955		318	0·476	103 55	175 15	116 0	− 5 53	Q.
	956		316	0·154	316 8	207 56	148 41	+ 9 28	R.
	957		315	0·247	292 45	215 6	155 51	+ 9 2	S.
	958		314	0·369	278 44	223 24	164 9	+ 7 10	T.
	959		314	0·506	280 2	231 56	172 41	+ 9 22	t.
30.	960	180·430	320	0·668	74 36	162 37	76 31	+10 26	P.
	961		319	0·623	128 57	174 10	88 4	−21 58	Q.
	962		319	0·550	129 16	178 39	92 33	−18 57	q.
	963		318	0·156	168 54	202 45	116 39	− 5 54	R.
	964		318	0·142	142 51	199 27	113 21	− 3 47	r.
	965		318	0·096	179 29	209 30	123 24	+ 2 43	s.
	966		316	0·523	280 6	234 58	148 52	+ 9 20	S.
	967		315	0·622	277 26	242 6	156 0	+ 8 48	T.
	968		314	0·735	273 28	251 9	165 3	+ 6 46	U.
July 1.	969	181·656	320	0·439	69 12	180 10	76 41	+10 43	P.
	970		320	0·775	72 15	155 3	51 34	+13 44	p.
	971		320	0·809	69 41	152 2	48 33	+16 12	p′.
	972		319	0·395	161 14	198 28	94 59	−19 6	Q.
	973		318	0·300	237 29	220 14	116 45	− 5 37	R.
	974		316	0·737	277 55	252 22	148 53	+ 9 44	S.
	975		315	0·810	276 28	259 7	155 38	+ 9 3	T.
	976		314	0·895	274 3	268 36	165 7	+ 7 13	U.
5.	977	185·618	322	0·972	102 18	133 56	334 15	−11 40	P.
	978		323	0·341	235 7	225 23	65 42	− 7 38	Q.
	979		323	0·520	225 28	232 3	72 22	−17 49	R.
	980		321	0·736	241 22	251 47	92 6	−17 35	S.
	981		321	0·717	235 34	248 22	88 41	−20 43	s.
	982		318	0·927	261 59	276 5	116 24	− 5 27	T.
	983		320	0·462	287 18	235 27	75 46	+11 19	U.
	984		324	0·173	285 15	218 30	58 49	+ 6 10	V.
	985		324	0·122	287 41	215 36	55 55	+ 5 42	W.
6.	986	186·509	322	0·900	104 44	147 24	335 4	−11 46	P.
	987		323	0·565	251 45	242 26	70 6	− 6 59	Q.
	988		323	0·504	247 32	237 47	65 27	− 7 46	q.
	989		324	0·647	235 58	244 6	71 46	−18 3	R.
	990		324	0·724	244 0	252 16	79 56	−15 38	r.
	991		324	0·805	244 37	259 26	87 6	−17 37	S.
	992		324	0·833	246 8	262 39	90 19	−17 17	s.
	993		318	0·985	263 44	289 17	116 57	− 5 16	T.
	994		320	0·628	283 15	248 6	75 46	+11 15	U.
	995		321	0·385	277 37	232 14	59 54	+ 6 21	V.
	996		321	0·311	278 2	227 42	55 22	+ 5 59	W.
	997	186·545	322	0·899	104 52	147 33	334 43	−11 51	P.
	998		323	0·573	252 3	243 0	70 10	− 6 59	Q.
	999		323	0·512	247 52	238 19	65 29	− 7 47	q.
	1000		324	0·652	236 25	244 37	71 47	−17 58	R.
	1001		324	0·729	244 23	252 48	79 50	−15 32	r.
	1002		324	0·811	244 56	260 7	87 17	−17 33	S.
	1003		320	0·634	282 55	248 37	75 47	+11 6	U.
	1004		321	0·321	277 13	228 21	55 31	+ 5 48	W.
8.	1005	188·632	326	0·827	83 20	156 1	313 35	+ 8 16	P.
	1006		326	0·845	83 17	154 8	311 42	+ 8 20	p.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1862.									
July 8.	1007	188·632	325	0·878	103° 10′	151° 49′	309° 23′	− 8° 59′	Q.
	1008		325	0·925	103 4	145 26	303 0	− 9 53	q′.
	1009		325	0·927	106 30	145 48	303 22	−13 2	q″.
	1010		325	0·576	115 57	179 51	337 25	−10 55	q.
	1011		321	0·845	259 42	268 16	65 50	− 7 10	R.
	1012		321	0·892	261 44	273 57	71 31	− 6 13	r.
	1013		323	0·928	251 26	277 21	74 55	−16 15	S.
	1014		323	0·952	251 53	281 36	79 10	−16 34	s.
	1015		320	0·914	280 59	277 58	75 32	+10 56	T.
11.	1016	191·600	326	0·283	75 46	198 46	314 14	+ 8 30	P.
	1017		326	0·318	77 30	196 33	312 1	+ 8 30	p.
	1018		331	0·977	92 12	137 12	252 40	+ 0 47	Q.
	1019		328	0·922	102 43	143 36	264 4	− 8 8	q.
	1020		329	0·473	126 51	191 20	306 48	−11 40	R.
	1021		329	0·412	124 6	194 1	309 29	− 8 42	r.
	1022		329	0·585	123 16	183 42	299 10	−14 2	?.
	1023		322	0·323	173 9	211 40	327 8	−14 25	S.
	1024		322	0·315	190 45	217 25	332 53	−13 55	T.
	1025		322	0·292	210 20	222 40	338 8	−10 40	U.
13.	1026	193·472	331	0·793	95 5	163 12	252 7	+ 9 14	P.
	1027		330	0·680	109 37	257 19	346 14	− 7 59	p.
	1028		325	0·277	189 45	218 19	307 14	−11 33	Q.
	1029		325	0·239	201 7	219 49	308 44	− 8 45	q.
	1030		325	0·344	174 29	213 25	302 20	−15 29	q′.
	1031		332	0·599	249 36	250 15	339 10	−10 6	R.
	1032		327	0·551	239 22	244 31	333 26	−13 52	r.
	1033		327	0·492	231 41	238 50	327 45	−14 53	r′.
	1034		327	0·381	224 39	236 22	325 17	−17 22	?.
	1035		326	0·178	296 12	225 51	314 46	+ 8 16	S.
	1036		326	0·137	305 8	223 6	312 1	+ 8 27	s.
14.	1037	194·465	331	0·634	97 14	178 17	253 7	+ 0 59	P.
	1038		330	0·503	117 49	189 54	264 44	− 8 4	Q.
	1039		325	0·378	227 14	233 23	308 13	−10 54	R.
	1040		325	0·376	236 6	234 52	309 42	− 8 53	r.
	1041		325	0·386	209 3	227 19	302 9	−15 55	?.
	1042		327	0·614	247 36	251 37	326 27	−11 48	S.
	1043		327	0·544	243 39	246 2	320 52	−11 48	s.
	1044		322	0·772	256 27	265 46	340 36	−10 1	T.
	1045		322	0·736	251 17	261 35	336 25	−12 52	t.
	1046		326	0·394	283 55	240 15	315 5	+ 8 11	U.
15.	1047	195·437	331	0·443	100 34	192 17	253 19	+ 1 4	P.
	1048		330	0·330	134 28	203 43	264 45	− 8 0	Q.
	1049		325	0·541	243 55	246 45	307 47	−11 57	R.
	1050		325	0·547	249 34	248 28	309 30	− 9 2	r.
	1051		327	0·699	251 53	259 33	320 35	−11 42	S.
	1052		327	0·771	253 43	265 52	326 54	−12 16	T.
	1053		322	0·900	260 5	280 45	341 47	−10 14	U.
	1054		326	0·585	281 17	253 58	315 0	+ 8 2	V.
16.	1055	196·444	331	0·227	108 49	206 30	253 15	+ 7 45	P.
	1056		331	0·308	147 3	208 22	255 7	− 9 39	Q.
	1057		331	0·365	139 45	204 12	250 57	−10 35	q.
	1058		332	0·217	179 24	218 10	264 55	− 7 49	R.
	1059		330	0·382	246 1	238 55	285 40	− 6 2	S.
	1060		330	0·412	245 35	240 28	287 13	− 7 3	s.
	1061		334	0·707	252 50	261 16	308 1	−11 31	T.
	1062		334	0·718	257 5	263 3	309 48	− 8 53	t.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1862.					° ′	° ′	° ′	° ′	
July 16.	1063	196·444	325	0·846	256 37	274 35	321 20	−12 7	U.
	1064		327	0·897	258 17	280 51	327 36	−12 5	V.
	1065		327	0·980	262 25	296 28	343 13	−10 35	W.
	1066		326	0·712	278 58	264 36	311 21	+ 6 36	X.
	1067		326	0·724	280 50	265 35	312 20	+ 7 57	x.
	1068		326	0·760	281 20	268 40	315 25	+ 8 22	Y.
17.	1069	197·432	336	0·986	86 26	139 18	172 2	+ 8 54	P.
	1070		336	0·988	84 32	138 31	171 15	+10 44	p.
	1071		331	0·064	193 22	220 43	253 27	+11 6	Q.
	1072		332	0·272	177 15	218 5	250 49	−15 31	R.
	1073		332	0·250	199 53	223 58	256 42	− 9 14	r.
	1074		330	0·300	228 9	232 9	264 53	− 7 57	S.
	1075		334	0·551	258 59	252 12	284 56	− 4 34	t.
	1076		334	0·589	254 25	253 56	286 40	− 7 51	r.
	1077		334	0·628	257 59	257 15	289 59	− 6 37	T.
	1078		325	0·844	257 12	275 19	308 3	−12 0	U.
	1079		327	0·939	259 17	288 9	320 53	−12 42	v.
	1080		327	0·972	260 47	294 53	327 37	−12 17	V.
	1081		326	0·871	279 38	280 50	313 34	+ 6 38	W.
	1082		326	0·859	281 10	279 25	312 9	+ 7 59	w.
	1083		335	0·384	304 31	240 15	272 59	+15 22	X.
	1084		335	0·351	303 34	238 33	271 17	+14 10	x.
18.	1085	198·417	336	0·965	83 19	145 51	164 37	+12 38	P.
	1086		337	0·925	87 47	153 19	172 5	+ 8 36	p.
	1087		331	0·238	261 3	234 12	252 58	+ 1 24	Q.
	1088		332	0·377	237 32	238 38	257 24	− 8 41	R.
	1089		333	0·320	219 44	231 43	250 29	−10 30	r.
	1090		330	0·465	248 38	245 50	264 36	− 7 36	S.
	1091		334	0·801	262 38	272 55	291 41	− 7 0	T.
	1092		334	0·734	262 35	267 2	285 48	− 5 49	t.
	1093		325	0·940	260 37	289 15	308 1	−11 50	r.
	1094		326	0·959	280 44	294 43	313 29	+ 6 45	U.
	1095		326	0·953	282 41	293 40	312 26	+ 8 39	u.
	1096		335	0·512	297 36	250 22	269 8	+15 26	V.
	1097		335	0·575	294 13	255 11	273 57	+14 47	W.
20.	1098	200·490	336	0·827	88 12	167 11	156 33	+ 9 15	p.
	1099		336	0·747	83 25	174 55	164 17	+12 41	P.
	1100		337	0·685	86 12	179 55	169 17	+10 19	Q.
	1101		337	0·625	87 9	184 29	173 51	+ 9 25	q.
	1102		331	0·657	272 24	263 53	253 15	+ 1 24	R.
	1103		332	0·649	254 48	260 45	250 7	− 9 37	S.
	1104		330	0·738	260 28	268 44	258 6	− 7 55	T.
	1105		330	0·768	265 50	272 13	261 35	− 4 33	t.
	1106		334	0·989	268 42	303 33	292 55	− 6 26	U.
	1107		335	0·835	290 59	279 32	268 54	+15 12	V.
	1108		335	0·876	289 36	284 22	273 44	+14 16	W.
	1109		338	0·945	294 32	294 21	283 43	+19 6	X.
21.	1110	201·458	336	0·686	88 21	180 43	156 21	+ 9 13	p.
	1111		336	0·581	81 52	188 57	164 35	+12 32	P.
	1112		337	0·506	85 10	193 52	169 30	+10 2	Q.
	1113		337	0·435	84 55	198 29	174 7	+ 9 32	q.
	1114		332	0·791	259 22	273 49	249 27	−10 10	R.
	1115		330	0·868	264 4	282 39	258 17	− 8 5	S.
	1116		331	0·811	274 41	277 58	253 36	+ 1 33	T.
	1117		335	0·963	289 35	298 46	274 24	+14 0	U.
	1118		335	0·932	290 44	293 3	268 41	+15 8	u.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Heliographical Longitude.	Heliographical Latitude.	Spot.
1862.									
July 21.	1119	201·458	338	0·982	294° 52′	303° 55′	279° 33′	+19° 1′	V.
	1120		339	0·638	306 34	260 37	236 15	+22 43	W.
	1121		339	0·635	303 32	261 2	236 40	+20 48	X.
25.	1122	205·447	340	0·748	81 27	179 47	98 50	+16 1	P.
	1123		342	0·820	86 6	172 41	91 44	+12 57	Q.
	1124		342	0·826	80 24	172 23	91 26	+17 39	R.
	1125		341	0·524	77 0	172 7	91 10	+20 31	S.
	1126		337	0·452	282 29	254 43	173 46	+ 6 45	T.
	1127		336	0·416	294 5	251 46	170 49	+11 27	U.
	1128		337	0·383	285 18	250 14	169 17	+ 7 42	V.
	1129		336	0·331	298 52	246 11	165 14	+11 49	W.
	1130	205·608	340	0·721	81 44	182 18	99 4	+15 35	P.
	1131		340	0·724	83 45	181 53	98 39	+14 10	P_1
	1132		342	0·801	86 23	174 41	91 27	+12 41	Q.
	1133		342	0·803	80 36	174 51	91 37	+17 19	R.
	1134		341	0·495	76 46	199 35	116 21	+15 9	S.
	1135		337	0·483	282 8	256 54	173 40	+ 6 37	T.
	1136		336	0·446	293 11	253 55	170 41	+11 26	U.
	1137		337	0·414	284 23	252 22	169 8	+ 7 27	V.
	1138		336	0·364	296 27	248 32	165 18	+11 34	W.
26.	1139	206·446	340	0·578	78 28	194 35	99 28	+15 57	P.
	1140		340	0·572	80 20	194 42	99 35	+14 50	P_1.
	1141		342	0·680	85 41	186 7	91 0	+12 44	Q.
	1142		342	0·704	82 24	184 29	89 22	+15 13	R.
	1143		342	0·673	78 23	187 25	92 18	+17 32	S.
	1144		343	0·972	112 22	154 9	59 2	−12 0	T.
	1145		337	0·644	282 24	268 55	173 48	+ 6 38	U.
	1146		336	0·605	290 22	265 46	170 39	+11 27	V.
	1147		336	0·532	291 26	260 35	165 28	+11 26	W.
	1148	206·625	340	0·544	77 52	197 9	99 30	+15 46	P.
	1149		340	0·540	79 40	197 14	99 35	+14 45	P_1.
	1150		342	0·652	86 0	188 27	90 48	+12 22	Q.
	1151		342	0·677	82 34	186 52	89 13	+14 52	R.
	1152		342	0·643	78 1	190 0	92 21	+17 21	S.
	1153		343	0·966	113 8	155 47	58 8	−12 26	T.
	1154		337	0·679	282 27	271 44	174 5	+ 6 36	U.
	1155		336	0·638	289 57	268 27	170 48	+11 24	V.
	1156		336	0·565	290 35	263 7	165 28	+11 14	W.
27.	1157	207·430	340	0·386	71 25	209 0	99 55	+15 27	P.
	1158		342	0·512	83 37	199 29	90 24	+12 34	Q.
	1159		342	0·538	79 35	198 8	89 3	+15 0	R.
	1160		342	0·501	72 38	201 43	92 38	+17 43	S.
	1161		343	0·899	115 3	167 57	58 52	−11 48	T.
	1162		343	0·946	115 51	161 7	52 2	−13 56	t.
	1163		345	0·969	112 50	155 56	46 51	−11 54	r.
	1164		337	0·801	282 49	283 3	173 58	+ 6 23	U.
	1165		336	0·770	289 11	280 8	171 3	+11 22	V.
	1166		336	0·710	289 8	274 56	165 51	+11 5	W.
28.	1167	208·458	340	0·201	41 39	224 21	100 42	+15 19	P.
	1168		342	0·304	79 39	213 49	90 10	+11 15	Q.
	1169		342	0·343	69 45	212 49	89 10	+15 5	R.
	1170		342	0·320	56 0	216 35	92 56	+18 7	S.
	1171		346	0·972	111 31	220 22	96 43	−10 20	T.
	1172		343	0·774	119 0	182 51	59 12	−11 16	U.
	1173		345	0·862	117 4	173 47	50 8	−12 5	V.
	1174		344	0·667	254 0	268 37	144 58	−12 10	W.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1862.					° ′	° ′	° ′	° ′	
July 28.	1175	208·458	344	0·625	252 9	265 10	141 31	−11 58	X.
	1176		336	0·923	283 51	298 16	174 37	+ 6 15	Y.
	1177		337	0·905	289 12	295 55	172 16	+11 14	Z.
29.	1178	209·444	340	0·200	332 59	238 44	101 6	+14 48	P.
	1179		342	0·120	44 20	227 40	90 2	+11 21	Q.
	1180		342	0·184	37 40	226 35	88 57	+14 59	R.
	1181		342	0·218	16 19	230 10	92 32	+18 8	S.
	1182		346	0·898	114 15	169 48	52 10	−10 15	T.
	1183		343	0·626	124 54	196 32	58 54	−10 42	U.
	1184		345	0·736	124 58	188 27	50 49	−14 3	v.
	1185		345	0·776	120 37	183 57	46 19	−12 8	V.
	1186		344	0·775	257 58	279 3	141 25	−12 54	W.
	1187		344	0·805	260 42	282 24	144 46	−11 40	X.
	1188		337	0·976	289 31	309 36	171 58	−10 41	Z.
30.	1189	210·470	347	0·949	111 27	162 27	10 15	− 8 43	P.
	1190		346	0·771	118 0	184 39	32 27	− 9 44	Q.
	1191		345	0·584	133 57	202 45	50 33	−13 51	R.
	1192		345	0·622	128 17	198 38	46 26	−12 10	r.
	1193		343	0·484	140 7	210 21	58 9	−12 41	S.
	1194		343	0·441	138 7	212 2	59 50	−10 14	s.
	1195		344	0·893	262 30	293 15	141 3	−12 59	T.
	1196		340	0·378	305 18	253 16	101 4	+14 36	U.
	1197		342	0·196	311 36	242 26	90 14	+11 32	V.
	1198		342	0·217	330 31	240 52	88 40	+15 17	W.
	1199		342	0·295	328 36	244 29	92 17	+18 19	X.
	1200	210·661	347	0·936	112 16	165 4	10 9	− 9 0	P.
	1201		346	0·747	119 18	187 11	32 16	−10 0	Q.
	1202		345	0·557	136 36	205 22	50 27	−14 2	R.
	1203		345	0·592	130 40	201 25	46 30	−12 25	r.
	1204		343	0·458	143 56	213 1	58 6	−12 54	S.
	1205		343	0·410	142 32	214 54	59 59	−10 26	s.
	1206		344	0·915	263 24	296 30	141 35	−12 54	T.
	1207		344a	0·523	226 9	255 40	100 45	−15 57	T.
	1208		340	0·413	303 18	255 50	100 55	−14 34	U.
	1209		342	0·234	306 27	245 8	90 13	+11 32	V.
	1210		342	0·244	323 29	243 24	88 29	+15 16	W.
	1211		342	0·324	323 2	247 15	92 20	− 8 14	X.
31.	1212	211·495	348	0·985	114 4	155 20	348 36	−12 3	P.
	1213		347	0·850	114 57	177 21	10 37	− 8 53	p.
	1214		346	0·614	124 40	199 4	32 20	− 9 38	Q.
	1215		345	0·456	144 14	213 54	47 10	−12 45	R.
	1216		345	0·420	145 18	215 56	49 12	−11 31	r.
	1217		345	0·431	150 9	216 57	50 13	−11 28	ρ.
	1218		343	0·354	166 39	225 6	58 22	−12 58	S.
	1219		343	0·298	170 56	227 46	61 2	−10 26	s.
	1220		349	0·571	236 41	259 6	92 22	−18 0	T.
	1221		340	0·564	297 56	267 17	100 33	+14 30	U.
	1222		342	0·484	301 26	261 26	94 42	+15 1	V.
	1223		342	0·408	296 27	257 6	90 22	+11 43	W.
	1224		342	0·391	306 8	254 57	88 13	+15 6	X.
Aug. 1.	1225	212·455	348	0·927	116 36	168 48	348 27	−12 0	P.
	1226		347	0·714	119 41	191 30	11 9	− 8 51	Q.
	1227		346	0·453	136 5	212 33	32 12	− 9 32	R.
	1228		345	0·345	169 54	227 14	46 53	−12 19	S.
	1229		345	0·311	175 54	229 48	49 27	−11 29	s.
	1230		345	0·344	178 10	230 2	49 41	−13 38	σ.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1862.									
Aug. 1.	1231	212·455	343	0·329	203° 50′	238° 48′	58° 27′	−12° 44′	T.
	1232		343	0·260	203 39	237 49	57 28	− 8 43	t.
	1233		343	0·309	215 2	241 50	61 29	−10 26	r.
	1234		349	0·730	247 49	274 37	94 16	−19 1	U.
	1235		340	0·721	295 19	280 28	100 7	+14 21	V.
	1236		342	0·674	297 6	276 30	96 9	+15 9	W.
	1237		342	0·591	294 36	270 29	90 8	+12 45	X.
	1238		342	0·569	299 18	268 27	88 6	+15 10	Y.
2.	1239	213·517	351	0·975	114 41	160 13	324 48	−11 25	P.
	1240		351	0·945	116 27	166 50	331 25	−12 1	p.
	1241		350	0·817	121 1	183 50	348 25	−12 9	Q.
	1242		348	0·625	124 10	199 59	4 34	− 9 4	R.
	1243		348	0·539	129 15	206 49	11 24	− 9 15	r.
	1244		346	0·302	163 51	227 25	32 0	− 9 32	S.
	1245		345	0·346	210 41	242 15	46 50	−13 4	T.
	1246		345	0·338	217 30	244 14	48 49	−11 42	t.
	1247		345	0·378	216 1	244 50	48 25	−14 7	r.
	1248		343	0·470	243 50	257 44	62 19	−11 3	U.
	1249		343	0·444	236 20	254 11	58 46	−12 35	u.
	1250		343	0·387	237 32	251 56	56 31	− 9 47	u′.
	1251		340	0·865	294 29	295 43	100 18	+14 15	V.
	1252		342	0·768	293 21	285 49	90 24	+12 57	W.
	1253		342	0·752	296 40	284 9	88 44	+15 21	X.
3.	1254	214·514	354	0·969	90 22	160 10	310 36	+12 36	p.
	1255		353	0·936	93 82	166 43	317 9	+ 9 55	P.
	1256		352	0·909	117 32	173 31	323 57	−11 30	Q.
	1257		352	0·865	119 14	179 14	329 40	−11 43	q.
	1258		348	0·681	127 3	197 37	348 3	−12 7	R.
	1259		348	0·655	129 37	200 12	350 38	−12 50	r.
	1260		350	0·453	138 40	215 1	5 27	−10 2	S.
	1261		350	0·376	146 47	230 55	11 21	− 9 30	s.
	1262		346	0·284	208 13	241 9	31 35	− 9 44	T.
	1263		345	0·463	236 53	256 6	46 32	−13 21	U.
	1264		345	0·469	242 17	258 4	48 30	−11 41	u.
	1265		345	0·498	238 26	258 19	48 45	−14 18	u′.
	1266		343	0·635	255 45	271 54	62 20	−11 14	V.
	1267		343	0·600	251 25	268 23	58 49	−12 31	v.
	1268		343	0·550	252 58	265 40	56 6	−10 4	v′.
	1269		340	0·952	294 57	309 22	99 48	+14 18	W.
	1270		342	0·890	293 15	299 49	90 15	+12 53	X.
	1271		342	0·878	296 6	298 10	88 36	+15 23	Y.
	1272	214·524	351*a*	0·370	11 52	236 31	26 49	+27 41	O.
	1273		351*a*	0·359	7 2	238 27	28 45	+26 54	o.
	1274		354	0·971	90 27	159 57	309 55	+12 30	p.
	1275		353	0·939	93 18	166 15	316 33	+10 7	P.
	1276		352	0·912	117 23	173 2	323 20	−11 27	Q.
	1277		352	0·867	119 13	179 3	329 21	−11 45	q.
	1278		348	0·679	127 15	197 48	348 6	−12 10	R.
	1279		348	0·653	129 30	200 16	350 34	−12 44	r.
	1280		350	0·450	138 44	215 9	5 27	−10 0	S.
	1281		350	0·374	146 55	221 4	11 22	− 9 27	s.
	1282		346	0·282	208 49	241 17	31 35	− 9 33	T.
	1283		345	0·463	237 8	256 12	46 30	−13 14	U.
	1284		345	0·471	242 21	258 12	48 30	−11 45	u.
	1285		345	0·500	238 50	258 36	48 54	−14 17	u′.
	1286		343	0·635	255 59	271 59	62 17	−11 8	V.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1862.					° ′	° ′	° ′	° ′	
Aug. 3.	1287	214·524	343	0·603	251 42	268 40	58 58	−12 28	v.
	1288		343	0·552	253 24	265 53	56 11	− 9 55	v′.
	1289		340	0·957	294 48	310 21	100 39	+14 7	W.
	1290		342	0·895	293 30	300 23	90 41	+13 6	X.
	1291		342	0·879	296 18	298 25	88 43	+15 33	Y.
4.	1292	215·536	354	0·418	344 50	249 59	25 56	+27 39	P.
	1293		354	0·439	338 22	253 23	29 20	+27 11	p.
	1294		353	0·825	94 18	181 36	317 33	+10 1	Q.
	1295		353	0·892	90 30	173 54	309 51	+13 14	q.
	1296		355	0·981	109 30	159 45	295 42	− 5 50	R.
	1297		351	0·793	121 52	188 2	323 59	−11 27	S.
	1298		351	0·724	124 6	194 24	330 21	−11 9	s.
	1299		348	0·518	138 7	212 9	348 6	−12 5	T.
	1300		348	0·481	138 39	214 22	350 19	−10 54	t.
	1301		350	0·307	267 4	229 54	5 51	−10 0	U.
	1302		346	0·408	241 20	255 38	31 35	− 9 37	V.
	1303		345	0·652	251 39	272 43	48 40	−14 20	W.
	1304		345	0·616	252 11	270 28	46 25	−12 47	w.
	1305		345	0·639	253 32	272 58	48 55	−11 40	w′.
	1306		343	0·758	259 51	283 10	59 7	−12 30	X.
	1307		343	0·720	261 41	280 33	56 30	−10 10	x.
	1308		343	0·790	262 38	286 39	62 36	−11 22	x′.
	1309		340	0·995	295 44	322 41	98 38	+14 0	Y.
	1310		342	0·970	294 4	313 53	89 50	+12 56	Z.
	1311		342	0·962	296 32	312 30	88 27	+15 23	z.
5.	1312	216·519	354	0·675	95 2	195 53	317 53	+ 9 41	P.
	1313		354	0·684	91 52	195 14	317 14	+11 51	p.
	1314		353	0·770	89 49	188 2	310 2	+13 47	Q.
	1315		353	0·763	91 45	188 35	310 35	+12 17	q.
	1316		356	0·912	111 56	173 54	295 54	− 5 50	R.
	1317		351	0·654	128 58	201 46	323 46	−11 53	S.
	1318		348	0·384	157 55	225 33	347 33	−12 18	T.
	1319		348	0·345	163 33	228 38	350 38	−11 24	t.
	1320		350	0·298	197 59	240 3	2 3	−11 1	U.
	1321		350	0·293	212 0	244 4	6 4	− 9 48	u.
	1322		346	0·575	255 55	269 44	31 44	− 9 40	V.
	1323		346	0·528	256 40	266 58	28 58	− 7 54	v.
	1324		345	0·767	259 34	284 44	46 44	−13 13	W.
	1325		345	0·784	262 20	286 54	48 54	−11 41	w.
	1326		345	0·790	258 51	286 35	48 35	−14 28	w′.
	1327		343	0·884	264 39	297 42	59 42	−12 42	X.
	1328		343	0·907	266 48	301 8	63 8	−11 31	x.
6.	1329	217·519	353	0·431	93 57	210 31	318 20	+ 9 50	P.
	1330		353	0·498	89 14	209 50	317 39	+12 15	p.
	1331		354	0·607	87 48	202 23	310 12	+14 9	Q.
	1332		354	0·603	90 36	202 28	310 17	+12 26	q.
	1333		355	0·797	114 53	188 0	295 49	− 5 32	R.
	1334		350	0·412	148 42	222 25	330 14	−11 6	S.
	1335		350	0·449	145 44	219 48	327 37	−11 46	s.
	1336		350	0·435	139 55	216 52	324 41	−11 13	σ.
	1337		351	0·320	193 2	239 24	347 13	−12 18	T.
	1338		351	0·313	203 31	242 43	350 32	−11 37	t.
	1339		347	0·382	232 54	253 48	1 37	−10 56	U.
	1340		346	0·735	263 2	283 46	31 35	−10 4	V.
	1341		346	0·897	264 37	300 14	48 3	−13 26	W.
	1342		346	0·902	266 16	301 12	49 1	−12 8	w.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1862.					° ′	° ′	° ′	° ′	
Aug. 6.	1343	217·519	345	0·964	267 37	311 41	59 30	−12 59	X.
	1344		343	0·974	269 17	314 25	62 14	−11 48	x.
	1345	217·692	353	0·448	94 45	212 59	318 21	+ 9 19	P.
	1346		353	0·461	89 46	212 25	317 47	+11 39	p.
	1347		354	0·576	88 14	204 46	310 8	+13 37	Q.
	1348		354	0·571	91 16	204 55	310 17	+11 51	q.
	1349		355	0·774	116 21	190 31	295 53	− 6 8	R.
	1350		350	0·389	154 39	225 19	330 41	−11 38	S.
	1351		350	0·429	150 11	222 16	327 38	−12 16	s.
	1352		350	0·466	143 40	218 27	323 49	−11 46	σ.
	1353		347	0·449	244 51	260 28	5 50	−10 13	U.
	1354		346	0·762	264 6	286 24	31 46	−10 4	V.
	1355		346	0·917	265 26	303 20	48 42	−13 25	W.
	1356		346	0·920	267 15	304 10	49 32	−11 54	w.
	1357		345	0·977	268 4	315 11	60 33	−13 11	X.
	1358		343	0·986	269 40	318 0	63 22	−12 2	x.
7.	1359	218·440	353	0·293	91 58	223 28	318 13	+ 9 20	P.
	1360		353	0·315	85 42	222 31	317 16	+11 28	p.
	1361		354	0·436	84 8	215 17	310 2	+13 57	Q.
	1362		354	0·427	86 57	215 35	310 20	+12 38	q.
	1363		355	0·656	119 56	201 16	296 1	− 5 52	R.
	1364		351	0·308	181 43	236 40	331 25	−11 12	S.
	1365		351	0·340	171 29	232 53	327 38	−12 4	s.
	1366		351	0·359	160 17	228 49	323 34	−11 15	σ.
	1367		350	0·385	226 18	252 38	347 23	−12 38	T.
	1368		350	0·454	232 7	257 18	352 3	−14 38	t.
	1369		346	0·860	267 33	297 18	32 3	−10 8	U.
10.	1370	221·515	356	0·547	109 35	210 19	261 27	+ 2 40	P.
	1371		356	0·568	106 7	208 40	259 48	+ 4 26	p.
	1372		355	0·216	203 7	245 6	296 14	− 5 47	Q.
	1373		351	0·646	257 29	279 8	330 16	−11 48	R.
	1374		351	0·608	256 39	276 21	327 29	−11 3	r.
	1375		351	0·543	250 53	270 36	321 44	−11 40	f.
	1376		353	0·398	292 58	266 41	317 49	+ 9 18	S.
	1377		353	0·387	300 3	265 34	316 42	+11 56	s.
	1378		354	0·295	309 3	259 12	310 20	+13 15	T.
	1379		354	0·284	300 4	259 22	310 30	+10 35	t.
15.	1380	226·710	358	0·968	97 15	172 9	149 36	+10 27	p.
	1381		358	0·896	95 56	184 3	161 30	+12 17	P.
	1382		357	0·678	99 30	205 23	182 50	+ 9 35	Q.
	1383		357	0·625	91 57	209 49	187 16	+14 12	q.
	1384		359	0·662	136 13	212 12	189 39	−13 46	R.
18.	1385	229·655	360	0·994	93 39	166 1	101 41	+14 16	P.
	1386		358	0·427	92 49	226 12	161 52	+12 20	Q.
	1387		358	0·535	92 46	219 6	154 46	+13 32	q.
	1388		358	0·586	87 54	216 0	151 40	+16 50	q'.
	1389		358	0·791	126 48	201 54	137 34	−10 50	R.
	1390		357	0·086	70 15	247 8	182 48	+ 9 49	S.
	1391		359	0·335	193 47	249 54	185 34	−12 35	T.
	1392		359	0·336	215 10	257 8	192 48	−11 44	U.
19.	1393	230·438	361	0·965	94 42	176 11	100 45	+14 11	P.
	1394		363	0·994	97 15	167 31	92 5	+11 0	p.
	1395		362	0·839	83 58	195 10	119 44	+23 28	Q.
	1396		362	0·809	83 57	198 26	123 0	+23 4	q.
	1397		358	0·271	85 32	237 2	161 36	+12 29	R.
	1398		358	0·389	89 49	229 36	154 10	+13 10	r.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1862.									
Aug. 19.	1399	230·438	360	0·671	131° 27′	213° 27′	138° 1′	−10° 21′	S.
	1400		360	0·680	134 1	213 29	138 3	−12 13	s.
	1401		360	0·642	134 40	216 21	140 55	−11 22	σ.
	1402		359	0·361	220 48	260 14	184 48	−12 28	T.
	1403		359	0·363	231 9	263 36	188 10	−10 46	t.
	1404		359	0·426	239 32	268 43	193 17	−11 47	U.
	1405		359	0·118	313 45	258 0	182 34	+ 9 49	V.
	1406		359	0·087	338 41	255 2	179 36	+10 44	v.
20.	1407	231·459	361	0·882	95 37	190 23	100 28	+14 4	P.
	1408		363	0·942	98 44	181 52	91 57	+10 56	p.
	1409		362	0·698	81 28	210 12	120 17	+23 13	Q.
	1410		358	0·094	34 7	251 20	161 25	+12 5	R.
	1411		358	0·174	67 0	245 7	155 12	+13 23	r.
	1412		360	0·520	142 3	227 0	137 5	−10 41	S.
	1413		360	0·477	149 40	231 36	141 41	−12 1	s.
	1414		359	0·484	245 28	274 14	184 19	−12 32	T.
	1415		359	0·594	256 5	283 50	193 55	−12 15	U.
	1416		357	0·333	296 20	272 16	182 21	+ 9 20	V.
21.	1417	232·425	361	0·759	95 25	204 14	100 38	+14 12	P.
	1418		363	0·845	99 4	195 44	92 8	+11 26	p.
	1419		362	0·549	75 16	223 49	120 13	+23 22	Q.
	1420		362	0·507	72 41	227 13	123 37	+23 18	q.
	1421		360	0·360	160 0	240 48	137 12	− 9 36	R.
	1422		360	0·351	175 14	245 47	112 11	−11 53	r.
	1423		360	0·399	168 28	242 6	138 30	−13 23	ε.
	1424		359	0·631	258 48	287 52	184 16	−12 13	S.
	1425		359	0·746	264 34	298 3	194 27	−12 24	T.
	1426		358	0·210	314 33	264 51	161 15	+12 10	U.
	1427		357	0·533	295 25	286 8	182 32	+ 9 44	V.
22.	1428	233·551	361	0·578	94 1	219 51	100 16	+14 4	P.
	1429		363	0·680	99 31	211 52	92 17	+11 15	p.
	1430		362	0·376	60 21	239 10	119 35	+22 58	Q.
	1431		362	0·340	55 45	242 7	122 32	+22 28	q.
	1432		360	0·321	204 31	256 50	137 15	−11 32	R.
	1433		360	0·318	210 58	258 52	139 17	−11 1	r.
	1434		359	0·792	267 9	303 39	184 4	−12 11	S.
	1435		359	0·891	270 14	314 50	195 15	−12 47	T.
	1436		358	0·430	302 24	280 5	160 30	+12 14	U.
	1437		358	0·371	307 35	275 56	156 21	+13 28	u.
	1438		357	0·730	295 41	302 1	182 26	+ 9 59	V.
23.	1439	234·447	361	0·404	89 31	232 43	100 25	+14 8	P.
	1440		363	0·513	98 18	224 56	92 38	+11 24	p.
	1441		362	0·289	31 26	251 50	119 32	+23 19	Q.
	1442		362	0·268	19 28	255 35	123 17	+22 30	q.
	1443		360	0·404	240 21	271 31	139 13	−10 46	R.
	1444		360	0·445	245 13	274 51	142 33	−11 9	r.
	1445		359	0·893	271 13	316 11	183 53	−12 14	S.
	1446		359	0·959	273 19	326 53	194 35	−12 39	T.
	1447		358	0·598	299 47	292 33	160 15	+12 10	U.
	1448		358	0·569	304 5	290 7	157 49	+14 25	u.
	1449		357	0·851	296 30	314 32	182 14	+10 13	V.
24.	1450	235·472	361	0·204	72 6	247 8	100 18	+14 0	P.
	1451		364	0·969	121 13	182 57	36 7	− 9 48	Q.
	1452		360	0·569	257 11	286 10	139 20	−11 20	R.
	1453		362	0·333	343 39	268 46	121 56	+22 23	S.
	1454		358	0·762	298 47	306 42	159 52	+11 55	T.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1862.									
Aug. 24.	1455	235·472	357	0·950	297° 19′	329° 13′	182° 23′	+ 9° 58′	U.
25.	1456	236·442	364	0·894	124 9	196 35	35 59	− 9 46	P.
	1457		358	0·882	299 23	320 10	159 34	+12 6	Q.
	1458		361	0·130	355 56	260 45	100 9	+13 53	R.
	1459		362	0·486	324 16	283 11	122 35	+22 30	S.
	1460		362	0·461	332 39	279 20	118 44	+24 56	s.
	1461	236·468	364	0·889	124 5	197 15	36 17	− 9 33	P.
	1462		358	0·883	299 24	320 19	159 21	+12 7	Q.
	1463		361	0·131	355 27	260 52	99 54	+13 56	R.
	1464		362	0·488	324 15	283 19	122 21	+22 31	S.
	1465		362	0·461	302 41	284 56	123 58	+12 22	s.
26.	1466	237·461	364	0·767	128 44	211 34	36 31	− 9 39	P.
	1467		365	0·693	121 51	216 8	41 5	− 3 8	Q.
	1468		361	0·296	314 11	274 45	99 42	+13 52	R.
	1469		361	0·240	321 30	270 47	95 44	+14 11	r.
	1470		360	0·651	316 37	297 31	122 28	+22 41	S.
28.	1471	239·460	364	0·442	148 28	240 7	36 43	− 9 7	P.
	1472		365	0·265	146 52	248 24	45 0	− 2 3	Q.
	1473		365	0·317	141 43	245 0	41 36	− 2 36	q.
	1474		361	0·669	303 33	302 40	99 16	+14 10	R.
	1475		361	0·785	306 37	312 30	109 6	+17 13	r.
29.	1476	240·518	367	0·971	101 8	184 45	326 20	+10 50	P.
	1477		364	0·303	178 58	255 12	36 47	− 9 11	Q.
	1478		365	0·162	206 25	262 37	44 12	− 2 5	R.
	1479		365	0·182	189 23	259 39	41 14	− 3 5	r.
	1480		361	0·822	302 51	317 22	98 57	+14 10	S.
30.	1481	241·451	367	0·901	102 35	197 42	326 4	+10 31	P.
	1482		364	0·302	220 54	268 35	36 57	− 9 14	Q.
	1483		365	0·271	256 0	275 24	43 46	− 1 51	R.
	1484		361	0·922	303 13	330 38	99 0	+14 11	S.
Sept. 2.	1485	244·597	367	0·612	97 38	228 1	311 45	+14 14	P.
	1486		367	0·665	95 55	224 7	307 51	+15 48	p.
	1487		370	0·468	93 22	238 25	322 9	+14 50	q.
	1488		370	0·384	104 38	243 2	326 46	+ 9 21	Q.
	1489		369	0·869	128 48	208 3	291 47	−11 3	R.
	1490		369	0·809	131 50	215 0	298 44	−11 41	r.
	1491		368	0·709	138 10	225 14	308 58	−12 57	S.
	1492		368	0·638	141 21	231 12	314 56	−12 28	s.
	1493		364	0·777	273 38	313 54	37 38	− 9 4	T.
	1494		365	0·824	283 39	320 14	43 58	− 2 22	U.
4.	1495	246·428	369	0·247	81 33	254 50	312 36	+14 14	P.
	1496		369	0·320	83 19	250 34	308 20	+15 45	p.
	1497		368	0·650	145 35	233 28	291 14	−15 0	Q.
	1498		368	0·632	140 9	232 55	290 41	−11 20	q.
	1499		368	0·529	148 31	241 47	299 33	−11 46	R.
	1500		367	0·371	177 15	258 19	316 5	−12 27	S.
	1501		367	0·440	162 42	250 53	308 39	−12 52	s.
	1502		364	0·963	280 24	339 58	37 44	− 9 6	T.
6.	1503	248·465	369	0·404	182 59	261 25	290 17	−15 7	P.
	1504		369	0·350	202 7	269 18	298 10	−13 8	p.
	1505		369	0·323	211 50	272 29	301 21	−11 17	p.
	1506		368	0·402	235 49	282 26	311 18	−12 29	Q.
	1507		368	0·443	244 50	287 8	316 0	−12 9	q.
	1508		367	0·270	321 14	283 32	312 24	+14 27	R.
	1509		367	0·202	333 57	278 24	307 16	+14 48	r.
9.	1510	251·488	370	0·743	141 28	229 50	215 49	−15 17	P.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1862.									
Sept. 9.	1511	251·488	370	0·721	144° 57′	232° 45′	218° 44′	−16° 43′	p.
	1512		369	0·622	256 8	303 18	289 17	−15 44	Q.
	1513		369	0·653	266 24	308 45	294 44	−11 16	R.
	1514		369	0·710	266 30	312 53	298 52	−13 4	r.
	1515		369	0·730	270 10	315 28	301 27	−11 17	S.
	1516		369	0·837	273 20	325 52	311 51	−12 13	T.
	1517		367	0·799	305 57	325 50	311 49	+14 32	U.
10.	1518	252·582	370	0·599	152 9	244 35	215 3	−15 37	P.
	1519		370	0·556	159 6	249 40	220 8	−16 44	p.
	1520		371	0·771	265 44	318 30	288 58	−15 52	Q.
	1521		369	0·800	275 1	323 44	294 12	−10 1	R.
	1522		369	0·853	273 22	328 33	299 1	−12 53	r.
	1523		369	0·871	275 52	331 7	301 35	−11 22	S.
	1524		369	0·924	306 16	341 47	312 15	+14 37	T.
11.	1525	253·473	370	0·476	167 17	257 24	215 14	−15 34	P.
	1526		370	0·448	178 47	263 4	220 54	−16 49	p.
	1527		370	0·616	281 13	311 13	269 3	− 1 47	Q.
	1528		369	0·910	276 14	336 54	294 44	−12 27	R.
	1529		369	0·953	279 26	344 31	302 21	−11 5	S.
12.	1530	254·525	370	0·397	194 58	271 46	214 41	−15 49	P.
	1531		370	0·406	209 55	277 58	220 53	−16 31	p.
15.	1532	257·445	372	0·983	99 52	197 26	98 55	+15 27	P.
	1533		373	0·994	122 54	195 43	97 12	− 7 40	Q.
	1534		373*a*	0·659	113 29	236 57	138 26	+ 5 57	R.
	1535		370	0·662	260 3	312 50	214 19	−15 55	S.
	1536		370	0·735	263 50	319 27	220 56	−16 30	T.
16.	1537	258·525	372	0·914	100 32	212 32	98 42	+15 37	P.
	1538		374	0·940	125 32	210 56	97 6	− 7 47	Q.
	1539		373	0·962	121 10	206 13	92 23	− 4 22	q.
	1540		373*a*	0·449	116 8	252 36	138 46	+ 5 39	R.
	1541		370	0·807	268 35	328 11	214 21	−15 58	S.
	1542		370	0·866	270 42	334 46	220 56	−16 25	T.
18.	1543	260·478	372	0·668	98 51	239 29	97 57	+15 54	P.
	1544		377	0·400	98 25	258 10	116 38	+13 0	Q.
	1545		377	0·463	95 22	254 26	112 54	+15 12	q.
	1546		375	0·966	111 17	205 58	64 26	+ 5 13	R.
	1547		374	0·707	133 14	238 50	97 18	− 7 40	S.
	1548		373	0·744	127 1	234 38	93 6	− 4 10	s.
	1549		376	0·278	231 14	288 23	146 51	− 7 19	T.
	1550		376*a*	0·629	262 13	314 12	172 40	−13 48	U.
	1551		370	0·980	276 59	356 53	215 21	−15 47	V.
	1552		369	0·487	303 52	310 23	168 51	+10 37	W.
	1553		369	0·532	304 28	313 24	171 52	+11 9	w.
19.	1554	261·505	372	0·488	94 17	254 1	97 55	+16 8	P.
	1555		375	0·160	77 0	274 49	118 43	+12 40	Q.
	1556		376	0·872	112 40	221 23	65 17	+ 5 28	R.
	1557		377	0·960	126 34	210 18	54 12	− 9 0	S.
	1558		377	0·984	129 54	204 42	48 36	−13 17	s.
	1559		374	0·530	141 1	253 41	97 35	− 7 13	T.
	1560		373	0·560	132 30	250 0	93 54	− 3 43	t.
	1561		373	0·490	147 58	257 47	101 41	− 8 59	r.
	1562		369	0·642	302 9	322 22	166 16	+10 3	U.
	1563		369	0·680	303 31	325 16	169 10	+11 0	V.
	1564		369	0·720	303 35	328 34	172 28	+11 7	W.
20.	1565	262·495	378	0·668	65 16	250 53	80 45	+36 36	P.
	1566		372	0·306	82 20	267 45	97 37	+16 17	Q.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1862.					° ′	° ′	° ′	° ′	
Sept. 20.	1567	262·495	372	0·134	32 27	282 12	112 4	+14 39	q.
	1568		375	0·734	114 14	235 53	65 45	+ 5 25	R.
	1569		377	0·870	130 9	225 0	54 52	− 9 23	S.
	1570		377	0·904	131 19	221 8	51 0	−11 22	T.
	1571		377	0·951	128 44	213 26	43 18	−10 35	U.
	1572		377	0·966	133 15	211 11	41 3	−15 27	V.
	1573		374	0·357	157 29	267 52	97 44	− 7 5	W.
	1574		373	0·370	144 29	264 28	94 20	− 3 44	w.
	1575		374	0·331	171 12	272 30	102 22	− 8 57	w′.
	1576		374	0·832	303 37	339 57	169 49	+11 0	X.
	1577	262·639	372	0·276	79 41	269 54	97 43	+16 1	Q.
	1578		372	0·131	17 30	284 23	112 12	+14 28	q.
	1579		372	0·192	326 18	293 2	120 51	+12 39	q′.
	1580		375	0·712	115 10	237 57	65 45	+ 4 55	R.
	1581		377	0·853	130 55	227 17	55 6	− 9 31	S.
	1582		377	0·891	131 59	223 3	50 52	−11 33	T.
	1582		377	0·940	129 22	215 28	43 17	−10 48	U.
	1583		377	0·959	133 48	212 56	40 45	−15 37	V.
	1584		374	0·340	162 6	269 54	97 43	− 7 29	W.
	1585		374	0·345	148 12	266 35	94 24	− 4 4	w.
	1586		374	0·320	176 56	274 36	102 25	− 9 19	w′.
25.	1587	267·682	375	0·338	63 21	275 30	31 47	+22 11	P.
	1588		375	0·194	61 45	281 34	37 51	+15 49	p.
	1589		378	0·977	133 27	213 30	329 47	−15 36	Q.
	1590		377	0·405	193 42	283 20	39 37	−16 23	R.
	1591		377	0·304	197 2	285 40	41 57	−10 33	r.
	1592		377	0·315	210 47	289 58	46 15	−11 19	ρ.
	1593		377	0·375	235 15	299 11	55 28	−12 11	S.
	1594		377	0·341	240 38	299 41	55 58	− 9 24	s.
	1595		377	0·726	231 58	311 6	67 23	−34 11	T.
	1596		377	0·364	292 13	309 38	65 55	+ 5 9	W.
	1597		373	0·825	282 1	342 1	98 18	− 7 12	U.
	1598		372	0·782	311 9	339 56	96 13	+16 20	W.
	1599		376	0·972	306 38	5 29	121 46	+12 12	X.
	1600		376	0·942	305 52	359 27	115 44	+11 51	x.
30.	1601	272·425	378	0·377	181 2	282 36	332 36	−13 18	P.
	1602		379	0·446	168 48	276 53	325 53	−14 17	p.
	1603		379	0·644	157 36	262 22	311 22	−19 27	p′.
	1604		379	0·142	325 48	300 12	349 12	+10 38	Q.
	1605		379	0·185	311 18	303 24	352 24	+ 9 20	q.
	1606		380	0·384	327 35	313 5	2 5	+17 48	R.
	1607		380	0·633	293 45	332 12	21 12	+ 3 41	S.
	1608		377	0·955	281 40	3 28	52 28	−11 37	T.
	1609		377	0·878	273 17	350 18	39 18	−16 22	U.
	1610		377	0·888	280 57	353 19	42 19	−10 10	V.
	1611		377	0·970	286 13	7 29	56 29	− 7 52	W.
	1612	272·444	378	0·377	181 37	283 50	332 34	−13 22	P.
	1613		379	0·446	169 25	277 8	325 52	−14 26	p.
	1614		379	0·645	158 11	262 34	311 18	−19 47	p′.
	1615		379	0·143	324 47	300 22	349 6	+10 33	Q.
	1616		379	0·162	310 7	302 9	351 53	+ 8 50	q′.
	1617		380	0·385	326 37	313 19	2 3	+17 29	R.
	1618		380	0·635	293 18	332 19	21 3	+ 3 23	S.
	1619		377	0·957	281 42	3 52	52 36	−11 39	T.
	1620		377	0·881	273 24	350 45	39 29	−16 23	W.
	1621		377	0·890	280 58	353 37	42 21	−10 13	V.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1862.					° ′	° ′	° ′	° ′	
Sept. 30.	1622	272·444	377	0·971	286 20	7 45	56 29	− 7 48	W.
Oct. 1.	1623	273·511	380	0·384	104 1	271 43	305 19	+10 46	P.
	1624		380	0·675	136 17	254 24	288 0	− 8 17	Q.
	1625		378	0·349	220 37	299 11	332 47	−13 3	R.
	1626		379	0·350	200 33	292 4	325 40	−13 40	r.
	1627		381	0·373	308 9	315 46	349 22	+10 34	S.
	1628		381	0·429	304 31	319 30	353 6	+ 9 32	s.
	1629		381*a*	0·584	317 49	328 44	2 20	+17 51	T.
	1630		377	0·964	277 3	5 34	39 10	−16 25	U.
	1631		377	0·972	284 10	8 49	42 25	−10 1	V.
3.	1632	275·446	378	0·595	262 47	326 32	332 41	−13 30	P.
	1633		379	0·518	254 9	319 17	325 26	−14 19	p.
	1634		381	0·780	302 10	347 30	353 39	+ 9 25	Q.
	1635		381	0·725	304 9	342 42	348 51	+10 10	q.
	1636		381	0·944	297 29	6 43	12 52	+ 3 16	R.
	1637	275·644	378	0·624	264 59	329 20	332 41	−13 27	P.
	1638		378	0·549	256 34	321 58	325 19	−14 38	p_1.
	1639		378	0·552	258 56	322 54	326 15	−13 42	p_2.
	1640		378	0·702	272 42	337 4	340 25	−11 24	p_3.
	1641		381	0·814	303 23	351 1	354 22	+ 9 31	Q.
	1642		381	0·761	304 28	345 57	349 18	+10 24	q.
5.	1643	277·500	382	0·717	80 46	256 12	233 13	+29 27	P.
	1644		382	0·629	82 24	262 49	239 50	+25 48	p.
	1645		383	0·771	91 24	248 46	225 47	+23 13	Q.
	1646		383	0·776	97 16	247 32	224 33	+18 50	q.
	1647		378	0·872	276 57	355 31	332 32	−13 27	R.
	1648		379	0·811	272 35	348 5	325 6	−15 2	r.
8.	1649	280·587	385	0·939	101 35	230 30	163 44	+16 6	P.
	1650		382	0·359	34 47	297 48	231 2	+26 53	Q.
	1651		382	0·326	14 6	305 30	238 44	+24 41	R.
	1652		384	0·635	309 31	340 22	273 36	+13 3	S.
	1653		384	0·567	310 2	335 24	268 38	+12 46	T.
9.	1654	281·453	385	0·855	101 5	243 3	164 0	+16 20	P.
	1655		382	0·387	8 56	309 31	230 28	+27 41	Q.
	1656		382	0·410	348 49	317 56	238 53	+24 48	R.
	1657		384	0·773	308 12	352 45	273 42	+12 57	S.
	1658		384	0·715	308 48	347 38	268 35	+13 5	T.
10.	1659	282·640	385	0·699	99 41	259 13	163 20	+16 2	P.
	1660		382	0·503	344 50	325 13	230 20	+27 38	Q.
	1661		382	0·579	331 2	334 32	238 39	+24 18	R.
	1662		384	0·918	307 51	10 14	274 21	+12 49	S.
11.	1663	283·870	385	0·542	95 52	272 22	163 17	+16 6	P.
	1664		382	0·627	334 35	337 47	228 42	+27 43	Q.
	1665		382	0·720	324 31	348 2	238 57	+24 6	R.
	1666		384	0·980	308 24	23 21	274 16	+12 51	S.
16.	1667	284·440	388	0·992	116 24	226 37	48 28	+ 0 42	P.
	1668		387	0·962	127 27	236 15	58 6	+ 9 2	Q.
	1669		386	0·539	93 27	277 47	99 38	+16 55	R.
	1670		386	0·594	95 7	273 40	95 31	+17 1	r.
	1671		385	0·355	308 38	329 26	151 17	+ 9 37	S.
17.	1672	289·604	388	0·925	117 58	242 38	47 58	+ 0 35	P.
	1673		387	0·863	130 9	252 18	57 38	+ 8 58	Q.
	1674		386	0·319	79 31	294 41	100 1	+16 21	R.
	1675		386	0·405	85 52	288 46	94 6	+16 58	r.
21.	1676	293·531	387	0·258	184 18	308 28	58 6	− 8 27	P.
	1677		388	0·279	130 2	298 22	48 0	+ 1 11	p.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1862.									
Oct. 21.	1678	293·531	389	0·398	170° 23′	300° 13′	49° 49′	−13° 44′	Q.
	1679		389	0·447	161 2	295 8	44 46	−13 24	q.
	1680		389*a*	0·794	299 0	6 28	116 6	+ 5 34	R.
23.	1681	295·483	390	0·932	133 17	249 35	331 32	−14 14	P.
	1682		387	0·418	261 34	336 17	58 14	− 8 49	Q.
	1683		387	0·446	253 52	335 43	57 40	−12 32	q.
	1684		387	0·373	267 2	335 1	56 58	− 5 32	q.
	1685		388	0·187	275 2	325 56	47 53	+ 1 12	R.
	1686		389	0·394	239 23	328 52	50 49	−14 11	S.
	1687		389	0·365	237 2	327 4	49 1	−13 8	s.
24.	1688	296·427	391	0·878	92 12	256 10	324 43	+22 58	P.
	1689		391	0·909	91 23	252 1	320 34	+24 8	p.
	1690		390	0·841	136 15	262 41	331 14	−14 14	Q.
	1691		387	0·579	272 52	349 26	57 59	− 8 47	R.
	1692		387	0·608	267 9	349 56	58 29	−12 42	r.
	1693		387	0·560	278 13	349 10	57 43	− 5 28	r.
	1694		387	0·535	270 33	346 6	54 39	− 8 47	r.
	1695		388	0·383	287 57	339 7	47 40	+ 1 38	S.
	1696		389	0·529	258 41	342 34	51 7	−14 3	T.
	1697		389	0·436	249 38	334 54	43 27	−13 30	t.
26.	1698	298·559	391	0·589	81 19	287 6	325 25	+23 18	P.
	1699		391	0·648	83 17	282 15	320 34	+23 59	p.
	1700		392	0·789	97 20	267 31	305 50	+17 11	Q.
	1701		393	0·902	141 35	258 58	297 17	−21 1	R.
	1702		390	0·541	152 4	292 28	330 47	−14 28	S.
	1703		387	0·884	283 11	19 39	57 58	− 8 31	T.
Nov. 11.	1704	314·470	397	0·427	79 18	312 9	125 46	+16 20	P.
	1705		398	0·373	196 11	332 25	145 2	−18 36	Q.
	1706		396	0·535	253 21	0 30	173 7	−16 53	R.
	1707		394	0·113	333 31	339 48	152 28	+ 7 19	S.
	1708		395	0·204	315 43	345 43	158 20	+ 7 37	T.
	1709	314·605	397	0·405	77 32	314 52	125 34	+16 15	P.
	1710		397	0·451	78 58	311 58	122 40	+17 11	p.
	1711		398	0·373	200 49	334 23	145 5	−18 42	Q.
	1712		398	0·416	192 9	330 29	141 11	−20 58	q.
	1713		396	0·555	255 4	2 22	173 4	−16 54	R.
	1714		396	0·446	232 39	348 44	159 26	−19 35	r.
	1715		394	0·136	323 43	341 44	152 26	+ 7 5	S.
	1716		395	0·236	311 57	347 55	158 37	+ 7 29	T.
12.	1717	315·500	399	0·675	89 31	295 22	93 23	+17 20	P.
	1718		399	0·556	89 28	304 2	102 3	+14 54	p.
	1719		397	0·700	125 32	292 45	90 46	− 7 5	Q.
	1720		397	0·678	127 44	294 48	92 49	− 8 10	q.
	1721		397	0·413	232 40	348 35	146 36	−17 54	R.
	1722		396	0·687	263 33	14 47	172 48	−16 53	S.
	1723		394	0·310	306 12	353 31	151 32	+ 7 6	T.
	1724		395	0·419	304 26	0 13	158 14	+ 7 46	U.
13.	1725	316·528	399	0·533	84 6	307 36	91 2	+16 52	P.
	1726		399	0·477	81 1	311 47	95 13	+16 43	p.
	1727		401	0·992	120 40	254 59	38 25	− 8 14	Q.
	1728		400	0·825	130 43	283 40	67 6	−13 38	R.
	1729		396	0·826	269 13	29 30	172 56	−16 51	S.
	1730		396	0·567	254 26	4 58	148 24	−17 36	s.
	1731		394	0·519	299 27	8 2	150 28	+ 6 18	T.
	1732		395	0·611	301 7	14 22	157 48	+ 7 50	U.
16.	1733	319·489	400	0·694	125 19	297 18	38 44	− 7 55	P.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1862.					° ′	° ′	° ′	° ′	
Nov. 16.	1734	319·489	400*a*	0·351	162 44	327 6	68 26	−13 29	Q.
	1735		396	0·919	270 18	44 14	145 40	−17 59	R.
	1736		394	0·941	296 32	50 2	151 28	+ 5 55	S.
	1737		395	0·970	298 43	55 51	157 17	+ 7 58	s.
	1738		399	0·269	340 11	350 22	91 48	+14 8	T.
	1739		399	0·332	335 57	354 4	95 30	+15 55	U.
17.	1740	320·468	402	0·889	101 29	278 24	5 57	+ 9 25	P.
	1741		402	0·914	100 34	275 5	2 38	+10 22	p.
	1742		401	0·516	129 17	311 30	39 3	− 7 16	Q.
	1743		400	0·273	202 38	341 25	68 58	−13 24	R.
	1744		396	0·991	296 55	63 12	150 45	+ 6 19	S.
	1745		399	0·440	317 55	4 35	92 8	+13 40	T.
	1746		399	0·504	319 35	8 12	95 45	+16 6	U.
23.	1747	326·500	405	0·465	146 11	324 35	326 34	−14 44	P.
	1748		404	0·444	158 15	329 21	331 20	−18 0	p.
	1749		403	0·315	195 32	345 51	347 50	−16 37	Q.
	1750		403	0·335	188 31	343 20	345 19	−17 31	q.
	1751		401	0·778	278 28	37 13	39 12	− 7 4	R.
	1752		402	0·194	345 50	353 8	5 7	+10 56	S.
	1753		402	0·244	325 49	358 22	0 21	+10 0	T.
	1754		402	0·389	310 56	8 22	10 21	+ 9 55	U.
24.	1755	327·525	404	0·317	168 21	338 34	326 1	−14 18	P.
	1756		404	0·318	154 38	335 2	322 29	−11 41	p.
	1757		404	0·330	186 27	343 51	331 18	−17 12	r.
	1758		403	0·371	231 13	0 1	347 28	−16 40	Q.
	1759		401	0·901	280 10	51 41	39 8	− 6 54	R.
	1760		402	0·369	317 26	7 12	354 39	+11 39	S.
	1761		402	0·442	309 16	12 46	0 13	+10 20	T.
	1762		402	0·591	305 5	23 5	10 28	+10 52	U.
27.	1763	330·505	404	0·542	257 50	19 59	325 10	−14 31	P.
	1764		405	0·489	262 41	17 42	322 53	−10 47	Q.
	1765		402	0·858	300 52	49 23	354 34	+11 59	R.
	1766		402	0·911	298 49	56 10	1 21	+10 44	S.
	1767		402	0·968	299 26	66 11	11 22	+11 48	T.
Dec. 12.	1768	345·530	406	0·855	111 27	307 52	39 55	− 8 59	P.
	1769		406	0·834	109 56	310 1	42 4	− 7 32	p.
	1770		406*a*	0·911	127 37	302 47	34 50	−24 3	Q.
	1771		406*b*	0·442	144 39	346 23	78 26	−18 18	R.
25.	1772	358·538	411	0·936	75 4	312 8	219 41	+17 56	P.
	1773		410	0·981	77 37	302 8	209 41	+16 47	p.
	1774		410	0·590	130 55	348 13	255 46	−22 6	Q.
	1775		410	0·627	129 0	345 9	252 42	−22 15	q.
	1776		407	0·603	250 0	53 52	321 25	−16 52	R.
	1777		408	0·733	252 10	64 21	331 54	−18 23	S.
	1778		409	0·766	257 55	68 10	335 43	−14 46	T.
	1779		409*a*	0·996	289 2	102 52	10 25	+13 24	U.
30.	1780	363·538	414	0·926	84 56	317 21	153 58	+ 6 9	P.
	1781		414	0·861	85 4	325 39	162 16	+ 5 9	p.
	1782		414	0·891	82 34	322 20	158 57	+ 7 45	r.
	1783		413	0·261	140 53	14 1	210 38	−14 6	Q.
	1784		413	0·548	232 3	51 8	247 45	−23 35	R.
	1785		412	0·420	32 57	11 36	208 13	+18 17	S.
	1786		410	0·347	12 35	20 52	217 29	+16 53	T.
	1787		411	0·332	355 36	26 51	223 28	+16 10	U.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1863.									
Jan. 2.	1788	1·600	414	0·511	66° 43′	359° 51′	152° 43′	+ 9° 17′	P.
	1789		414	0·285	59 58	13 22	166 34	+ 5 15	Q.
	1790		414	0·396	62 58	6 57	160 9	+ 7 41	R.
	1791		412*a*	0·558	255 19	60 31	213 43	−11 40	S.
	1792		412*a*	0·454	243 19	51 54	205 6	−15 21	s.
	1793		411	0·741	296 45	71 25	244 37	+16 3	T.
	1794		410	0·688	299 27	66 30	219 42	+16 14	t.
4.	1795	3·538	414	0·244	15 18	25 44	151 26	+10 1	P.
	1796		414	0·268	308 31	41 34	167 16	+ 5 59	Q.
	1797		414	0·219	344 18	32 53	158 35	+ 8 30	R.
	1798		413	0·847	259 4	87 7	212 49	−11 31	S.
	1799		413	0·763	253 47	78 20	204 2	−14 57	s.
	1800		411	0·948	289 17	98 30	224 12	+16 33	T.
7.	1801	6·525	415	0·905	96 9	327 33	50 53	− 8 9	P.
	1802		414	0·630	291 45	68 32	151 52	+10 55	Q.
	1803		414	0·830	278 59	87 32	170 52	+ 6 4	R.
	1804		414	0·673	281 39	73 33	156 53	+ 5 32	S.
8.	1805	7·541	415	0·778	95 28	342 26	51 21	− 7 57	P.
	1806		414	0·762	283 52	81 21	150 16	+ 8 58	Q.
	1807		414	0·936	276 39	102 1	170 56	+ 6 11	R.
	1808		414	0·820	278 20	87 32	156 27	+ 5 32	S.
18.	1809	17·491	418	0·902	63 18	342 20	270 7	+15 57	P.
	1810		418	0·857	70 38	347 11	274 55	+ 8 31	p.
	1811		417	0·498	103 0	14 46	302 33	−13 49	Q.
	1812		417	0·484	96 17	15 2	302 49	−10 25	q.
	1813		416	0·279	126 8	31 18	319 5	−15 41	R.
	1814		416	0·298	117 21	28 51	316 38	−14 17	r.
	1815		415	0·984	257 16	123 29	51 16	− 7 16	S.
	1816	17·527	418	0·897	62 43	343 3	270 19	+16 19	P.
	1817		418	0·854	70 10	346 40	273 56	+ 8 49	p.
	1818		417	0·489	102 59	15 23	302 39	−13 40	Q.
	1819		417	0·472	95 37	15 46	303 2	−10 0	q.
	1820		416	0·269	126 29	31 51	319 7	−15 24	R.
	1821		416	0·285	117 14	29 32	316 48	−13 53	r.
20.	1822	19·497	418	0·654	50 35	10 38	269 58	+16 6	P.
	1823		419	0·775	52 46	0 36	259 56	+19 8	p.
	1824		418	0·551	57 11	15 35	274 55	+ 9 14	r.
	1825		416	0·285	224 37	59 0	318 20	−15 11	Q.
	1826		416	0·244	220 25	56 17	315 37	−14 31	q.
	1827		416	0·329	235 57	63 7	322 27	−13 29	q.
25.	1828	24·505	420	0·928	71 12	343 49	172 7	+ 6 30	P.
	1829		421	0·983	84 6	331 4	159 22	− 4 29	p.
	1830		422	0·557	125 29	24 16	212 34	−28 7	Q.
	1831		422	0·631	249 10	89 41	277 59	−11 32	R.
	1832		416	0·981	246 27	130 20	318 38	−14 56	S.
	1833		417	0·882	269 15	111 31	299 49	+ 4 54	T.
	1834		418	0·588	298 29	79 28	267 46	+16 10	U.
	1835		419	0·510	309 56	71 0	259 18	+17 24	V.
28.	1836	27·530	423	0·740	56 59	9 35	154 58	+12 4	P.
	1837		420	0·492	54 19	27 16	172 39	+ 6 41	Q.
	1838		420	0·567	58 44	21 40	167 3	+ 6 28	q.
	1839		421	0·626	77 46	15 8	160 31	− 3 38	R.
	1840		422	0·452	254 16	80 43	226 6	− 7 29	S.
	1841		422	0·392	251 22	76 47	222 10	− 8 28	T.
	1842		418	0·949	279 16	122 20	267 43	+16 61	U.
29.	1843	28·527	423	0·574	47 51	24 46	156 1	+12 0	P.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1863.					° ′	° ′	° ′	° ′	
Jan. 29.	1844	28·527	420	0·304	33 39	42 25	153 40	+ 6 36	Q.
	1845		420	0·380	46 15	36 7	167 22	+ 6 8	q.
	1846		421	0·417	74 15	30 21	161 36	− 3 30	R.
	1847		422	0·657	254 51	96 1	227 16	− 7 5	S.
	1848		422	0·607	252 18	92 15	223 30	− 8 41	T.
Feb. 8.	1849	38·524	423	0·198	96 22	54 14	43 41	−10 37	P.
	1850		423	0·128	111 28	59 1	48 28	−10 54	p.
	1851		421	0·256	112 15	52 53	42 20	−15 20	Q.
	1852		425	0·392	247 57	88 6	77 33	− 8 44	R.
	1853		425	0·445	250 44	91 28	80 55	− 7 43	r.
	1854		425	0·448	234 14	90 36	80 3	−14 58	S.
	1855		426	0·907	243 52	130 38	120 5	−12 42	T.
9.	1856	39·458	427	0·960	50 22	356 31	332 42	+20 53	P.
	1857		427	0·989	89 20	343 24	319 35	−15 37	Q.
	1858		424	0·068	179 43	66 58	43 9	−10 21	R.
	1859		424	0·152	225 39	73 43	49 54	−10 46	r.
	1860		424	0·147	162 45	65 41	41 52	−15 3	s.
	1861		425	0·568	248 28	100 41	76 52	− 8 52	S.
	1862		425	0·625	240 8	104 32	80 43	−14 7	s.
	1863		425	0·586	240 23	101 34	77 45	−13 36	σ.
	1864		426	0·978	243 15	144 39	120 50	−12 21	T.
May 6.	1865	125·646	441	0·827	71 24·9	95 55	289 37	− 5 49	P.
	1866		441	0·888	72 8·2	89 3	282 45	− 6 24	p.
	1867		440	0·561	40 54·6	120 48	314 30	+11 7	Q.
	1868		440	0·636	42 21·4	115 24	309 6	+12 23	q.
	1869		440	0·709	44 59·2	109 19	303 1	+12 38	R.
	1870		440	0·624	42 16·2	116 17	309 59	+12 5	r.
	1871		437	0·486	237 4·3	180 31	14 13	− 7 42	S.
	1872		437	0·547	238 7·7	184 29	18 11	− 7 34	s.
	1873		438	0·763	240 18·4	201 15	34 57	− 7 8	T.
	1874		438	0·752	230 43·2	199 42	33 24	−14 16	t.
	1875		438	0·758	233 10·2	200 24	34 6	−12 29	u.
7.	1876	126·562	441	0·691	71 38·3	108 53	289 34	− 5 41	P.
	1877		440	0·408	28 10·2	133 40	314 21	+11 32	Q.
	1878		440	0·478	32 23·3	128 49	309 30	+12 35	q.
	1879		440	0·561	39 6·0	122 10	302 51	+12 13	R.
	1880		440	0·444	31 6·2	131 1	311 42	+11 53	r.
	1881		437	0·652	239 45·9	193 26	14 7	− 7 14	S.
	1882		437	0·717	239 57·1	198 12	18 53	− 7 20	s.
	1883		438	0·887	254 3·1	214 20	35 1	+ 4 43	T.
	1884		438	0·885	241 29·1	214 39	35 20	− 6 23	t.
	1885		439	0·436	270 3·3	176 12	356 53	+ 6 44	v.
	1886		439	0·463	271 57·3	177 28	358 9	+ 8 12	x.
	1887		439	0·461	268 50·5	177 56	358 37	+ 6 50	u.
	1888		439	0·463	267 37·9	178 14	358 55	+ 6 21	w.
	1889		439	0·597	259 4·1	188 12	8 53	+ 4 28	y.
	1890	126·625	441	0·688	71 41·5	109 11	288 58	− 5 42	P.
	1891		440	0·405	28 6·8	133 53	313 40	+11 28	Q.
	1892		440	0·478	32 13·1	128 55	308 42	+12 41	q.
	1893		440	0·557	39 4·3	122 29	302 16	+12 8	R.
	1894		440	0·441	30 57·2	131 18	311 5	+11 49	r.
	1895		437	0·658	239 37·7	193 35	13 22	− 7 20	S.
	1896		437	0·723	239 48·0	198 42	18 29	− 7 28	s.
	1897		438	0·887	253 45·3	214 25	34 12	+ 4 27	T.
	1898		438	0·884	241 21·3	214 36	34 23	− 6 30	t.
	1899		439	0·437	269 33·3	176 26	356 13	+ 6 34	v.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1863.									
May 7.	1900	126·625	439	0·466	271° 28′·8	177° 48′	357° 35′	+ 8° 4′	x.
	1901		439	0·465	268 26·8	178 18	358 5	+ 6 46	u.
	1902		439	0·465	267 7·9	178 31	358 18	+ 6 11	w.
	1903		439	0·547	258 43·2	188 24	8 11	+ 3 35	y.
8.	1904	127·554	441	0·516	72 34·6	122 30	289 7	− 5 35	P.
	1905		441	0·463	70 59·7	125 58	292 35	− 4 40	p.
	1906		441	0·608	75 8·7	116 12	282 49	− 7 26	Π.
	1907		441	0·639	75 17·6	113 59	280 36	− 7 41	π.
	1908		441	0·884	79 30·7	91 29	258 6	−12 26	φ.
	1909		440	0·275	358 40·3	147 35	314 12	+11 31	Q.
	1910		440	0·338	14 46·3	141 19	307 56	+12 18	q.
	1911		440	0·397	23 13·4	136 30	303 7	+12 48	τ.
	1912		440	0·395	25 48·4	135 53	302 30	+11 58	R.
	1913		437	0·972	258 43·4	228 41	35 18	+10 25	T.
	1914		437	0·805	245 40·7	206 57	13 34	− 3 7	t.
	1915		437	0·926	251 37·1	220 45	27 22	+ 2 53	θ.
	1916		437	0·860	246 45·5	212 36	19 13	− 2 0	τ.
	1917		439	0·643	267 4·7	191 12	357 49	+10 3	q.
	1918		439	0·599	271 50·9	187 4	353 41	+11 43	U.
	1919		438*a*	0·508	214 23·4	180 11	346 48	−18 48	S.
	1920		438*a*	0·528	214 25·5	181 28	348 5	−19 25	s.
13.	1921	132·494	443	0·585	48 17·7	124 32	221 4	+ 9 17	P.
	1922		443	0·630	49 8·2	121 16	217 48	+ 9 46	p.
	1923		443	0·626	44 50·7	122 26	218 58	+12 13	Π.
	1924		444	0·592	24 19·2	131 57	228 29	+16 42	Q.
	1925		444	0·579	25 58·1	131 8	227 40	+20 31	q.
	1926		444	0·626	24 39·2	128 51	225 23	+23 11	x.
	1927		444	0·589	28 44·4	129 31	226 3	+19 38	K.
	1928		441	0·450	244 24·1	184 50	281 22	− 4 12	ρ.
	1929		441	0·492	246 53·2	187 36	284 8	− 3 5	r'.
	1930		441	0·481	245 45·7	186 52	283 24	− 3 37	r^2.
	1931		441	0·471	246 3·1	186 12	282 44	− 3 29	r^3.
	1932		441	0·468	242 52·9	185 58	282 30	− 4 57	r^4.
	1933		441	0·506	247 4·1	188 30	285 2	− 3 0	R_s.
	1934		441	0·506	242 9·7	188 25	284 57	− 5 28	R_t.
	1935		441	0·485	242 0·6	187 4	283 36	− 5 26	R_u.
	1936		441	0·583	243 58·7	193 43	290 15	− 4 45	S.
	1937		441	0·631	247 26·0	197 14	293 46	− 2 42	Σ_s.
	1938		441	0·612	245 19·7	195 52	292 24	− 4 0	Σ_t.
	1939		441	0·643	246 1·5	198 5	294 37	− 3 35	S′.
	1940		441	0·610	244 6·9	195 39	292 11	− 4 44	S″.
	1941		441	0·608	242 43·3	195 28	292 0	− 5 34	σ′.
	1942		441	0·593	240 10·9	194 17	290 49	− 7 1	σ″.
14.	1943	133·492	443	0·408	38 32·1	138 16	220 40	+ 9 21	P.
	1944		444	0·469	7 43·8	145 6	227 30	+21 3	Q.
	1945		441	0·698	247 8·3	203 19	285 43	− 2 54	R_s.
	1946		441	0·686	246 30·6	202 20	284 44	− 3 21	R_t.
	1947		441	0·638	245 37·6	198 39	281 3	− 3 55	ρ.
	1948		441	0·664	246 48·7	200 40	283 4	− 3 9	r'.
	1949		441	0·664	243 41·9	200 36	283 0	− 5 13	r^2.
	1950		441	0·672	243 21·5	201 14	283 38	− 5 28	r^3.
	1951		441	0·680	243 43·1	201 51	284 15	− 5 14	T_t.
	1952		441	0·691	243 44·4	202 43	285 7	− 5 15	T_u.
	1953		441	0·800	246 36·5	212 7	294 31	− 3 11	S_t.
	1954		441	0·774	245 57·7	209 45	292 9	− 3 43	S_u.
	1955		441	0·755	244 22·8	208 4	290 28	− 4 55	Σ.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1863.									
May 14.	1956	133·492	441	0·766	242° 51′·9	208° 59′	291° 23′	− 6° 5′	S_1.
	1957		441	0·775	244 11·4	209 50	292 14	− 5 5	S_2.
23.	1958	142·508	446	0·986	84 40·1	87 25	41 55	−13 24	P_1.
	1959		446	0·986	85 56·0	87 15	41 45	−14 39	P_2.
	1960		446	0·995	84 6·7	83 44	38 14	−12 52	p.
	1961		445	0·959	61 37·6	95 3	49 33	+ 8 50	Q.
	1962		445	0·979	60 21·5	90 18	44 48	+10 24	q'.
	1963		441	0·797	240 1·6	220 5	174 35	− 9 55	T.
	1964		441	0·795	240 54·7	220 0	174 30	− 9 12	t.
	1965		441	0·781	240 20·0	218 37	173 7	− 9 31	t.
	1966		441	0·800	241 30·3	220 27	174 57	− 8 46	u.
	1967		441	0·732	239 16·7	214 14	168 44	− 9 50	S.
	1968		441	0·758	240 19·0	216 32	171 2	− 9 19	s.
	1969	142·608	446	0·992	84 52·7	85 26	38 31	−13 35	P_1.
	1970		446	0·990	86 15·9	86 19	39 24	−14 58	P_2.
	1971		445	0·965	62 31·4	93 46	46 51	+ 8 6	Q.
	1972		445	0·962	62 29·0	111 14	64 19	+ 8 6	q'.
	1973		445	0·981	61 44·7	89 42	42 47	+ 9 7	q^2.
	1974		445	0·577	60 56·4	133 25	86 30	+ 4 47	K.
	1975		441	0·783	241 5·1	218 58	172 3	− 8 59	T.
	1976		441	0·783	242 8·6	218 59	172 4	− 8 9	t.
	1977		441	0·768	241 28·0	217 36	170 41	− 8 33	t.
	1978		441	0·788	242 41·5	219 30	172 35	− 7 45	u.
	1979		441	0·717	240 25·1	213 9	166 14	− 8 53	S.
24.	1980	143·515	445	0·976	62 58·3	91 49	32 1	+ 8 10	P.
	1981		446	0·869	61 15·7	109 26	49 38	+ 8 16	p.
	1982		447	0·969	56 50·2	94 7	34 19	+13 58	Q.
	1983		447	0·953	53 26·8	97 58	38 10	+16 51	q.
	1984		449	0·980	82 47·1	90 34	30 46	−11 10	R.
	1985		449	0·929	86 11·9	101 1	41 13	−14 0	r.
	1986		449	0·929	87 10·9	101 11	41 23	−14 54	ρ'.
	1987		449	0·919	86 3·0	102 32	42 44	−13 45	ρ^2.
	1988		441*a*	0·178	190 36·9	173 45	113 57	−10 20	S.
	1989		441	0·912	241 44·9	234 14	174 26	− 9 37	T.
	1990		441	0·910	242 27·4	233 58	174 10	− 8 58	τ.
25.	1991	144·653	447	0·955	63 23·5	97 43	21 47	+ 7 52	P.
	1992		447	0·946	61 24·3	99 31	23 35	+ 9 37	p.
	1993		445	0·887	61 43·8	108 9	32 13	+ 8 31	Q_1.
	1994		445	0·877	60 59·6	109 21	33 25	+ 9 9	Q_2.
	1995		446	0·704	56 23·6	126 29	50 33	+ 9 59	R.
	1996		446	0·693	57 47·4	127 9	51 13	+ 8 51	r.
	1997		446	0·739	57 38·4	123 28	47 32	+ 9 41	ρ.
	1998		448	0·938	78 57·8	100 20	24 24	− 6 55	S.
	1999		449	0·884	83 47·5	108 13	32 17	−10 57	s.
	2000		449	0·809	87 17·9	116 47	40 51	−13 2	T.
	2001		449	0·801	89 0·1	117 49	41 53	−14 15	τ.
	2002		449	0·591	76 29·4	133 49	57 53	− 3 39	u.
26.	2003	145·629	447	0·866	63 5·5	111 28	21 41	+ 7 25	P.
	2004		447	0·858	60 38·8	112 39	22 52	+ 9 24	p.
	2005		447	0·844	60 48·3	114 7	24 20	+ 9 5	Π.
	2006		447	0·830	61 40·9	115 28	25 41	+ 8 11	π.
	2007		447	0·826	63 36·8	115 42	25 55	+ 6 33	o.
	2008		445	0·765	60 23·6	121 47	32 0	+ 8 21	Q_1.
	2009		445	0·755	59 33·1	122 46	32 59	+ 8 51	Q_2.
	2010		446	0·589	53 47·7	136 22	46 35	+ 9 49	R_1.
	2011		446	0·581	53 30·2	136 58	47 11	+ 9 50	R_2.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1863.					° ′	° ′	° ′	° ′	
May 26.	2012	145·629	446	0·569	53 1·8	137 53	48 6	+ 9 51	r.
	2013		446	0·534	51 39·9	140 27	50 40	+ 9 50	S_1.
	2014		446	0·518	52 27·6	141 20	51 33	+ 9 7	S_2.
	2015		448	0·838	80 8·4	114 13	24 26	− 7 7	T.
	2016		449	0·753	86 10·8	122 45	32 58	−11 6	t.
	2017		449	0·712	90 11·3	126 47	37 0	−13 23	U.
	2018		449	0·702	89 36·4	127 30	37 43	−12 49	u.
	2019		449	0·655	92 23·9	131 34	41 47	−13 49	v.
	2020		449	0·996	5 42·5	168 35	78 48	+ 3 52	V.
	2021	145·636	447	0·866	63 9·5	111 27	21 34	+ 7 22	P.
	2022		447	0·857	60 43·0	112 46	22 53	+ 9 20	p.
	2023		447	0·842	61 2·9	114 16	24 23	+ 8 52	Π.
	2024		447	0·829	61 40·2	130 20	40 27	+ 6 8	π.
	2025		445	0·766	60 40·6	121 44	31 51	+ 8 9	Q_1.
	2026		445	0·754	59 46·4	122 51	32 58	+ 8 40	Q_2.
	2027		446	0·586	53 41·7	136 36	46 43	+ 9 50	R_1.
	2028		446	0·581	54 26·0	136 50	46 57	+ 9 19	R_2.
	2029		446	0·567	53 1·5	138 2	48 9	+ 9 49	r.
	2030		446	0·533	51 43·8	140 30	50 37	+ 9 47	S_1.
	2031		446	0·517	52 37·8	141 25	51 32	+ 9 0	S_2.
	2032		448	0·838	80 21·5	114 11	24 18	− 7 18	T.
	2033		449	0·754	86 11·6	122 42	32 49	−11 7	t.
	2034		449	0·655	92 37·4	131 39	41 46	−13 57	v.
	2035		449	0·094	70 3·7	168 38	78 45	+ 3 29	V.
27.	2036	146·562	447	0·735	63 55·7	124 57	21 57	+ 5 42	P.
	2037		447	0·796	61 37·7	119 43	16 43	+ 8 7	p.
	2038		447	0·727	60 15·7	126 0	23 0	+ 8 15	Π.
	2039		447	0·735	60 34·8	125 15	22 15	+ 8 8	π.
	2040		447	0·688	61 31·2	128 59	25 59	+ 6 52	O.
	2041		447	0·716	60 10·0	126 55	23 55	+ 8 10	o.
	2042		445	0·612	59 15·2	134 55	31 55	+ 7 18	Q_1.
	2043		445	0·600	57 57·6	135 54	32 54	+ 7 54	Q_2.
	2044		446	0·413	49 40·6	149 8	46 8	+ 8 17	R.
	2045		446	0·401	44 55·7	150 39	47 39	+ 9 44	r.
	2046		446	0·353	46 51·8	153 1	50 1	+ 7 49	ρ.
	2047		446	0·357	41 54·4	153 36	50 36	+ 9 29	S_1.
	2048		446	0·345	43 43·5	153 56	50 56	+ 8 34	S_2.
	2049		446*a*	0·204	274 52·2	182 29	79 29	+ 3 21	W.
	2050		446*a*	0·205	264 39·8	183 10	80 10	+ 1 23	w.
	2051		446*a*	0·158	280 9·6	179 40	76 40	+ 3 6	m.
	2052		448	0·697	84 10·5	128 4	25 4	− 8 43	T.
	2053		448	0·592	91 15·4	136 49	33 49	−11 41	t.
	2054		449	0·484	99 51·0	145 27	42 27	−13 39	U.
	2055		449	0·481	97 58·3	145 17	42 17	−12 44	u.
	2056		449	0·556	98 23·2	140 32	37 32	−14 48	v.
	2057	146·578	447	0·730	63 52·0	125 20	22 6	+ 5 43	P.
	2058		447	0·785	59 2·8	121 32	18 18	+ 7 47	p.
	2059		447	0·725	60 4·7	126 14	23 0	+ 8 21	Π.
	2060		447	0·733	61 23·7	125 21	22 7	+ 7 31	π.
	2061		447	0·683	61 19·4	129 25	26 11	+ 6 57	O.
	2062		447	0·712	60 0·1	127 17	24 3	+ 8 14	o.
	2063		445	0·609	59 10·4	135 9	31 55	+ 7 19	Q_1.
	2064		445	0·599	57 51·5	136 2	32 48	+ 7 57	Q_2.
	2065		446	0·409	48 49·5	149 34	46 20	+ 8 31	R.
	2066		446	0·399	44 28·2	150 50	47 36	+ 9 51	r.
	2067		446	0·354	41 18·0	153 55	50 41	+ 9 34	S_1.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1863.									
May 27.	2068	146·578	446	0·342	43° 5·9	154° 14′	51° 0′	+ 8° 41′	S_2.
	2069		448	0·695	83 58·8	128 18	25 4	− 8 33	T.
	2070		448	0·589	91 8·4	137 1	33 47	−11 34	t.
	2071		449	0·481	100 0·4	145 42	42 28	−13 38	U.
	2072		449	0·476	98 0·3	145 37	42 23	−12 38	u.
	2073		446*a*	0·208	274 56·1	182 45	79 31	+ 3 28	W.
	2074		446*a*	0·164	279 37·8	180 2	76 48	+ 3 11	w.
28.	2075	147·458	447	0·547	60 34·2	140 11	24 28	+ 5 56	P.
	2076		447	0·559	58 38·4	139 33	23 50	+ 7 9	p.
	2077		447	0·546	54 54·0	140 57	25 14	+ 8 56	Π.
	2078		447	0·562	57 29·0	139 31	23 48	+ 7 50	π.
	2079		447	0·525	55 23·5	142 15	26 32	+ 8 18	O.
	2080		447	0·532	54 12·6	141 57	26 14	+ 9 2	o.
	2081		447	0·539	56 6·3	141 15	25 32	+ 8 10	υ.
	2082		445	0·411	51 2·1	149 57	34 14	+ 8 3	Q_1.
	2083		446	0·203	8 5·7	167 31	51 48	+ 9 37	S_1.
	2084		446	0·184	9 33·0	167 45	52 2	+ 8 30	S_2.
	2085		446	0·173	9 40·8	168 1	52 18	+ 7 55	S_3.
	2086		446	0·203	359 40·5	169 8	53 25	+10 12	Z'_1.
	2087		446	0·189	358 47·4	169 32	53 49	+ 9 27	Z'_2.
	2088		446	0·177	2 33·8	169 5	53 22	+ 8 36	Z'_3.
	2089		446	0·186	3 14·2	168 47	53 4	+ 9 2	Z'_4.
	2090		446	0·250	23 39·0	163 2	47 19	+ 9 57	σ_1.
	2091		446	0·219	24 36·7	164 5	48 22	+ 8 26	σ_2.
	2092		446	0·205	23 21·3	164 49	49 6	+ 8 1	σ_3.
	2093		446	0·218	20 17·8	164 50	49 7	+ 9 1	σ_4.
	2094		449*a*	0·461	263 11·7	199 26	83 43	+ 3 48	R.
	2095		449	0·395	112 21·0	154 5	38 22	−15 21	ρ.
	2096		449	0·370	110 9·1	154 55	39 12	−13 45	ρ.
	2097		448	0·505	87 54·3	143 6	27 23	− 8 15	T.
	2098		448	0·515	87 21·2	142 23	26 40	− 8 7	t.
	2099		448	0·336	115 0·8	157 39	41 56	−13 51	t_1.
	2100		448	0·275	118 24·2	161 11	45 28	−12 8	t_2.
	2101	147·630	447	0·581	61 21·4	137 55	19 46	+ 6 0	P.
	2102		447	0·591	59 43·5	137 26	19 17	+ 7 3	p.
	2103		447	0·580	55 58·2	138 42	20 33	+ 9 1	Π.
	2104		447	0·591	57 49·6	137 40	19 31	+ 8 9	π.
	2105		447	0·552	56 24·0	140 29	22 20	+ 8 18	O.
	2106		447	0·566	55 26·3	139 42	21 33	+ 9 4	o.
	2107		445	0·571	57 8·0	139 8	20 59	+ 8 13	υ.
	2108		445	0·446	52 58·1	147 47	29 38	+ 7 56	Q_1.
	2109		446	0·435	51 10·3	148 41	30 32	+ 8 27	Q_2.
	2110		446	0·225	17 20·7	165 18	47 9	+ 9 46	S_1.
	2111		446	0·199	21 45·1	165 29	47 20	+ 7 59	S_2.
	2112		446	0·190	19 18·2	166 12	48 3	+ 7 50	S_3.
	2113		446	0·216	10 7·9	166 58	48 49	+10 7	Σ'_1.
	2114		446	0·204	9 55·9	167 20	49 11	+ 9 30	Σ'_2.
	2115		446	0·194	14 18·1	166 51	48 42	+ 8 34	Σ'_3.
	2116		446	0·202	14 17·2	166 37	48 28	+ 8 58	Σ'_4.
	2117		446	0·280	29 52·9	160 46	42 37	+10 4	σ'_1.
	2118		446	0·248	31 23·1	161 55	43 46	+ 8 32	σ'_2.
	2119		446	0·231	30 13·4	162 52	44 43	+ 8 4	σ'_3.
	2120		446	0·246	27 41·8	162 38	44 29	+ 9 7	σ'_4.
	2121		449	0·421	264 8·8	197 57	79 48	+15 19	R.
	2122		449	0·433	107 50·2	151 8	32 59	−15 6	ρ.
	2123		449	0·394	107 40·1	153 16	35 7	−13 44	S.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1863.									
May 28.	2124	147·630	448	0·539	87 0·4	140 53	22 44	− 8 13	T.
	2125		448	0·548	86 25·6	140 13	22 4	− 8 1	τ_1.
	2126		448	0·365	110 58·3	155 30	37 21	−13 46	τ_2.
	2127		448	0·306	112 58·3	158 50	40 41	−12 10	t.
29.	2128	148·455	447	0·383	55 12·2	152 3	22 11	+ 6 3	P.
	2129		447	0·389	55 12·2	152 30	22 38	+ 8 18	p.
	2130		447	0·378	49 21·4	153 19	23 27	+ 8 29	Π.
	2131		447	0·374	46 7·6	153 52	24 0	+ 9 2	π.
	2132		447	0·370	47 29·1	153 51	23 59	+ 8 29	σ.
	2133		445	0·252	35 53·9	161 52	32 0	+ 7 57	Q_1.
	2134		445	0·248	31 53·7	162 40	32 48	+ 8 36	Q_2.
	2135		445	0·222	46 7·6	162 5	32 13	+ 4 59	q.
	2136		446	0·196	337 16·2	174 39	44 47	+10 20	σ_1.
	2137		446	0·163	328 9·0	175 54	46 2	+ 8 9	σ_2.
	2138		446	0·173	324 47·4	176 37	46 45	+ 8 32	σ_3.
	2139		448	0·337	96 6·9	155 13	25 21	− 8 15	T.
	2140		448	0·350	95 10·3	154 23	24 31	− 8 13	t.
	2141		446	0·240	308 8·6	181 31	51 39	+10 26	S_1.
	2142		446	0·209	308 11·4	180 24	50 32	+ 8 57	S_2.
	2143		446	0·189	304 30·4	180 18	50 26	+ 7 37	S_3.
	2144		446	0·232	304 46·5	181 50	51 58	+ 9 34	Σ_1.
	2145		446	0·208	303 43·7	181 8	51 16	+ 8 21	Σ_2.
	2146		446	0·633	261 30·3	212 16	82 24	+ 4 28	R.
	2147	148·663	447	0·340	53 35·6	154 59	22 11	+ 5 50	P.
	2148		447	0·351	46 16·5	155 20	22 32	+ 8 25	p.
	2149		447	0·336	44 27·0	156 26	23 38	+ 8 34	Π.
	2150		447	0·336	42 31·7	156 46	23 58	+ 9 7	π.
	2151		447	0·329	43 57·3	156 52	24 4	+ 8 31	o.
	2152		445	0·216	28 38·7	164 46	31 58	+ 7 56	Q_1.
	2153		445	0·215	24 9·2	165 43	32 55	+ 8 20	Q_2.
	2154		445	0·185	40 36·4	164 44	31 56	+ 4 56	q.
	2155		446	0·208	323 54·8	177 41	44 53	+10 26	σ_1.
	2156		446	0·181	313 14·5	178 55	46 7	+ 8 11	σ_2.
	2157		446	0·190	312 42·1	179 15	46 27	+ 8 33	σ_3.
	2158		448	0·297	99 42·2	158 5	25 17	− 8 18	T.
	2159		448	0·310	98 26·5	157 12	24 24	− 8 17	t.
	2160		446	0·269	299 57·9	184 28	51 40	+10 26	S_1.
	2161		446	0·242	298 44·3	183 32	50 44	+ 9 4	S_2.
	2162		446	0·223	294 47·7	183 18	50 30	+ 7 37	S_3.
	2163		446	0·261	297 25·3	184 35	51 47	+ 9 36	Σ_1.
	2164		446	0·241	295 17·5	184 3	51 15	+ 8 25	Σ_2.
	2165		446	0·674	261 13·8	215 31	82 43	+ 4 37	R.
June 1.	2166	151·500	451	0·931	79 20·6	108 7	295 4	− 4 40	P.
	2167		451	0·946	78 57·5	105 33	292 30	− 4 21	p.
	2168		451	0·942	81 49·9	106 21	293 18	− 7 3	π.
	2169		450	0·752	86 5·3	128 17	315 14	− 8 59	T_1.
	2170		450	0·747	85 49·5	128 45	315 42	− 8 43	T_2.
	2171		450	0·740	86 27·9	129 24	316 21	− 9 7	.
	2172		450	0·784	85 46·6	125 22	312 19	− 9 6	τ_1.
	2173		450	0·768	85 38·7	126 50	313 47	− 8 49	τ_2.
	2174		450	0·756	85 22·6	127 56	314 53	− 8 29	τ_3.
	2175		447	0·231	300 52·9	185 35	12 32	+ 9 7	σ_1.
	2176		447	0·221	298 27·8	185 32	11 29	+ 8 19	σ_2.
	2177		447	0·248	289 56·5	188 2	14 9	+ 7 46	σ_3.
	2178		447	0·344	274 10·6	195 17	22 14	+ 6 11	Σ_1.
	2179		447	0·314	271 55·4	206 42	33 39	+ 8 37	Σ_2.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1863.									
June 1.	2180	151·500	445	0·504	271° 17′·8	205° 23′	32° 20′	+ 7° 56′	Q_1.
	2181		445	0·515	271 56·7	205 58	32 55	+ 8 26	Q_2.
	2182		445	0·475	267 58·9	203 54	30 51	+ 5 55	q.
	2183		448	0·400	234 48·8	198 35	25 32	− 8 8	R.
	2184		446	0·716	266 25·2	221 13	48 10	+ 8 9	S.
	2185		446	0·755	266 4·5	224 32	51 29	+ 8 23	U.
	2186		446	0·771	268 1·1	225 45	52 42	+10 3	V.
2.	2187	152·487	451	0·828	80 29·1	121 43	294 40	− 4 49	P.
	2188		451	0·853	80 0·4	119 5	292 2	− 4 43	p.
	2189		451	0·846	82 34·8	119 57	292 54	− 6 41	π.
	2190		451	0·885	76 56·7	115 14	288 11	− 1 59	v.
	2191		451	0·925	60 4·5	110 35	283 32	+13 31	w.
	2192		450	0·577	90 26·0	143 8	316 5	− 9 10	T_1.
	2193		450	0·580	89 32·0	142 51	315 48	− 8 42	T_2.
	2194		450	0·591	89 3·1	142 1	314 58	− 8 34	t.
	2195		450	0·625	88 39·3	139 32	312 29	− 8 48	t_1.
	2196		450	0·626	89 27·8	139 33	312 30	− 9 19	t_2.
	2197		445	0·683	266 51·1	219 32	32 29	+ 7 52	Q_1.
	2198		445	0·692	267 30·9	220 10	33 7	+ 8 26	Q_2.
	2199		445	0·635	265 15·6	216 4	29 1	+ 6 17	q_1.
	2200		445	0·655	264 9·9	217 37	30 34	+ 5 47	q_2.
	2201		447	0·411	280 26·6	199 18	12 15	+ 9 51	S.
	2202		447	0·434	273 4·5	201 46	14 43	+ 7 27	σ_1.
	2203		447	0·453	271 6·0	201 54	14 51	+ 6 36	σ_2.
	2204		448	0·593	241 30·4	212 45	25 42	− 8 8	R.
	2205		446	0·893	266 0·6	239 45	52 42	+ 9 44	V.
3.	2206	153·487	451	0·672	81 58·8	136 24	295 10	− 4 40	P.
	2207		451	0·702	81 40·1	134 2	292 48	− 4 38	p.
	2208		451	0·705	82 48·7	133 53	292 39	− 4 22	π.
	2209		451	0·805	58 7·1	126 7	284 53	+13 33	w.
	2210		450	0·372	99 48·7	158 16	317 2	− 9 4	T.
	2211		450	0·392	96 15·1	156 36	315 22	− 8 15	t_1.
	2212		450	0·386	96 59·2	157 5	315 51	− 8 23	t_2.
	2213		450	0·432	94 3·1	153 55	312 41	− 8 10	τ_1.
	2214		450	0·441	95 7·6	153 31	312 17	− 8 47	τ_2.
	2215		450	0·453	96 4·6	152 57	311 43	− 9 25	t_3.
	2216		447	0·600	272 50·5	213 39	12 25	+10 13	S.
	2217		447	0·605	273 2·6	213 55	12 41	+10 25	σ.
	2218		447	0·668	266 10·8	219 23	18 9	+ 7 4	s.
	2219		445	0·836	264 52·6	234 18	33 4	+ 7 53	Q_1.
	2220		445	0·841	265 29·2	234 51	33 37	+ 8 26	Q_2.
	2221		445	0·846	262 50·9	235 30	34 16	+ 6 16	q.
4.	2222	154·517	451	0·484	84 50·3	150 42	294 51	− 4 30	P_1.
	2223		451	0·492	85 20·1	150 13	294 22	− 4 49	P_2.
	2224		451	0·522	84 39·7	148 12	292 21	− 4 45	p_1.
	2225		451	0·518	84 1·7	148 26	292 35	− 4 23	p_2.
	2226		451	0·572	90 50·5	145 25	289 34	− 8 40	Π.
	2227		451	0·518	87 47·6	148 42	292 51	− 6 19	π_1.
	2228		451	0·516	89 19·5	148 59	293 8	− 7 4	π_2.
	2229		451	0·506	88 28·7	149 34	293 43	− 6 31	π_3.
	2230		451	0·471	89 17·0	151 53	296 2	− 6 26	ξ.
	2231		450	0·254	112 32·6	167 24	311 33	− 8 50	S_1.
	2232		450	0·264	113 46·7	167 7	312 6	− 9 26	S_2.
	2233		450	0·259	117 29·6	167 59	312 8	− 9 59	σ.
	2234		450	0·280	116 57·3	166 55	311 4	−10 41	s.
	2235		450	0·237	111 13·6	168 2	312 11	− 7 59	v_1.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1863.					° ′	° ′	° ′	° ′	
June 4.	2236	154·517	450	0·228	113 56·0	168 51	313 0	− 8 11	n_2.
	2237		450	0·215	116 35·0	169 48	313 57	− 8 9	n_3.
	2238		450	0·193	127 56·1	172 23	316 2	− 8 50	o_1.
	2239		450	0·194	132 26·1	173 3	317 12	− 9 24	o_2.
	2240		445	0·939	265 47·9	248 27	32 36	+ 9 27	Q_1.
	2241		445	0·944	266 31·2	249 19	33 28	+10 12	Q_2.
	2242		447	0·765	271 55·4	227 45	11 54	+12 16	T_1.
	2243		447	0·761	271 26·2	227 29	11 38	+11 51	T_2.
	2244	154·769	451	0·430	85 52·0	154 24	294 59	− 4 23	P_1.
	2245		451	0·439	86 17·1	153 52	294 27	− 4 39	P_2.
	2246		451	0·467	84 39·7	151 59	292 34	− 4 12	p.
	2247		451	0·492	88 51·9	150 46	291 11	− 6 26	π_1.
	2248		451	0·463	90 29·2	152 45	293 20	− 6 48	π_2.
	2249		451	0·421	90 40·5	155 24	295 59	− 6 16	π_3.
	2250		451	0·524	92 17·3	149 4	289 39	− 8 36	Π.
	2251		450	0·213	121 22·2	170 48	311 23	− 8 48	S.
	2252		450	0·191	120 40·7	171 37	312 12	− 7 48	s.
	2253		450	0·160	143 8·2	175 53	316 28	− 8 32	O_1.
	2254		450	0·165	147 34·7	176 28	317 3	− 9 3	O_2.
	2255		445	0·962	264 15·3	253 4	33 39	+ 8 8	Q_1.
	2256		445	0·962	265 4·1	253 2	33 37	+ 8 55	Q_2.
6.	2257	156·471	451	0·090	132 16·2	178 11	294 37	− 4 6	P_1.
	2258		451	0·094	133 45·8	178 10	294 36	− 4 17	P_2.
	2259		452	0·358	23 43·0	168 3	284 29	+16 35	R.
	2260		452	0·339	25 28·6	168 19	284 45	+15 20	r.
	2261		452	0·326	27 16·4	168 24	284 50	+14 21	U.
	2262		452	0·319	30 1·9	168 3	284 29	+13 25	u.
	2263		452	0·345	26 22·5	167 50	284 16	+15 24	v.
	2264		450	0·273	221 55·8	194 8	310 34	− 8 43	S.
	2265		450	0·287	219 15·7	194 25	310 51	− 9 48	s.
	2266		450	0·374	232 38·8	201 15	317 41	− 8 31	Q_2.
	2267		450	0·370	230 5·3	200 35	317 1	− 9 16	Q_1.
8.	2268	158·492	453	0·767	65 53·0	133 38	221 24	+ 8 48	P.
	2269		453	0·913	66 26·1	117 34	205 20	+10 0	p.
	2270		452	0·349	310 16·4	195 34	283 20	+16 30	S_1.
	2271		452	0·351	308 47·3	196 3	283 49	+16 15	S_2.
	2272		452	0·341	304 29·2	196 47	284 33	+14 49	T.
	2273		452	0·357	304 15·5	197 31	285 17	+15 25	t.
	2274		452	0·374	303 41·5	198 27	286 13	+16 1	S.
	2275		452	0·351	299 23·3	198 27	286 13	+13 37	u.
	2276		451	0·409	246 25·2	206 36	294 22	− 4 2	R.
	2277		450	0·736	244 29·6	229 9	316 55	− 7 8	O.
	2278		450	0·757	244 47·3	231 14	319 0	− 7 7	o.
	2279	158·522	453	0·759	65 43·3	134 22	221 43	+ 8 57	P.
	2280		453	0·910	66 10·6	118 8	205 29	+10 13	p.
	2281		452	0·356	309 20·3	196 10	283 31	+16 38	S_1.
	2282		452	0·357	307 59·8	196 35	283 56	+16 22	S_2.
	2283		452	0·350	303 38·8	197 25	284 46	+15 0	T.
	2284		452	0·364	303 24·1	198 7	285 28	+15 32	t.
	2285		452	0·381	303 18·8	198 55	286 16	+16 14	s.
	2286		452	0·361	298 40·5	199 9	286 30	+14 8	u.
	2287		451	0·416	246 43·1	207 7	294 28	− 4 0	R.
	2288		450	0·744	244 46·4	230 8	317 29	− 8 58	O.
	2289		450	0·763	244 58·1	231 51	319 12	− 9 4	o.
9.	2290	159·489	453	0·599	63 27·3	148 4	221 42	+ 8 49	P.
	2291		453	0·788	65 55·5	132 38	206 16	+ 9 30	p.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1863.									
June 9.	2292	159·489	452	0·503	296° 6′·7	210° 0′	283° 38′	+16° 2′	S_1.
	2293		452	0·506	290 42·0	208 54	282 32	+16 24	S_2.
	2294		452	0·513	289 42·1	210 44	284 22	+16 11	s.
	2295		452	0·525	285 56·5	212 23	286 1	+14 48	T.
	2296		452	0·522	287 38·8	211 47	285 25	+15 31	t.
	2297		452	0·529	283 45·1	213 5	286 43	+13 50	U.
	2298		452	0·538	287 53·0	212 43	286 21	+16 5	u.
	2299		452	0·242	286 2·5	213 25	287 3	+15 19	v.
	2300		451	0·608	250 8·4	220 55	294 33	− 4 8	R.
	2301		451	0·582	241 47·3	218 18	291 56	− 8 43	r.
	2302		451	0·602	241 5·3	219 36	292 14	− 9 27	ę.
	2303		450	0·875	247 0·7	244 15	317 53	− 9 2	Q.
	2304		450	0·889	246 51·8	245 54	319 32	− 9 19	q.
	2305		450	0·876	248 21·5	244 25	318 3	− 7 52	o.
	2306	159·615	453	0·576	63 16·1	149 50	221 41	+ 8 39	P.
	2307		453	0·770	65 48·0	134 28	206 19	+ 9 26	p.
	2308		452	0·522	288 40·3	211 40	283 31	+15 59	S_1.
	2309		452	0·524	289 13·5	211 38	283 29	+16 17	S_2.
	2310		452	0·531	288 13·5	212 20	284 11	+16 2	s.
	2311		452	0·545	284 36·2	214 3	285 54	+14 39	T.
	2312		452	0·542	286 11·0	213 31	285 22	+15 22	t.
	2313		452	0·547	282 44·4	214 36	286 27	+13 47	U.
	2314		452	0·556	286 15·0	214 25	286 16	+15 48	u.
	2315		452	0·557	284 58·4	214 45	286 36	+15 10	v.
	2316		451	0·627	250 31·1	222 30	294 21	− 4 5	R.
	2317		451	0·600	242 32·3	219 45	291 36	− 8 35	r.
	2318		451	0·622	241 26·9	221 10	293 1	− 9 35	ę.
	2319		450	0·888	247 4·1	245 55	317 46	− 9 10	Q.
	2320		450	0·901	246 56·0	247 35	319 26	− 9 26	q.
	2321		450	0·886	248 24·6	245 45	317 36	− 7 57	o.
10.	2322	160·604	483	0·387	56 21·4	163 45	221 34	+ 8 48	P.
	2323		452	0·688	281 8·6	226 2	283 51	+16 3	S.
	2324		452	0·698	280 27·1	226 57	284 46	+15 48	s.
	2325		452	0·692	279 15·0	226 44	284 33	+14 53	T.
	2326		452	0·700	279 10·8	227 20	285 9	+14 59	t.
	2327		452	0·716	277 6·5	229 1	286 50	+13 53	U_1.
	2328		452	0·713	277 45·2	228 39	286 28	+14 17	U_2.
	2329		452	0·708	279 55·0	227 50	285 39	+15 39	u.
	2330		452	0·713	279 14·9	228 25	286 14	+15 19	v.
	2331		451	0·792	252 33·8	236 57	294 46	− 3 59	R.
	2332		450	0·968	248 14·4	259 49	317 38	− 9 22	Q.
11.	2333	161·586	452	0·840	277 23·1	241 32	285 25	+16 9	S.
	2334		452	0·828	277 43·2	240 12	284 5	+16 12	s.
	2335		452	0·856	276 22·3	243 28	287 21	+15 35	T.
	2336		452	0·850	274 38·4	242 59	286 52	+14 1	U.
	2337		451	0·911	254 3·0	251 14	295 7	− 3 45	R.
	2338	161·608	453	0·198	33 37·6	177 48	221 22	+ 8 45	P.
	2339		452	0·838	277 18·0	241 22	284 56	+16 2	S.
	2340		452	0·830	277 35·9	240 29	284 3	+16 8	s.
	2341		452	0·857	276 12·7	243 36	287 10	+15 27	T.
	2342		452	0·851	274 27·2	243 11	286 45	+13 53	U.
	2343		451	0·912	254 10·9	251 23	294 57	− 3 39	R.
13.	2344	163·484	453	0·323	283 37·2	204 59	221 57	+ 8 33	P.
	2345		454	0·398	231 18·4	208 31	225 29	− 9 49	S.
	2346		454	0·423	233 30·9	210 21	227 19	− 9 40	T_1.
	2347		454	0·415	233 53·5	209 56	226 54	− 9 19	T_2.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1863.									
June 13.	2348	163·484	454	0·451	237° 8·1	212° 35′	229° 33′	− 8° 52′	O.
	2349		454	0·461	237 42·7	213 19	230 17	− 8 52	o.
	2350	163·668	453	0·345	281 53·8	206 35	220 57	+ 8 30	P.
	2351		454	0·415	232 31·2	209 52	224 14	− 9 51	S.
	2352		454	0·442	233 55·5	211 43	226 5	−10 0	T_1.
	2353		454	0·453	235 22·1	212 37	226 59	− 9 41	T_2.
	2354		454	0·474	238 6·8	214 20	282 42	− 8 58	O.
	2355		454	0·483	238 43·0	215 1	229 23	− 8 54	o.
19.	2356	169·677	455*a**	0·846	22 55·9	153 24	82 31	+47 44	P.
	2357		455*a**	0·832	31 40·8	149 5	78 12	+40 55	p.
	2358		455	0·669	100 17·0	153 26	82 33	−10 46	S.
	2359		455	0·646	100 52·4	155 16	84 23	−10 40	s.
	2360		455	0·625	101 20·9	156 54	86 1	−10 31	T.
	2361		485	0·641	101 2·1	155 38	84 45	−10 41	t.
	2362		483*a*	0·620	285 17·1	229 39	158 46	+15 31	O.
20.	2363	170·595	456	0·963	75 10·6	120 16	36 25	+ 7 27	P.
	2364		455	0·450	109 40·3	170 33	86 39	−10 11	S.
	2365		455	0·434	104 21·4	170 36	86 42	− 7 38	s.
	2366		455	0·351	149 30·9	186 16	102 22	−17 0	R.
	2367		455	0·955	262 33·1	266 55	183 1	+ 0 41	O.
	2368		455	0·769	281 20·3	243 16	159 25	+15 36	T.
21.	2369	171·501	456	0·891	75 39·8	132 35	35 50	+ 7 14	P.
	2370	171·512	456	0·887	75 43·6	132 59	36 4	+ 7 11	P.
23.	2371	173·472	456	0·591	74 39·2	161 18	36 35	+ 7 3	P.
	2372		456	0·564	72 44·6	163 19	38 36	+ 7 55	p.
	2373		457	0·680	61 29·4	156 21	31 38	+16 28	S.
	2374		457	0·660	61 29·2	157 53	33 10	+16 3	s.
	2375		457	0·597	60 32·5	162 36	37 53	+15 19	T.
	2376		457	0·658	63 33·6	157 39	32 56	+14 43	t.
	2377	173·725	456	0·547	73 56·4	164 41	36 23	+ 7 12	P.
	2378		456	0·518	71 28·9	166 47	38 29	+ 8 12	p.
	2379		457	0·646	60 15·1	159 23	31 5	+16 38	S.
	2380		457	0·621	59 52·2	161 19	33 1	+16 18	s.
	2381		457	0·548	58 54·3	166 34	38 16	+15 10	T.
	2382		457	0·611	61 51·1	161 38	33 20	+14 57	t.
25.	2383	175·665	457	0·924	58 20·1	133 21	337 32	+25 10	O.
	2384		457	0·508	57 48·0	171 23	15 34	+15 23	Q.
	2385		456	0·146	50 4·1	192 20	36 31	+ 7 8	P.
	2386		456	0·166	52 43·2	191 8	35 19	+ 7 24	p.
	2387		457	0·220	8 49·6	196 5	40 16	+14 41	T.
	2388		457	0·255	23 47·7	191 54	36 5	+15 12	t.
	2389		457	0·299	30 19·2	188 50	33 1	+16 23	R
	2390		457	0·232	12 5·8	195 3	39 14	+15 9	ς.
	2391		455	0·741	287 14·9	245 10	89 21	+18 9	V.
	2392		455	0·744	285 41·9	245 40	89 51	+17 5	u.
	2393		455	0·792	286 23·2	249 57	93 8	+18 31	v.
26.	2394	176·539	456	0·112	312 32·6	204 25	36 12	+ 7 9	P.
	2395		456	0·100	325 28·6	202 55	34 42	+ 7 26	p.
	2396		457*a*	0·864	243 23·1	257 2	88 49	−17 15	T.
	2397		457*a*	0·902	243 52·6	261 56	93 43	−17 52	t.
	2398		457*a*	0·878	242 13·7	258 28	90 15	−18 37	t.
	2399		457*a*	0·234	330 8·2	205 55	37 42	+14 40	S.
	2400		457*a*	0·249	335 21·7	205 4	36 51	+15 56	s.
	2401		457*b*	0·240	336 36·5	204 35	36 22	+15 32	σ.
	2402		457*b*	0·735	63 47·1	154 29	86 16	+17 10	O.
	2403		457*b*	0·890	284 50·9	261 58	93 45	+18 39	Q.

455*a** } are both doubtful spots.
455*a** }

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1863.					° ′	° ′	° ′	° ′	
June 27.	2404	177·524	456	0·343	279 15·8	220 31	38 26	+ 7 6	P.
	2405		456	0·314	282 27·7	218 35	36 24	+ 7 43	p.
	2406		456	0·294	275 43·2	217 51	35 40	+ 5 28	π.
	2407		457	0·956	246 38·7	271 57	89 46	−17 10	T.
	2408		457	0·989	246 45·6	281 7	98 56	−18 8	t.
	2409		458	0·318	164 56·1	297 32	15 21	−15 33	Q.
	2410		458	0·274	357 55·1	200 20	18 9	+18 25	S.
	2411		458	0·288	6 59·8	197 33	15 22	+18 57	s.
	2412		458	0·288	8 52·8	197 0	16 49	+18 48	σ.
29.	2413	179·576	458	0·709	60 37·3	160 22	315 5	+20 3	O.
	2414		458	0·681	60 52·9	162 31	311 14	+19 14	o_1.
	2415		458	0·682	60 11·5	162 35	310 18	+19 42	o_2.
	2416		458	0·620	109 13·5	167 23	316 6	−11 32	Q.
	2417		456	0·715	273 1·3	248 26	37 9	+ 6 40	P.
	2418		456	0·681	274 55·2	245 39	34 22	+ 7 49	p.
	2419		456	0·680	271 54·3	245 38	34 21	+ 5 46	π.
	2420		457*a*	0·690	287 45·3	244 57	33 40	+16 35	T_1.
	2421		457*a*	0·699	287 11·5	245 46	34 29	+16 22	T_2.
	2422		457*a*	0·719	287 28·8	247 19	36 22	+16 56	t.
	2423		457*a*	0·742	284 37·7	249 44	38 27	+15 17	U.
	2424		457*a*	0·750	285 47·4	250 24	39 7	+15 31	u.
	2425		457*a*	0·755	283 34·0	250 59	39 42	+14 41	V.
	2426		457*a*	0·767	283 17·0	252 6	40 49	+14 39	v.
	2427		457*a*	0·675	288 46·3	243 34	32 17	+16 58	t.
	2428	179·585	458	0·709	60 39·9	160 20	308 55	+20 1	O.
	2429		458	0·680	61 2·2	162 35	311 10	+19 6	o_1.
	2430		458	0·681	60 21·6	162 42	311 17	+19 33	o_2.
	2431		458*a*	0·622	109 12·7	167 18	315 53	−11 33	Q.
	2432		456	0·715	272 49·2	248 26	37 1	+ 6 31	P.
	2433		456	0·680	275 3·3	245 33	34 8	+ 7 54	p.
	2434		456	0·659	272 0·3	244 3	32 38	+ 5 47	π.
	2435		457*a*	0·690	287 36·3	244 58	33 33	+16 29	T_1.
	2436		457*a*	0·699	287 8·9	245 46	34 21	+16 20	T_2.
	2437		457*a*	0·712	287 11·3	246 48	35 23	+16 36	t.
	2438		457*a*	0·740	284 30·0	238 15	26 50	+15 10	U.
	2439		457*a*	0·749	285 49·8	250 9	38 44	+16 16	u.
	2440		457*a*	0·753	283 14·2	250 56	39 31	+14 26	V.
	2441		457*a*	0·766	283 7·8	252 1	40 36	+14 31	v.
	2442		457*a*	0·676	288 40·6	243 39	32 14	+16 55	t.
30.	2443	180·651	458	0·522	52 54·9	176 48	310 16	+19 33	Q.
	2444		458	0·549	53 7·3	175 2	308 30	+20 19	q.
	2445		458	0·726	68 47·6	158 32	292 0	+15 8	O.
	2446		458	0·748	69 38·4	156 33	290 1	+14 50	o.
	2447		457*a*	0·661	278 38·3	244 56	18 24	+ 9 59	R.
	2448		457*a*	0·678	276 19·9	246 23	19 51	+ 8 31	r.
	2449		457*a*	0·636	276 49·7	243 6	16 34	+ 8 32	ρ.
	2450		456	0·866	272 31·6	263 49	37 17	+ 6 18	P.
	2451		456	0·835	274 18·3	260 30	33 58	+ 7 46	p.
	2452		457*a*	0·835	285 14·3	259 41	33 9	+16 50	U.
	2453		457*a*	0·844	284 30·2	260 48	34 16	+16 21	u.
	2454		457*a*	0·889	281 49·4	266 18	39 46	+14 33	V.
	2455		457*a*	0·853	284 34·8	261 48	35 16	+16 32	v.
	2456	180·661	458	0·521	52 32·3	176 55	310 14	+19 43	Q.
	2457		458	0·548	52 52·4	175 9	308 28	+20 25	q.
	2458		458	0·726	68 43·7	158 36	291 55	+15 10	O.
	2459		458	0·748	69 32·8	156 35	289 54	+14 54	o.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1863.									
June 30.	2460	180·661	457*a*	0·663	278° 43·9	245° 4′	18° 23′	+ 9° 59′	R.
	2461		457*a*	0·681	276 18·9	246 36	19 55	+ 8 31	r.
	2462		457*a*	0·643	276 55·3	243 41	17 0	+ 8 39	ρ.
	2463		456	0·867	272 48·9	236 59	10 18	+ 5 40	P.
	2464		456	0·836	274 36·2	260 33	33 52	+ 8 0	p.
	2465		457*a*	0·837	285 34·4	259 53	33 12	+17 8	U.
	2466		457*a*	0·846	284 42·5	261 1	34 20	+16 33	u.
	2467		457*a*	0·891	282 0·9	266 34	39 53	+14 44	V.
	2468		457*a*	0·855	284 42·9	261 58	35 17	+16 40	v.
July 1.	2469	181·522	459	0·468	96 43·9	177 26	298 32	− 1 38	O.
	2470		459	0·486	92 53·6	176 2	297 8	+ 0 0	o.
	2471		457	0·820	275 57·3	259 42	20 48	+ 8 47	r.
	2472		457	0·775	276 10·0	255 22	16 28	+ 8 44	ρ.
	2473		456	0·946	273 24·4	275 47	36 53	+ 6 43	P.
	2474		457	0·961	281 24·3	278 43	39 49	+14 20	V.
	2475		457	0·919	284 55·2	271 13	32 19	+17 19	v.
	2476		457	0·920	285 28·2	271 20	32 26	+17 50	u.
2.	2477	182·501	460	0·729	63 5·5	161 9	268 22	+19 56	O.
	2478		460	0·713	62 44·8	162 33	269 46	+19 50	o.
	2479		459	0·690	87 54·3	162 13	269 26	+ 2 13	Q
	2480		459	0·737	86 18·5	158 22	265 35	+ 3 14	q.
	2481		458	0·273	96 56·2	190 8	297 21	+ 0 34	R.
	2482	182·513	460	0·719	62 45·5	162 2	269 5	+19 58	O.
	2483		460	0·700	62 26·2	163 40	270 43	+19 46	o.
	2484		459	0·677	87 55·8	163 16	270 19	+ 2 15	Q.
	2485		459	0·727	86 12·5	159 16	266 19	+ 3 20	q.
	2486		458	0·254	97 35·8	191 17	298 20	+ 0 35	R.
3.	2487	183·517	460	0·570	57 13·6	175 34	268 23	+19 49	O.
	2488		460	0·567	56 17·0	175 59	268 48	+20 13	o.
	2489		460	0·550	55 56·2	177 11	270 0	+19 54	o.
	2490		459	0·493	88 59·5	173 41	266 30	+25 45	Q.
	2491		459	0·557	86 21·1	172 56	265 45	+ 3 47	q_1.
	2492		459	0·562	86 47·2	172 36	265 25	+ 3 32	q_2.
	2493	183·683	460	0·540	55 35·5	178 6	269 59	+19 49	O.
	2494		460	0·536	54 37·2	178 37	270 30	+20 9	o.
	2495		460	0·518	54 5·0	179 56	271 49	+19 48	o.
	2496		459	0·455	89 25·3	179 52	271 45	+ 2 27	Q.
	2497		459	0·520	86 19·8	175 35	267 28	+ 3 51	q_1.
	2498		459	0·526	86 47·6	175 12	267 5	+ 3 36	q_2.
4.	2499	184·466	460	0·286	91 46·9	191 2	270 23	+ 2 21	Q.
	2500		460	0·643	82 24·5	167 45	247 6	+ 6 37	q.
	2501		460	0·223	342 22·5	211 18	290 39	+15 39	O.
	2502		460	0·219	335 24·9	212 42	292 3	+14 54	o.
	2503		459	0·411	296 46·0	229 26	308 47	+14 13	S.
	2504		459	0·407	297 42·4	229 3	308 24	+14 28	s.
	2505		459	0·429	293 14·6	230 8	309 29	+13 19	T.
	2506		459	0·435	293 25·6	231 27	310 48	+13 31	t.
	2507	184·478	460	0·287	91 26·5	191 13	270 24	+ 2 30	Q.
	2508		460	0·555	80 36·1	174 19	253 30	+ 7 21	q.
	2509		460	0·222	343 19·4	211 18	290 29	+15 43	O.
	2510		460	0·219	336 52·1	212 39	291 50	+15 4	o.
	2511		459	0·408	297 21·9	229 25	308 36	+14 22	S.
	2512		459	0·402	298 8·0	228 56	308 7	+14 29	s.
	2513		459	0·424	293 36·8	230 59	310 10	+13 20	T.
	2514		459	0·431	293 36·1	231 24	310 35	+13 29	t.
5.	2515	185·517	462	0·994	73 9·4	124 7	188 33	+16 18	P.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1863.					° ′	° ′	° ′	° ′	
July 5.	2516	185·817	462	0·987	74 24·2	127 31	191 57	+15 8	R.
	2517		462	0·971	76 4·5	132 20	196 46	+13 33	Q.
	2518		459a	0·395	75 33·0	185 52	250 18	+ 8 32	O.
	2519		459a	0·307	76 37·8	191 7	255 33	+ 7 8	o.
	2520		459b	0·505	120 57·6	182 38	247 4	−12 15	U.
	2521		459b	0·460	120 42·5	185 10	249 36	−10 41	u.
	2522		461	0·581	289 1·9	242 47	307 13	+14 14	S.
	2523		461	0·599	287 11·8	244 17	308 43	+13 28	s.
	2524		461	0·634	285 40·4	247 3	311 29	+13 4	T.
	2525		461	0·674	285 57·4	250 4	314 30	+13 48	t.
6.	2526	186·486	463	0·944	74 9·3	138 52	189 34	+15 46	P.
	2527		463	0·928	75 32·5	141 24	192 6	+14 23	R.
	2528		462	0·895	76 24·2	146 8	196 50	+13 25	Q.
	2529		459	0·338	56 23·1	192 40	243 22	+14 3	U.
	2530		459	0·194	62 14·8	199 31	250 13	+ 8 38	O.
	2531		459	0·188	60 40·7	199 59	250 41	+ 8 44	o.
	2532		459	0·179	58 58·5	200 34	251 16	+ 8 46	v.
	2533		459	0·151	60 43·3	201 52	252 34	+ 7 44	W.
	2534		459	0·162	64 55·4	201 0	251 42	+ 7 25	w.
	2535		459	0·081	47 54·5	206 3	256 45	+ 6 39	U.
	2536		459	0·099	50 37·3	205 5	255 47	+ 7 8	u.
	2537		461	0·730	285 6·0	255 43	306 25	+13 42	S_1.
	2538		461	0·725	285 40·6	255 15	305 57	+14 3	S_2.
	2539		461	0·754	284 0·7	257 56	308 38	+13 10	s.
	2540		461	0·789	282 53·7	261 13	311 55	+12 39	T.
	2541		461	0·783	283 8·3	260 39	311 21	+12 47	t.
7.	2542	187·495	463	0·844	73 28·3	153 26	189 5	+15 57	P.
	2543		462	0·771	75 36·8	160 33	196 57	+13 28	Q.
	2544		459	0·215	283 34·5	222 34	258 58	+ 6 28	U.
	2545		459	0·195	284 49·3	221 20	257 44	+ 6 27	u.
	2546		459	0·136	296 36·2	217 29	253 53	+ 7 8	v.
	2547		461	0·865	283 48·4	270 3	306 27	+13 42	S.
	2548		461	0·915	282 11·5	276 36	313 0	+12 33	T.
	2549	187·503	463	0·842	73 40·4	153 41	189 57	+15 55	P.
	2550		462	0·770	75 34·0	160 30	196 46	+13 29	Q.
	2551		459	0·217	282 29·6	222 16	258 32	+ 6 24	U.
	2552		459	0·195	283 42·2	221 15	257 31	+ 6 27	u.
	2553		459	0·137	295 8·1	217 19	253 35	+ 7 7	v
	2554		461	0·866	283 39·6	270 0	306 16	+13 40	S.
	2555		461	0·916	282 5·7	276 28	312 44	+12 37	T.
9.	2556	189·458	463	0·546	68 7·4	181 1	189 33	+15 31	P_1.
	2557		463	0·551	67 13·1	180 49	189 21	+16 6	P_2.
	2558		463	0·525	71 33·1	181 55	190 27	+12 48	p.
	2559		462	0·428	69 8·0	188 25	196 57	+12 41	Q_1.
	2560		462	0·439	68 14·3	187 50	196 22	+13 17	Q_2.
	2561		462	0·447	66 15·6	187 39	196 11	+14 17	Q_3.
	2562		464	0·638	116 44·0	176 20	184 52	−12 54	U.
	2563		464	0·634	116 14·0	176 32	185 4	−12 29	u.
	2564		464	0·664	116 21·4	174 23	182 55	−13 25	v.
	2565		464	0·671	113 35·9	173 12	181 44	−11 54	w.
	2566		459	0·692	278 9·1	256 2	264 34	+ 7 46	T.
	2567		459	0·656	275 51·8	253 15	261 47	+ 6 8	t.
	2568		459	0·637	280 49·8	251 38	260 10	+ 8 50	t.
	2569		459	0·658	279 19·7	253 18	261 50	+ 8 25	O.
	2570		459	0·645	279 6·5	252 21	260 53	+ 8 12	o.
	2571	189·467	463	0·548	66 51·7	180 57	189 22	+15 29	P_1.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1863.					° ′	° ′	° ′	° ′	
July 9.	2572	189·467	463	0·553	65 56·8	179 59	188 24	+15 54	P_3.
	2573		463	0·527	70 21·0	181 40	190 5	+12 32	p.
	2574		462	0·429	67 49·7	188 13	196 38	+12 37	Q_1.
	2575		462	0·440	66 50·6	187 41	196 6	+13 10	Q_2.
	2576		462	0·451	64 52·1	186 52	195 17	+14 16	Q_3.
	2577		464	0·634	116 27·6	176 15	184 40	−12 52	U.
	2578		464	0·628	115 50·9	176 19	184 44	−12 26	u.
	2579		464	0·661	116 1·8	174 15	182 40	−13 20	v.
	2580		464	0·666	113 14·2	173 10	181 35	−11 51	u.
	2581		459	0·690	279 4·8	256 20	264 45	+ 7 41	T.
	2582		459	0·650	276 46·2	253 43	262 8	+ 6 9	t.
	2583		459	0·655	280 16·7	252 55	261 20	+ 8 32	O.
	2584		459	0·643	280 7·5	252 0	260 25	+ 8 16	o.
10.	2585	190·496	463	0·257	57 57·0	195 26	189 15	+15 6	P_1.
	2586		463	0·356	56 14·7	195 51	189 40	+15 33	P_2.
	2587		463	0·361	55 36·7	195 39	189 28	+15 57	P_3.
	2588		463	0·370	56 23·6	195 1	188 50	+16 0	P_4.
	2589		463	0·364	57 23·8	195 12	189 1	+15 28	P_5.
	2590		463	0·348	52 11·3	197 6	190 55	+16 28	p_1.
	2591		463	0·340	51 58·2	197 31	191 20	+16 16	p_2.
	2592		463	0·325	51 28·1	198 23	192 12	+15 50	π_1.
	2593		463	0·336	54 46·6	197 11	191 0	+15 19	π_2.
	2594		462	0·240	52 28·2	202 22	196 11	+12 31	Q_1.
	2595		462	0·251	51 0·2	202 3	195 52	+13 14	Q_2.
	2596		462	0·267	48 47·4	201 41	195 30	+14 16	Q_3.
	2597		462	0·294	54 32·8	199 22	193 11	+13 58	q.
	2598		464	0·505	123 57·1	187 55	181 44	−11 56	V_1.
	2599		464	0·511	125 15·3	187 36	181 25	−12 53	V_2.
	2600		464	0·510	127 10·6	188 12	182 1	−13 40	v.
	2601		464	0·468	125 2·7	189 59	183 48	−11 19	w.
	2602		464	0·431	131 23·6	194 6	187 55	−12 16	U.
	2603		464	0·447	128 41·8	192 6	185 55	−11 56	u.
	2604		459	0·789	278 12·3	265 20	259 9	+ 7 45	S_1.
	2605		459	0·796	278 24·8	265 59	259 48	+ 7 56	S_2.
	2606		459	0·810	277 56·5	267 21	261 10	+ 7 27	s.
	2607		459	0·855	277 58·9	272 2	265 51	+ 7 38	T.
	2608		459	0·840	277 1·9	270 29	264 18	+ 6 50	t.
	2609		459	0·820	275 38·0	268 24	262 13	+ 5 41	O.
	2610		459	0·778	276 5·8	264 20	258 9	+ 6 6	o.
	2611	190·512	463	0·352	57 1·8	195 48	189 23	+15 3	P_1.
	2612		463	0·351	55 9·4	195 50	189 25	+15 29	P_2.
	2613		463	0·358	54 28·2	195 28	189 3	+15 55	P_3.
	2614		463	0·367	55 31·8	195 5	188 40	+15 58	P_4.
	2615		463	0·360	56 21·7	194 59	188 34	+15 30	P_5.
	2616		463	0·342	51 20·6	197 16	190 51	+16 26	p_1.
	2617		463	0·336	50 58·6	197 38	191 13	+16 21	p_2.
	2618		463	0·322	50 29·3	198 25	192 0	+15 48	π_1.
	2619		463	0·332	53 53·0	197 29	191 4	+15 18	π_2.
	2620		462	0·238	51 1·6	202 43	196 18	+12 25	Q_1.
	2621		462	0·248	49 32·2	202 25	196 0	+13 4	Q_2.
	2622		462	0·265	47 19·5	201 51	195 26	+14 15	Q_3.
	2623		462	0·292	53 17·6	199 32	193 7	+13 57	q.
	2624		464	0·499	123 54·9	188 6	181 41	−11 59	V_1.
	2625		464	0·505	125 12·1	187 35	181 10	−12 51	V_2.
	2626		464	0·505	127 15·3	188 25	182 0	−13 30	v.
	2627		464	0·465	128 4·0	190 45	184 20	−11 15	w.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Heliographical Longitude.	Heliographical Latitude.	Spot.
1863.									
July 10.	2628	190·512	464	0·428	131° 16′·7	194° 14′	187° 49′	−12° 15′	V.
	2629		464	0·440	128 52·4	192 10	185 45	−11 42	u.
	2630		459	0·795	278 14·2	265 43	259 18	+ 7 42	S_1.
	2631		459	0·800	278 33·0	266 8	259 43	+ 7 52	S_2.
	2632		459	0·815	278 2·7	267 33	261 8	+ 7 27	s.
	2633		459	0·858	278 9·9	273 12	265 47	+ 7 36	T.
	2634		459	0·845	277 12·4	270 41	264 16	+ 6 49	t.
	2635		459	0·823	275 38·9	268 36	262 11	+ 5 50	O.
	2636		459	0·782	276 12·7	264 32	258 7	+ 5 59	o.
12.	2637	192·502	463	0·265	349 16·8	218 51	184 13	+19 4	O.
	2638		463	0·288	3 12·3	214 58	180 20	+20 50	o.
	2639		463	0·272	3 51·9	214 48	180 10	+19 55	v_1.
	2640		463	0·269	2 55·2	215 10	180 32	+19 44	v_2.
	2641		463	0·246	324 53·2	224 8	189 30	+15 22	P_1.
	2642		463	0·254	323 4·3	224 48	190 10	+15 26	P_2.
	2643		462	0·303	307 19·5	230 1	195 23	+14 2	Q_1.
	2644		462	0·296	305 6·9	229 58	195 20	+13 14	Q_2.
	2645		462	0·311	312 22·6	229 28	194 50	+15 32	q.
	2646		462	0·253	312 6·1	226 42	192 4	+13 22	R.
	2647		464	0·306	215 50·5	225 5	190 27	−10 37	U.
	2648		464	0·283	204 26·1	221 25	186 47	−10 57	u.
	2649		464	0·282	188 7·2	216 53	182 15	−12 0	V.
	2650		464	0·276	185 44·0	216 11	181 33	−11 49	v.
	2651		459	0·983	279 28·6	294 83	260 15	+ 7 46	T.
	2652	192·511	463	0·263	350 8·0	218 53	184 7	+19 5	O.
	2653		463	0·286	3 53·7	214 36	179 50	+20 47	o.
	2654		463	0·272	4 40·3	214 49	180 3	+19 49	v_1.
	2655		463	0·269	3 29·5	215 26	180 40	+19 46	v_2.
	2656		463	0·244	325 35·2	224 17	189 31	+15 20	P_1.
	2657		463	0·250	323 28·9	224 44	189 58	+13 21	P_2.
	2658		463	0·197	319 38·1	224 24	199 38	+10 58	p.
	2659		462	0·299	307 37·1	229 46	195 0	+13 57	Q_1.
	2660		462	0·291	305 34·1	230 3	195 17	+13 15	Q_2.
	2661		464	0·304	215 11·2	225 8	190 22	−10 36	U.
	2662		464	0·279	303 37·3	221 16	186 30	−10 54	u.
	2663		464	0·278	187 42·9	216 57	182 11	−11 59	V.
	2664		464	0·275	185 8·2	216 0	181 14	−11 50	v.
	2665		459	0·981	279 38·0	294 53	260 7	+ 7 42	T.
14.	2666	194·500	465	0·874	117 37·1	160 4	97 5	−18 45	T.
	2667		462	0·608	292 54·1	253 21	190 22	+15 18	P.
	2668		462	0·590	289 43·9	252 29	189 30	+13 10	p.
	2669		462	0·677	290 20·3	258 58	195 59	+14 43	Q.
	2670		464	0·642	289 13·6	256 21	193 22	+13 33	R.
	2671		463	0·555	298 38·6	248 42	185 43	+17 28	O_1.
	2672		463	0·565	297 6·1	249 40	186 41	+16 52	O_2.
	2673		463	0·536	303 58·4	246 20	183 21	+19 41	o.
	2674		463	0·515	302 18·6	245 21	182 22	+18 18	v_1.
	2675		463	0·519	301 6·2	245 50	182 51	+17 50	v_2.
	2676		463α	0·664	251 52·1	255 56	192 57	−10 33	U.
	2677		463α	0·548	242 47·4	245 54	182 55	−12 14	V.
15.	2678	195·510	465	0·767	122 51·8	173 16	96 6	−18 43	T.
	2679		463	0·747	290 5·1	265 52	188 42	+15 10	P.
	2680		463	0·749	285 22·6	266 26	189 16	+11 41	p.
	2681		462	0·807	288 58·4	271 32	194 22	+14 56	Q.
	2682		462	0·783	287 33·6	269 18	192 8	+13 36	R.
	2683		463	0·728	294 9·2	263 40	186 30	+17 51	O_1

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Nodp.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1863.									
July 15.	2684	195·510	463	0·725	293° 34′·3	263° 31′	186° 21′	+17° 24′	O_2.
	2685		463	0·680	297 27·0	259 9	181 59	+19 14	o.
	2686		463	0·668	295 23·5	258 33	181 23	+17 40	v_1.
	2687		463	0·676	294 35·7	259 20	182 10	+17 17	v_2.
	2688		464	0·799	256 46·2	268 59	191 49	−10 37	V.
	2689	195·519	465	0·771	122 37·3	173 26	96 0	−18 45	T.
	2690		463	0·745	290 5·3	266 3	188 37	+15 13	P.
	2691		463	0·736	287 1·1	266 19	188 53	+11 29	p.
	2692		462	0·806	288 59·3	271 45	194 19	+14 56	Q.
	2693		462	0·782	287 12·9	269 43	112 17	+13 34	R.
	2694		463	0·719	293 10·8	263 45	186 19	+17 50	O_1.
	2695		463	0·717	292 23·9	263 33	186 7	+17 17	O_2.
	2696		463	0·679	297 31·0	259 20	181 54	+19 16	o.
	2697		463	0·666	295 34·9	258 46	181 20	+17 36	v_1.
	2698		463	0·675	294 28·3	259 34	182 8	+17 18	v_2.
	2699		464	0·801	255 51·1	269 19	191 53	−10 34	V.
17.	2700	197·474	465	0·510	145 10·9	200 2	94 52	−18 45	T.
	2701		463	0·956	289 14·1	293 19	188 9	+15 22	P_1.
	2702		463	0·960	288 43·4	294 5	188 55	+14 52	P_2.
	2703		463	0·980	289 25·5	299 12	194 2	+15 27	Q.
	2704		463	0·951	292 6·6	292 22	187 12	+18 6	O.
	2705		464	0·981	263 19·2	297 28	192 18	−10 4	V.
	2706		464	0·967	262 23·1	293 48	188 38	−10 31	V.
	2707		464*a*	0·134	351 28·6	221 45	116 35	+12 12	S.
	2708		464*a*	0·281	101 28·5	203 47	98 37	+ 2 37	S.
	2709		464*a*	0·262	97 59·0	204 50	99 40	+ 3 37	σ.
	2710	197·483	465	0·509	145 25·3	200 5	94 48	−18 41	T.
	2711		463	0·960	289 16·1	293 28	188 11	+15 20	P_1.
	2712		463	0·964	288 36·7	294 4	188 47	+14 54	P_2.
	2713		463	0·984	289 24·2	299 17	194 0	+15 19	Q.
	2714		463	0·955	291 57·0	292 31	187 14	+18 6	O.
	2715		462	0·984	263 16·0	297 34	192 17	−10 3	U.
	2716		462	0·970	261 29·8	293 46	188 29	−10 29	V.
	2717		464*a*	0·134	349 2·7	221 26	116 9	+12 10	S.
	2718		464*a*	0·279	101 1·8	203 57	98 40	+ 2 43	s.
	2719		465	0·177	168 29·7	217 9	111 52	− 5 6	r.
18.	2720	198·601	465	0·411	172 49·7	215 47	94 38	−18 48	T.
	2721		465	0·337	169 54·5	215 51	94 42	−14 8	t.
	2722		466	0·286	299 53·6	236 19	115 10	+11 28	S_1.
	2723		466	0·291	298 58·1	236 41	115 32	+11 20	S_2.
	2724		466	0·323	297 9·5	238 45	117 36	+11 30	σ.
19.	2725	199·469	465	0·417	200 23·1	228 21	94 53	−18 52	P.
	2726		465	0·412	217 11·2	234 48	101 20	−15 41	p.
	2727		467*a*	0·932	78 18·2	153 0	19 32	+19 35	O.
	2728		466	0·468	291 11·3	249 7	115 39	+11 35	S_1.
	2729		466	0·474	293 2·8	249 18	115 50	+12 31	S_2.
	2730		466	0·503	290 30·8	251 29	118 1	+11 42	s_1.
	2731		466	0·509	289 58·2	251 56	118 8	+11 30	s_2.
	2732		466	0·521	289 56·3	253 45	120 17	+11 38	s_3.
	2733	199·503	465	0·414	200 7·7	228 46	94 49	−18 49	P.
	2734		465	0·488	204 48·9	231 55	97 58	−22 33	π.
	2735		466	0·467	291 44·7	249 38	115 41	+11 36	S_1.
	2736		466	0·472	293 25·6	249 36	115 39	+12 29	S_2.
	2737		466	0·499	291 5·8	251 46	117 49	+11 44	s_1.
	2738		466	0·506	290 32·6	251 54	117 57	+11 35	s_2.
	2739		466	0·519	290 33·4	254 7	120 10	+11 36	s_3.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1863.					° ′	° ′	° ′	° ′	
July 24.	2740	204·474	465	0·888	252 48·0	285 2	80 35	−19 9	P.
	2741		465	0·908	261 15·8	289 32	85 5	−12 27	p.
	2742		467*a*	0·273	276 9·8	242 28	38 1	+ 4 41	o.
	2743	204·623	465	0·904	253 14·8	287 1	80 27	−19 11	P.
	2744		465	0·922	261 50·4	291 37	85 3	−12 25	p.
	2745		467*a*	0·305	276 30·6	244 31	37 57	+ 4 41	o.
25.	2746	205·458	465	0·964	256 19·2	298 59	80 34	−19 16	P.
	2747		467	0·909	87 29·7	161 52	303 27	+11 49	O.
	2748	205·489	465	0·968	256 20·5	299 30	80 39	−19 11	P.
	2749		467	0·908	87 45·3	162 12	303 21	+11 33	O.
26.	2750	206·500	467	0·989	90 21·2	146 39	273 28	+ 8 50	R.
	2751		467*b*	0·949	283 57·2	300 26	67 15	+ 6 54	S.
	2752		467*b*	0·946	283 21·9	299 50	66 39	+ 6 23	s.
	2753		467*b*	0·964	283 15·2	303 24	70 13	+ 6 2	O.
	2754		467	0·760	57 38·0	184 59	311 48	+33 45	π.
	2755		467	0·753	57 1·7	185 56	312 45	+33 53	ρ.
28.	2756	208·460	467	0·845	92 25·6	172 36	271 37	+ 8 48	R.
	2757		467	0·889	98 0·7	167 47	266 48	+ 3 44	P.
	2758		467	0·985	86 15·5	149 36	248 37	+13 48	O.
	2759		467	0·968	88 25·8	154 25	253 26	+11 55	σ.
	2760	208·501	467	0·835	92 29·6	173 8	271 34	+ 8 46	R.
	2761		467	0·878	98 14·8	168 25	266 51	+ 3 40	P.
	2762		467	0·978	86 58·4	150 14	248 40	+13 44	O.
	2763		467	0·960	88 45·3	155 1	253 27	+12 0	o.
29.	2764	209·460	467	0·693	92 54·7	187 26	272 16	+ 8 49	R.
	2765		467	0·749	99 59·5	182 58	267 48	+ 3 34	P.
	2766		468	0·913	87 44·6	165 4	249 54	+13 14	O.
	2767		468	0·865	89 0·4	171 14	256 4	+12 6	o.
	2768	209·507	467	0·691	93 8·5	188 12	272 21	+ 8 48	R.
	2769		467	0·746	100 6·7	183 42	267 51	+ 3 33	P.
	2770		468	0·913	87 45·5	165 48	249 55	+13 14	O.
	2771		468	0·863	89 50·0	171 57	256 6	+12 0	o.
30.	2772	210·460	467	0·509	92 18·0	201 50	272 28	+ 8 55	R.
	2773		467	0·571	102 10·1	197 38	268 16	+ 3 34	P.
	2774		468	0·790	88 27·9	180 6	250 44	+12 43	O_1.
	2775		468	0·796	89 50·9	179 29	250 7	+11 39	O_2.
	2776		468	0·784	89 45·9	180 39	251 17	+11 41	O_3.
	2777		468	0·773	91 12·6	181 35	252 13	+10 32	o.
	2778		468	0·755	89 2·1	183 19	253 57	+12 8	Q_1.
	2779		468	0·752	89 30·0	183 32	254 10	+11 46	Q_2.
	2780		468	0·744	89 56·6	184 15	254 53	+11 24	q.
	2781		468	0·722	90 0·8	186 6	256 44	+11 16	T.
	2782		468	0·737	89 55·5	184 51	255 29	+11 24	t.
	2783	210·465	467	0·516	92 24·2	202 1	272 35	+ 8 54	R.
	2784		467	0·580	102 12·7	197 46	268 20	+ 3 34	P.
	2785		468	0·793	88 29·0	180 17	250 51	+12 41	O_1.
	2786		468	0·800	89 55·1	179 41	250 15	+11 40	O_2.
	2787		468	0·787	89 49·2	180 48	251 22	+11 43	O_3.
	2788		468	0·777	91 23·6	181 50	252 24	+10 34	o.
	2789		468	0·760	89 3·5	183 28	254 2	+12 9	Q_1.
	2790		468	0·757	89 36·5	183 40	254 14	+11 49	Q_2.
	2791		468	0·747	90 4·8	184 23	254 57	+11 27	q.
	2792		468	0·726	90 11·7	186 12	256 46	+11 18	T.
	2793		468	0·741	90 6·1	184 59	255 33	+11 27	t.
31.	2794	211·469	467	0·305	89 20·3	215 47	272 7	+ 8 56	R.
	2795		467	0·370	105 47·8	211 45	268 5	+ 3 29	P.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1863.									
July 31.	2796	211·469	468	0·640	88 0·2	193 43	250 3	+12 28	O_1.
	2797		468	0·654	88 41·5	192 36	248 56	+12 7	O_2.
	2798		468	0·630	89 25·9	194 21	250 41	+11 30	O_3.
	2799		468	0·594	89 56·8	196 59	253 19	+10 57	o.
	2800		468	0·629	87 29·6	194 35	250 55	+12 41	p.
	2801		468	0·581	89 5·9	197 59	254 19	+11 21	Q.
	2802		468	0·549	89 44·5	200 9	256 29	+10 46	T_1.
	2803		468	0·548	89 3·5	200 16	256 36	+11 8	T_2.
	2804		468	0·555	89 11·4	199 45	256 5	+11 7	t.
	2805		468	0·563	89 16·3	199 13	255 33	+11 8	q.
	2806	211·505	467	0·299	89 23·6	216 25	272 14	+ 8 59	R.
	2807		467	0·365	105 55·1	212 22	268 11	+ 3 31	P.
	2808		468	0·634	88 9·4	194 22	250 11	+12 27	O_1.
	2809		468	0·650	89 43·6	193 6	248 55	+12 13	O_2.
	2810		468	0·625	89 18·5	195 1	250 50	+11 29	O_3.
	2811		468	0·586	91 3·7	197 45	253 34	+10 58	σ.
	2812		468	0·623	87 27·0	195 0	250 49	+12 40	p.
	2813		468	0·575	89 7·5	198 41	254 30	+11 24	Q.
	2814		468	0·544	89 53·4	200 51	256 40	+10 47	T_1.
	2815		468	0·542	80 0·2	200 20	256 9	+11 3	T_2.
	2816		468	0·549	89 25·1	200 11	256 0	+11 1	t.
	2817		468	0·556	89 25·5	199 41	255 30	+11 12	q.
Aug. 1.	2818	212·493	467	0·086	64 59·0	230 19	272 7	+ 8 47	R.
	2819		467	0·143	116 46·2	226 27	268 5	+ 3 37	P.
	2820		468	0·335	85 38·5	215 14	257 2	+10 38	T_1.
	2821		468	0·341	84 46·5	214 55	256 43	+11 0	T_2.
	2822		468	0·371	87 41·1	212 52	254 40	+10 22	t.
	2823		468	0·399	87 40·2	211 8	252 56	+10 40	σ.
	2824		468	0·450	85 17·9	208 7	249 55	+12 15	Q.
	2825		468	0·433	87 6·8	208 58	250 46	+11 51	σ.
	2826		468	0·390	85 30·7	211 43	253 31	+11 24	q.
	2827	212·503	467	0·080	63 4·8	231 31	272 11	+ 8 45	R.
	2828		467	0·134	116 35·8	226 29	268 9	+ 3 39	P.
	2829		468	0·332	85 23·7	215 20	257 0	+10 37	T_1.
	2830		468	0·333	84 12·3	215 19	256 59	+11 1	T_2.
	2831		468	0·365	87 24·5	212 55	254 35	+10 17	t.
	2832		468	0·391	87 26·8	210 59	252 39	+10 35	σ.
	2833		468	0·443	85 4·9	208 12	249 52	+12 18	O.
	2834		468	0·428	86 41·3	209 1	250 41	+11 52	o.
	2835		468	0·385	84 56·6	211 57	253 37	+11 28	q.
3.	2836	214·669	467	0·416	290 11·2	260 56	271 52	+ 8 58	R.
	2837		467	0·365	275 10·6	257 36	268 32	+ 3 14	P.
	2838		468	0·190	310 11·3	246 12	257 8	+11 8	T.
	2839		468	0·168	307 31·4	245 14	256 10	+10 9	t.
	2840		468	0·142	359 59·7	238 7	249 3	+14 1	O.
	2841		469	0·916	97 13·2	169 53	180 49	+ 6 38	S.
	2842		469	0·904	96 53·6	171 32	182 28	+ 7 2	s.
	2843		469	0·938	94 20·2	208 31	219 27	+ 5 36	Σ.
	2844		469	0·922	94 43·5	168 50	179 46	+ 8 52	σ.
	2845		469	0·935	95 13·1	166 49	177 45	+ 8 18	π.
	2846		469	0·977	93 51·6	158 19	169 15	+ 9 3	V.
	2847		469	0·968	93 18·9	160 21	171 17	+ 9 46	v.
	2848		469	0·967	92 23·4	160 34	171 30	+10 38	U.
4.	2849	215·465	467	0·576	288 45·0	272 26	272 5	+ 8 48	R.
	2850		467	0·535	277 39·8	269 19	268 58	+ 2 47	P.
	2851		468	0·368	298 21·8	258 15	257 54	+11 35	T.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1863.									
Aug. 4.	2852	21ʰ·465	468	0·513	280° 5′·7	267° 59′	267° 38′	+ 4° 18′	p_1.
	2853		468	0·506	279 29·3	267 30	267 9	+ 3 57	p_2.
	2854		469	0·823	98 35·1	181 36	181 15	+ 6 22	S.
	2855		469	0·805	97 44·6	183 27	183 6	+ 7 9	s.
	2856		469	0·852	96 7·2	178 30	178 9	+ 8 18	Σ.
	2857		469	0·832	95 45·7	180 39	180 18	+ 8 40	σ.
	2858		469	0·903	93 20·5	172 18	171 57	+10 32	U.
	2859		469	0·897	94 2·0	173 1	172 40	+ 9 56	u.
	2860		469	0·897	96 12·6	173 10	172 49	+ 8 0	V.
	2861		469	0·902	95 8·2	172 25	172 4	+ 8 55	v.
5.	2862	21ʰ·576	467	0·761	289 15·5	287 58	271 51	+ 9 6	R.
	2863		467	0·734	281 5·9	285 21	269 14	+ 3 9	P.
	2864		467	0·713	282 53·4	283 41	267 34	+ 4 35	p.
	2865		468	0·590	294 47·3	274 12	258 5	+12 10	T.
	2866		468	0·572	296 10·5	272 52	256 45	+12 49	t.
	2867		468	0·572	290 28·4	273 6	256 59	+ 9 17	τ.
	2868		468	0·574	292 20·7	273 13	257 6	+10 39	τ.
	2869		468	0·552	293 33·0	271 38	255 31	+11 12	t.
	2870		468	0·545	292 14·4	271 12	255 5	+10 26	θ.
	2871		469	0·766	95 31·4	188 4	171 57	+ 9 21	U_1.
	2872		469	0·769	94 57·5	187 46	171 39	+ 9 47	U_2.
	2873		469	0·768	93 55·6	187 50	171 43	+10 34	u.
	2874		469	0·753	93 22·7	189 15	173 8	+10 58	V.
	2875		470	0·674	96 21·8	195 42	179 35	+ 8 44	Σ.
	2876		470	0·664	96 21·1	196 29	180 22	+ 8 44	σ.
	2877		470	0·674	98 14·1	195 43	179 36	+ 7 28	x.
	2878		470	0·667	98 12·2	196 17	180 10	+ 7 30	y.
	2879		469	0·646	99 11·0	197 54	181 47	+ 6 53	S_1.
	2880		469	0·658	99 40·1	197 2	180 55	+ 6 33	S_2.
	2881		469	0·624	98 25·7	199 30	183 23	+ 7 24	s.
	2882	21ʰ·583	467	0·764	289 17·4	287 58	271 45	+ 9 3	R.
	2883		467	0·735	281 5·8	285 30	269 17	+ 3 11	P.
	2884		467	0·714	282 49·5	283 43	267 30	+ 4 38	p.
	2885		468	0·591	294 43·0	274 25	258 12	+12 11	T.
	2886		468	0·572	296 4·1	272 47	256 34	+12 47	t.
	2887		468	0·573	292 42·7	273 8	256 55	+ 9 20	τ.
	2888		468	0·574	290 8·9	273 15	257 2	+10 35	τ.
	2889		468	0·554	293 24·8	271 49	255 36	+11 11	t.
	2890		468	0·548	291 46·8	271 12	254 59	+10 25	θ.
	2891		469	0·767	95 25·9	188 9	171 56	+ 9 22	U_1.
	2892		469	0·769	94 3·8	187 53	171 40	+ 9 50	U_2.
	2893		470	0·675	96 28·3	195 46	179 33	+ 8 43	Σ.
	2894		470	0·665	96 28·5	196 37	180 24	+ 8 45	σ.
	2895		470	0·674	97 51·0	195 48	179 35	+ 7 25	x.
	2896		470	0·669	97 43·4	196 25	180 12	+ 7 31	y.
	2897		469	0·646	99 12·2	197 59	181 46	+ 6 53	S_1.
	2898		469	0·658	99 47·6	197 10	180 57	+ 6 31	S_2.
	2899		469	0·624	98 35·7	199 35	183 22	+ 7 22	s.
	2900		469	0·739	95 22·0	190 25	174 12	+ 9 28	V.
	2901		469	0·546	85 3·0	205 48	189 35	+14 40	Q.
	2902		469	0·589	80 13·7	206 56	190 43	+17 5	q.
7.	2903	21ʰ·458	467	0·967	291 1·3	315 53	273 5	+ 9 4	R.
	2904		467	0·962	284 7·9	314 4	271 16	+ 2 34	P.
	2905		468	0·901	292 18·2	304 44	261 56	+10 51	T.
	2906		468	0·882	292 56·8	302 19	259 31	+11 28	t.
	2907		468	0·872	293 50·1	301 10	258 22	+12 16	τ.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1863.									
Aug. 7.	2908	218·458	468	0·884	295° 49′·3	302° 40′	259° 52′	+14° 1′	t.
	2909		468	0·856	293 1·2	299 14	256 26	+11 35	δ.
	2910		469	0·412	91 51·7	215 55	173 7	+10 24	U_1.
	2911		469	0·404	91 6·7	216 27	173 39	+10 38	U_2.
	2912		469	0·389	89 51·1	217 29	174 41	+10 58	u.
	2913		469	0·402	93 47·9	216 27	173 39	+ 9 33	v.
	2914		470	0·280	92 50·1	223 58	181 10	+ 8 56	Σ_1.
	2915		470	0·272	92 19·9	224 26	181 38	+ 9 1	Σ_2.
	2916		470	0·241	95 18·9	226 10	183 22	+ 8 1	x_1.
	2917		470	0·247	94 11·0	225 51	183 3	+ 8 20	x_2.
	2918		470	0·234	94 38·8	226 35	183 47	+ 8 7	y_1.
	2919		470	0·241	93 4·6	226 14	183 26	+ 8 33	y_2.
	2920		469	0·250	95 52·0	225 37	182 49	+ 7 56	ζ.
	2921		469	0·260	96 35·7	225 1	182 13	+ 7 48	ξ.
	2922		469	0·272	97 29·6	224 19	181 31	+ 7 37	μ.
	2923		469	0·245	98 33·1	225 53	183 5	+ 7 15	S_1.
	2924		469	0·230	97 8·3	226 48	184 0	+ 7 31	S_2.
	2925		469	0·350	106 2·9	219 35	176 47	+ 4 56	s.
	2926		469	0·241	75 45·1	227 30	184 42	+12 30	Q.
	2927	218·479	467	0·970	290 58·7	316 7	273 1	+ 9 6	R.
	2928		467	0·964	284 6·7	314 21	271 15	+ 2 37	P.
	2929		468	0·909	292 12·3	304 55	261 49	+10 49	T.
	2930		468	0·890	292 49·0	302 36	259 30	+11 25	t.
	2931		468	0·880	293 44·2	301 26	258 20	+12 13	τ.
	2932		468	0·890	295 47·9	303 0	259 54	+13 59	t.
	2933		468	0·861	292 53·7	299 35	256 29	+11 33	δ.
	2934		469	0·398	91 48·6	216 6	173 0	+10 22	U_1.
	2935		469	0·393	90 57·5	216 38	173 32	+10 37	U_2.
	2936		469	0·381	89 24·3	217 45	174 39	+10 57	u_1.
	2937		469	0·374	89 43·3	217 28	174 22	+10 49	u_2.
	2938		469	0·387	93 49·2	216 47	173 41	+ 9 31	v.
	2939		470	0·265	92 2·1	224 5	180 59	+ 8 53	Σ_1.
	2940		470	0·257	91 48·7	224 40	181 34	+ 8 58	Σ_2.
	2941		470	0·237	95 41·2	226 23	183 17	+ 8 0	x_1.
	2942		470	0·234	94 7·8	226 8	183 2	+ 8 16	x_2.
	2943		470	0·228	95 8·7	226 51	183 45	+ 8 10	y_1.
	2944		470	0·227	92 43·6	226 36	183 30	+ 8 30	y_2.
	2945		469	0·223	98 14·7	226 15	183 9	+ 7 11	S_1.
	2946		469	0·214	97 49·0	227 4	183 58	+ 7 33	S_2.
	2947		469	0·243	96 51·9	225 50	182 44	+ 7 53	ζ.
	2948		469	0·256	97 30·4	225 15	182 9	+ 7 49	ξ.
	2949		469	0·266	98 52·0	224 50	181 44	+ 7 33	μ_1.
	2950		469	0·269	97 41·8	224 37	181 31	+ 7 40	μ_2.
	2951		469	0·333	106 45·2	219 46	176 40	+ 4 50	s_1.
	2952		469	0·348	107 53·1	219 17	176 11	+ 3 51	s_2.
	2953		469	0·470	51 20·5	221 4	177 58	+27 36	V.
	2954		469	0·227	73 46·9	228 26	185 20	+12 28	Q.
8.	2955	219·583	470	0·199	82 35·8	230 20	171 34	+10 19	U_1.
	2956		470	0·202	77 51·9	230 30	171 44	+11 17	U_2.
	2957		470	0·192	85 6·8	230 33	171 47	+ 9 44	v_1.
	2958		470	0·197	86 24·7	230 12	171 26	+ 9 35	v_2.
	2959		470	0·174	73 16·8	232 21	173 35	+11 20	u_1.
	2960		470	0·150	74 22·9	233 33	174 47	+10 29	u_2.
	2961		469	0·065	55 29·9	238 38	179 52	+ 9 7	Σ_1.
	2962		469	0·058	53 52·6	238 58	180 12	+ 8 54	Σ_2.
	2963		469	0·041	28 56·8	240 31	181 45	+ 8 37	x.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude. from Node.	Heliographical Longitude.	Heliographical Latitude.	Spot.
1863.									
Aug. 8.	2964	219·583	469	0·034	12° 52·3	241° 10′	182° 24′	+ 8° 18′	γ.
	2965		469	0·027	327 54·7	242 16	183 30	+ 7 37	S_1.
	2966		469	0·028	313 29·2	242 33	183 47	+ 7 9	S_2.
	2967		468	0·952	293 42·3	313 58	255 12	+11 27	δ.
	2968		468	0·976	292 46·5	319 9	260 23	+10 13	T.
	2969		471	0·958	112 15·5	168 54	110 8	− 6 18	O_1.
	2970		471	0·959	112 38·4	168 52	110 6	− 6 40	O_2.
	2971	219·614	470	0·194	82 47·2	230 42	171 30	+10 17	U_1.
	2972		470	0·187	84 58·1	231 1	171 49	+ 9 45	v_1.
	2973		470	0·192	86 4·8	230 32	171 20	+ 9 32	v_2.
	2974		470	0·172	73 46·8	232 39	173 27	+11 24	u_1.
	2975		470	0·145	75 3·5	233 58	174 46	+10 36	u_2.
	2976		469	0·060	52 46·9	238 53	179 41	+ 9 13	Σ_1.
	2977		469	0·054	50 26·7	239 31	180 19	+ 8 59	Σ_2.
	2978		468	0·951	294 10·3	314 23	255 11	+11 25	δ.
	2979		468	0·984	293 43·1	319 17	260 5	+10 21	T.
	2980		471	0·956	112 1·9	169 25	110 13	− 6 20	O_1.
	2981		471	0·956	112 38·5	169 19	110 7	− 6 39	O_2.
10.	2982	221·482	473	0·983	100 39·3	163 28	77 46	+ 4 53	P.
	2983		472	0·930	117 31·4	176 26	90 44	− 9 39	Q.
	2984		472	0·922	118 51·3	177 57	92 15	−10 36	q.
	2985		472	0·926	120 3·5	177 28	91 46	−11 51	K.
	2986		472	0·957	118 1·2	171 45	86 3	−11 0	r_1.
	2987		472	0·957	118 40·7	172 0	86 18	−11 36	r_2.
	2988		472	0·936	127 28·3	177 39	91 57	−18 53	R.
	2989		471	0·731	118 44·6	196 48	111 6	− 6 57	O.
	2990		471	0·790	119 3·4	195 5	109 23	− 7 23	o_1.
	2991		471	0·793	117 37·7	194 52	109 10	− 7 11	o_2.
	2992		470	0·248	298 26·9	257 26	171 44	+10 33	U_1.
	2993		470	0·258	295 30·7	257 6	171 24	+10 58	U_2.
	2994		470	0·328	283 59·6	257 36	171 54	+ 9 58	v.
	2995		469	0·385	293 18·1	265 30	179 48	+ 9 17	Σ_1.
	2996		469	0·386	291 26·9	266 23	180 41	+ 8 54	Σ_2.
	2997		469	0·374	292 15·4	265 13	179 31	+ 8 12	σ.
	2998		469	0·392	288 15·7	272 11	186 29	+ 6 11	S.
	2999		469	0·449	287 40·8	283 45	198 3	+ 6 48	s.
	3000	221·597	473	0·978	101 4·9	165 13	77 53	+ 4 47	P.
	3001		472	0·918	117 57·7	178 11	90 51	− 9 44	Q.
	3002		472	0·906	119 5·6	179 37	92 17	−10 40	q.
	3003		472	0·914	120 33·3	179 6	91 46	−11 47	K.
	3004		472	0·949	110 37·7	173 35	86 15	−11 3	r.
	3005		472	0·925	128 7·2	179 1	91 41	−18 56	R.
	3006		471	0·707	119 31·9	198 37	111 17	− 6 55	O.
	3007		471	0·769	119 40·3	196 45	109 25	− 7 27	o_1.
	3008		471	0·777	118 14·7	196 31	109 11	− 7 16	o_2.
	3009		470	0·291	294 45·9	258 59	171 39	+10 35	U_1.
	3010		470	0·284	296 57·0	258 41	171 21	+11 0	U_2.
	3011		470	0·359	283 59·1	259 18	171 58	+10 5	v.
	3012		469	0·415	292 59·2	266 53	179 33	+ 9 14	Σ_1.
	3013		469	0·416	291 19·3	268 4	180 44	+ 8 55	Σ_2.
	3014		469	0·406	292 1·3	266 46	179 26	+ 8 11	σ.
	3015		469	0·421	288 10·0	273 54	186 34	+ 5 59	S.
11.	3016	222·479	473	0·930	86 28·3	174 51	75 0	+19 24	Π.
	3017		473	0·920	102 7·4	176 47	76 56	+ 5 1	P_1.
	3018		473	0·912	102 47·7	178 6	78 15	+ 4 31	P_2.
	3019		472	0·827	121 22·2	190 35	90 44	− 9 39	Q.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1863.					° ′	° ′	° ′	° ′	
Aug. 11.	3020	22·479	472	0·811	122 37·7	192 10	92 19	−10 33	q.
	3021		472	0·813	124 0·4	191 41	91 50	−11 40	K.
	3022		472	0·873	122 4·2	186 2	86 11	−11 14	r.
	3023		472	0·845	131 54·0	191 50	91 59	−18 46	R.
	3024		471	0·569	125 15·0	212 2	112 11	− 7 0	O_1.
	3025		471	0·584	125 28·6	211 45	111 54	− 7 11	O_2.
	3026		471	0·642	124 36·8	209 28	109 37	− 7 32	o.
	3027		470	0·622	125 49·9	271 19	171 28	+10 39	U_1.
	3028		470	0·471	291 11·9	271 4	171 13	+10 57	U_2.
	3029		469	0·583	290 36·6	279 38	179 47	+ 9 18	Σ_1.
	3030		469	0·580	291 26·2	280 22	180 31	+ 8 59	Σ_2.
	3031		469	0·561	284 19·4	279 47	179 56	+ 2 21	v.
	3032	22·492	473	0·931	86 30·7	174 48	74 46	+19 28	Π.
	3033		473	0·921	101 51·4	176 53	76 51	+ 5 2	P_1.
	3034		473	0·912	102 44·1	178 21	78 19	+ 4 36	P_2.
	3035		472	0·827	121 20·6	190 58	90 56	− 9 40	Q.
	3036		472	0·812	122 38·7	192 19	92 17	−10 38	q.
	3037		472	0·811	123 56·1	191 50	91 48	−11 43	K.
	3038		472	0·875	122 38·1	186 14	86 12	−11 17	r.
	3039		472	0·843	132 7·8	192 5	92 3	−18 47	R.
	3040		471	0·559	125 21·4	212 12	112 10	− 6 59	O_1.
	3041		471	0·579	125 44·7	212 1	111 59	− 7 7	O_2.
	3042		471	0·638	124 43·3	209 41	109 39	− 7 39	o_1.
	3043		471	0·619	125 48·1	209 52	109 50	− 7 42	o_2.
	3044		470	0·471	291 27·8	271 35	171 33	+10 44	U_1.
	3045		470	0·466	292 28·8	271 16	171 14	+10 59	U_2.
	3046		469	0·589	290 35·5	279 46	179 44	+ 9 18	Σ_1.
	3047		469	0·583	291 37·1	280 29	180 27	+ 9 2	Σ_2.
	3048		469	0·564	284 24·5	280 5	180 3	+ 2 11	v.
	3049		469*a*	0·230	41 20·8	237 46	137 44	+18 16	N.
	3050		469*a*	0·197	38 84·1	239 9	139 7	+16 49	n.
12.	3051	23·494	473	0·815	86 34·9	189 27	75 13	+19 23	Π.
	3052		473	0·864	85 29·5	185 3	70 49	+20 13	π.
	3053		473	0·808	103 1·9	191 13	76 59	+ 5 59	P_1.
	3054		473	0·803	103 47·8	191 16	77 2	+ 5 10	P_2.
	3055		473	0·794	104 36·1	191 25	77 11	+ 4 58	P_3.
	3056		472	0·687	127 9·0	205 21	91 7	− 9 42	Q.
	3057		472	0·657	128 57·9	207 5	92 51	−10 39	q.
	3058		472	0·656	131 3·3	205 58	91 44	−11 50	K.
	3059		472	0·738	127 13·3	200 33	86 19	−11 11	r.
	3060		471	0·376	139 24·6	227 15	113 1	− 7 1	O.
	3061		472	0·724	139 20·6	208 31	94 17	−18 44	R.
	3062		471	0·469	134 31·6	223 54	109 40	− 7 38	o_1.
	3063		471	0·441	135 37·6	224 1	109 47	− 7 49	o_2.
	3064		469	0·670	290 58·2	286 36	172 22	+10 43	U.
	3065		469	0·713	292 0·8	294 33	180 19	+ 9 33	Σ.
	3066		470	0·243	344 31·0	252 16	138 2	+18 27	N.
	3067		470	0·250	331 3·8	254 3	139 49	+16 59	n.
	3068	23·510	473	0·814	86 45·1	189 42	75 21	+19 29	Π.
	3069		473	0·863	85 28·5	185 4	70 43	+20 17	π.
	3070		473	0·804	103 0·3	191 19	76 58	+ 5 54	P_1.
	3071		473	0·800	103 51·9	191 34	77 13	+ 5 9	P_2.
	3072		473	0·791	104 41·1	191 40	77 19	+ 4 52	P_3.
	3073		472	0·683	127 20·8	205 23	91 2	− 9 51	Q.
	3074		472	0·655	129 13·7	207 16	92 55	−10 42	q.
	3075		472	0·654	131 19·5	206 4	91 43	−11 55	K.

Table III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1863.									
Aug. 12.	3076	223·510	472	0·737	127° 16′·4	200° 33′	86° 12′	−11° 19′	r.
	3077		471	0·374	139 34·5	227 30	113 9	− 7 2	O.
	3078		472	0·722	139 34·9	208 36	94 15	−18 45	R.
	3079		471	0·467	134 44·4	224 10	109 49	− 7 35	o_1.
	3080		471	0·438	135 45·0	224 12	109 51	− 7 46	o_2.
	3081		471	0·671	290 54·5	286 52	172 31	+10 40	U.
	3082		469	0·714	292 1·6	294 45	180 24	+ 9 39	Σ.
	3083		470	0·241	344 2·0	252 30	138 9	+18 22	N.
	3084		470	0·249	330 42·2	254 18	139 57	+17 4	n.
14.	3085	225·590	473	0·482	79 34·4	219 28	75 30	+19 28	Π.
	3086		473	0·589	79 35·6	214 30	70 32	+20 22	π.
	3087		473	0·453	107 7·1	220 57	76 59	+ 5 39	P_1.
	3088		473	0·449	108 33·3	221 17	77 19	+ 4 53	P_2.
	3089		473	0·438	110 3·2	221 23	77 25	+ 4 22	P_3.
	3090		473	0·387	156 14·4	235 11	91 13	−10 0	Q.
	3091		473	0·345	160 6·8	236 48	92 50	−10 52	q.
	3092		473	0·342	164 16·2	235 46	91 48	−12 1	K.
	3093		472	0·388	160 19·1	230 11	86 13	−11 36	r.
	3094		472	0·476	168 8·4	239 0	95 2	−18 49	R.
	3095		471	0·221	204 26·5	253 51	109 53	− 7 46	o_1.
	3096		471	0·235	203 1·3	253 57	109 59	− 7 59	o_2.
	3097		471	0·637	303 20·0	284 5	140 7	+17 11	n.
	3098		470	0·954	293 0·5	318 5	174 7	+10 51	U.
	3099		470	0·945	292 2·5	317 39	173 41	+10 31	u.
15.	3100	226·458	473	0·307	63 38·9	232 36	76 19	+19 24	Π.
	3101		473	0·427	69 40·7	227 17	71 0	+20 25	π.
	3102		473	0·241	111 23·9	233 52	77 35	+ 5 17	P_1.
	3103		473	0·241	114 59·0	234 2	77 45	+ 4 25	P_2.
	3104		473	0·232	117 1·5	234 37	78 20	+ 4 3	P_3.
	3105		474	0·952	123 5·8	178 21	22 4	−13 48	S.
	3106		474	0·985	124 2·9	170 34	14 17	−16 13	s.
	3107		474	0·606	138 25·0	216 10	59 53	−13 8	Σ.
	3108		474	0·324	201 14·4	261 58	105 41	−12 0	Q.
	3109		474	0·364	210 51·6	253 13	96 56	−13 52	K.
	3110		472	0·425	194 0·0	246 49	90 32	−18 19	R.
	3111		471	0·409	254 20·7	268 9	111 52	− 6 7	O.
	3112		471	0·339	247 41·0	266 19	110 2	− 7 0	o.
	3113	226·507	473	0·306	63 41·1	233 12	76 13	+19 28	Π.
	3114		473	0·425	69 28·3	228 3	71 4	+20 17	π.
	3115		473	0·241	111 49·7	234 27	77 28	+ 5 19	P_1.
	3116		473	0·240	114 53·4	234 54	77 55	+ 4 23	P_2.
	3117		473	0·230	117 5·2	235 22	78 23	+ 4 0	P_3.
	3118		474	0·952	123 12·0	179 7	22 8	−13 44	S.
	3119		474	0·979	123 39·5	171 18	14 19	−16 15	s.
	3120		474	0·605	138 26·0	216 18	59 19	−13 20	Σ.
	3121		474	0·326	201 4·1	262 36	105 37	−12 4	Q.
	3122		474	0·327	210 2·9	262 29	105 30	− 9 2	q.
	3123		474	0·364	210 58·3	253 59	97 0	−13 54	K.
	3124		472	0·426	194 16·7	247 34	90 35	−18 21	R.
	3125		471	0·411	254 19·3	268 54	111 55	− 6 9	O.
	3126		471	0·340	247 45·2	267 12	110 13	− 7 1	o.
17.	3127	228·481	475	0·593	87 56·4	214 4	29 5	+16 36	Q.
	3128		475	0·544	87 16·2	217 37	32 38	+16 15	q.
	3129		475	0·522	77 12·8	220 47	35 48	+20 54	K.
	3130		474	0·724	133 23·3	208 6	23 7	−13 47	S.
	3131		474	0·320	146 9·3	234 25	49 26	− 5 3	Σ.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1863.									
Aug. 17.	3132	228·481	474	0·387	140° 35·2	216° 13′	31° 14′	−11° 17′	σ_1.
	3133		474	0·381	141 43·3	217 12	32 13	−10 58	σ_2.
	3134		474	0·350	142 44·3	218 26	33 27	− 9 43	x.
	3135		474	0·370	136 46·9	216 3	31 4	− 6 17	y.
	3136		473	0·220	281 20·3	263 10	78 11	+ 5 23	P_1.
	3137		473	0·225	276 24·5	263 29	78 30	+ 4 39	P_2.
	3138		473	0·243	275 13·4	263 58	78 59	+ 4 7	P_3.
	3139		472	0·571	242 3·9	275 32	90 33	−18 24	R.
18.	3140	229·677	476*a*	0·484	90 29·5	228 29	20 32	+14 2	Q.
	3141		476*a*	0·421	90 53·0	226 30	24 33	+13 2	q_1.
	3142		476*a*	0·451	93 26·3	225 16	23 19	+12 29	q_2.
	3143		475	0·349	77 13·3	237 24	35 27	+20 51	K.
	3144		475	0·377	78 29·8	235 8	33 11	+19 4	r.
	3145		475	0·382	76 31·2	234 59	33 2	+21 1	s.
	3146		475	0·342	72 53·5	238 50	36 53	+20 2	t.
	3147		474	0·549	146 27·3	225 16	23 19	−13 49	S.
	3148		474	0·209	207 44·3	251 25	49 28	− 5 19	Σ_1.
	3149		474	0·208	201 38·1	251 27	49 30	− 5 3	Σ_2.
	3150		474	0·226	182 19·4	233 25	31 28	−11 10	σ_1.
	3151		474	0·222	187 19·6	234 47	32 50	−10 54	σ_2.
	3152		474	0·210	181 21·5	232 40	30 43	−10 49	σ_3.
	3153		473	0·476	285 34·4	280 57	79 0	+ 5 29	P_1.
	3154		473	0·480	282 54·0	281 10	79 13	+ 4 48	P_2.
	3155		473	0·496	282 23·0	281 12	79 15	+ 4 13	P_3.
	3156		472	0·735	255 38·3	292 8	90 11	−18 22	R.
19.	3157	230·690	476*a*	0·292	81 53·4	236 11	19 52	+13 54	Q_1.
	3158		476*a*	0·287	83 58·9	236 15	19 56	+13 13	Q_2.
	3159		476*a*	0·210	76 52·3	241 16	24 57	+12 53	q_1.
	3160		476*a*	0·215	79 15·8	240 45	24 26	+12 35	q_2.
	3161		476*a*	0·225	84 49·1	239 41	23 22	+11 43	o.
	3162		475	0·197	52 6·8	251 20	35 1	+20 50	K.
	3163		475	0·244	46 4·7	249 19	33 0	+19 2	r.
	3164		475	0·234	38 25·7	249 4	32 45	+21 7	s.
	3165		475	0·193	31 32·6	252 17	35 58	+20 9	t.
	3166		475	0·216	50 56·8	251 33	35 14	+20 9	l.
	3167		474	0·419	166 16·4	239 23	23 4	−13 46	S_1.
	3168		474	0·413	167 38·8	239 30	23 11	−13 37	S_2.
	3169		474	0·420	167 28·7	239 26	23 7	−13 41	S_3.
	3170		474	0·339	251 44·8	265 38	49 19	− 5 11	Σ_1.
	3171		474	0·323	248 31·1	265 43	49 24	− 4 59	Σ_2.
	3172		474	0·290	237 10·0	247 39	31 20	−11 9	σ_1.
	3173		474	0·271	237 0·6	248 34	32 15	−10 59	σ_2.
	3174		474	0·261	238 54·3	246 59	30 40	−10 44	σ_3.
	3175		473	0·665	287 35·4	295 8	78 49	+ 5 31	P_1.
	3176		473	0·673	285 50·2	295 18	78 59	+ 4 57	P_2.
	3177		473	0·683	285 27·9	295 17	78 58	+ 4 11	P_3.
	3178		472	0·856	262 15·3	306 23	90 4	−18 20	R.
22.	3179	233·557	474*a*	0·696	212 34·7	266 37	9 38	−35 28	S.
	3180		476*a*	0·459	303 10·9	281 36	24 37	+12 55	Q_1.
	3181		476*a*	0·483	302 26·7	283 13	26 14	+12 50	Q_2.
	3182		476*a*	0·540	301 0·6	287 11	30 12	+12 37	q_1.
	3183		476*a*	0·521	299 59·5	285 57	28 38	+11 57	q_2.
	3184		473*a*	0·817	300 1·7	309 51	52 52	+13 28	Σ_1.
	3185		473*a*	0·765	298 31·8	304 53	47 54	+12 11	Σ_2.
	3186		473*a*	0·908	303 32·8	320 35	63 36	+16 35	O.
24.	3187	235·680	475*a*	0·555	102 49·0	222 52	295 47	+ 9 21	N.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1863.									
Aug. 24.	3188	235·680	476*b*	0·612	300° 48·3	294° 27′	7° 22′	+12° 42′	Q_1.
	3189		476*b*	0·627	300 38·9	295 34	8 29	+12 42	Q_2.
	3190		476*b*	0·695	300 25·2	300 55	13 50	+12 54	q_1.
	3191		476*b*	0·677	299 15·6	299 31	12 26	+12 2	q_2.
	3192		* 476*a*	0·906	300 33·3	322 18	35 13	+13 19	Σ.
	3193		476*a*	0·958	303 6·0	330 57	43 52	+15 22	P.
28.	3194	239·456	476	0·567	300 28·9	295 0	314 21	+11 42	N_1.
	3195		476	0·582	300 1·8	296 3	315 24	+11 31	N_2.
	3196		476	0·572	299 43·2	295 22	314 43	+11 17	N_3.
	3197		476	0·484	288 15·3	289 18	308 39	+ 5 20	O.
	3198		476	0·440	275 8·2	285 27	304 48	− 0 4	P.
29.	3199	240·444	477*a*	0·950	125 39·6	330 38	335 58	−11 56	S.
	3200		476	0·976	294 37·3	339 0	344 20	+ 5 37	Q.
	3201		476	0·689	291 34·5	305 0	310 20	+ 6 4	N_1.
	3202		476	0·695	291 28·6	305 21	310 41	+ 5 52	N_2.
	3203		476	0·692	290 59·5	305 12	310 32	+ 5 33	N_3.
	3204		476	0·722	289 11·3	307 29	312 49	+ 4 3	O.
	3205		476	0·191	273 25·5	271 49	277 9	+ 3 52	Π.
	3206		476	0·447	281 43·7	287 28	292 48	+ 2 34	p.
31.	3207	242·479	477	0·743	108 41·4	215 13	191 41	+ 6 33	P.
	3208		477	0·746	107 46·3	214 55	191 23	+ 7 13	π.
	3209		477	0·739	103 44·8	215 21	191 49	+10 12	p.
	3210		478*a*	0·585	144 38·7	233 32	210 0	−12 36	S.
Sept. 1.	3211	243·437	477*b*	0·731	135 31·4	221 26	184 19	−12 12	M.
3.	3212	245·451	477	0·086	59 7·9	262 15	196 34	+10 13	O.
7.	3213	249·510	477*c*	0·927	109 49·5	202 2	78 46	+ 5 24	M.
10.	3214	252·424	477*c*	0·773	101 50·1	222 7	57 31	+13 27	O.
	3215		477*c*	0·495	115 28·2	243 25	78 49	+ 5 14	M.
	3216		478	0·049	167 9·7	272 46	118 10	+ 4 27	Q.
	3217		478	0·068	200 11·8	272 52	108 16	+ 3 33	q.
	3218		479	0·122	249 35·0	278 7	113 31	+ 2 23	P.
	3219		479	0·115	248 15·9	277 42	113 6	+ 2 34	p.
	3220		479	0·108	243 17·1	277 2	112 26	+ 2 29	π_1.
	3221		479	0·104	243 50·3	276 55	111 19	+ 2 43	π_2.
11.	3222	253·434	477*c*	0·607	100 30·8	235 14	56 59	+13 28	O.
	3223		477*c*	0·274	119 51·0	256 57	78 2	+ 5 21	M.
	3224		478	0·240	283 20·9	286 59	108 4	+ 4 21	Q.
	3225		478	0·242	279 9·1	287 18	108 23	+ 3 37	q.
	3226		479	0·349	280 15·7	292 34	113 39	+ 2 28	P.
	3227		479	0·342	280 32·1	291 56	113 1	+ 2 29	p.
	3228		479	0·334	280 35·4	291 23	112 28	+ 2 28	π_1.
	3229		479	0·332	279 52·8	290 2	111 7	+ 2 46	π_2.
16.	3230	258·436	479*a*	0·528	119 30·8	247 21	357 28	+ 3 26	Q.
	3231		479*b*	0·586	185 38·5	245 15	355 22	− 5 1	N.
17.	3232	259·507	479*a*	0·303	126 37·6	262 50	357 46	+ 3 13	Q.
	3233		479*c*	0·687	112 14·9	236 27	331 23	+ 6 39	x.
	3234		479*c*	0·678	115 3·7	237 19	332 15	+ 4 17	y.
18.	3235	260·479	479*a*	0·101	157 40·0	233 21	314 30	+ 3 10	Q.
	3236		479*c*	0·525	105 1·2	249 17	330 26	+11 9	x.
	3237		479*c*	0·522	103 5·6	249 35	330 44	+12 8	y.
19.	3238	261·440	479*a*	0·169	272 49·2	290 54	358 25	+ 3 21	Q.
21.	3239	263·583	479*c*	0·259	344 17·5	294 16	331 23	+18 9	M.
22.	3240	264·524	480*a*	0·376	120 28·9	263 1	286 47	+ 4 34	N.
	3241		480*b*	0·936	101 32·5	274 26	298 12	+ 9 26	P.
23.	3242	265·453	480	0·844	101 9·8	227 52	238 28	+15 46	P.
	3243		480	0·868	101 46·4	225 11	235 47	+15 18	p.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1863.									
Sept. 23.	3244	265·453	480	0·897	102° 20·6	221° 31′	232° 7′	+14° 48′	π.
	3245		480	0·888	103 29·2	222 30	233 6	+13 47	o.
	3246		480	0·877	104 4·0	224 3	234 39	+13 18	w.
	3247		480	0·641	100 21·9	246 13	256 49	+14 58	x.
	3248		480	0·593	101 6·8	249 42	260 18	+14 4	y.
	3249		480	0·487	103 35·3	256 54	267 30	+11 50	z.
	3250		480c	0·342	280 34·1	305 8	315 44	+ 1 34	M.
	3251		480c	0·362	282 43·6	306 29	317 5	+ 1 59	m.
	3252		480c	0·412	281 26·5	309 22	319 58	+ 0 42	N.
24.	3253	266·653	480	0·689	99 18·8	244 24	237 58	+15 43	P.
	3254		480	0·707	100 2·0	241 46	235 20	+15 16	p.
	3255		480	0·742	102 47·1	238 3	231 37	+14 51	π.
	3256		480	0·752	103 36·6	239 17	232 51	+13 50	o.
	3257		480	0·744	104 10·2	240 43	234 17	+13 11	w.
	3258		480	0·423	96 54·7	262 25	255 59	+14 57	x.
	3259		480	0·362	92 20·3	266 5	259 39	+14 1	y.
	3260		480	0·274	97 24·0	273 19	266 53	+11 47	z.
	3261		480c	0·892	287 50·7	349 4	342 38	− 1 50	W.
	3262		480	0·719	100 24·9	241 13	234 47	+15 4	a.
	3263		480	0·706	101 23·8	241 41	235 15	+14 56	b.
25.	3264	267·455	480	0·541	96 26·6	253 4	237 41	+15 40	P.
	3265		480	0·560	97 25·4	255 30	235 0	+15 15	p.
	3266		480	0·608	101 51·4	252 49	231 39	+13 57	o.
	3267		480	0·190	68 56·8	249 28	258 47	+13 59	y.
	3268		480	0·097	46 26·7	283 46	265 57	+13 58	Z.
	3269	267·622	480	0·516	95 48·0	257 45	237 35	+15 30	P.
	3270		480	0·534	96 58·2	255 21	235 11	+15 7	p.
	3271		480	0·582	101 42·2	251 50	231 40	+14 1	o.
	3272		480	0·170	60 43·4	278 53	258 43	+14 4	y.
	3273		480	0·104	18 33·6	276 14	256 4	+14 55	x.
26.	3274	268·464	480	0·360	88 32·3	269 18	237 11	+15 37	P_1.
	3275		480	0·353	86 44·4	269 47	237 40	+15 28	P_2.
	3276		480	0·370	89 43·2	267 13	235 6	+15 13	p.
	3277		480	0·418	97 22·9	263 1	230 54	+13 55	O_1.
	3278		480	0·414	95 54·2	263 24	231 17	+14 4	O_2.
	3279		480	0·170	350 56·9	290 57	258 50	+14 2	y.
	3280	268·594	480	0·340	87 27·9	271 12	237 14	+15 35	P_1.
	3281		480	0·334	85 29·1	271 46	237 38	+15 27	P_2.
	3282		480	0·352	89 10·6	269 9	235 11	+15 14	p.
	3283		480	0·398	96 55·6	264 49	230 51	+13 56	o_1.
	3284		480	0·404	95 24·9	265 19	231 21	+14 4	o_2.
	3285		480	0·128	10 55·2	293 9	259 11	+11 2	q.
28.	3286	270·546	481	0·917	123 11·2	235 21	173 43	− 3 50	S.
	3287		481	0·945	122 4·6	220 49	159 11	− 3 29	s.
	3288		481	0·939	123 33·2	222 9	160 31	− 5 10	σ.
	3289		480	0·161	85 50·2	282 22	220 44	+11 27	Π_1.
	3290		480	0·150	81 32·1	283 40	222 2	+11 30	Π_2.
	3291		480	0·111	62 34·9	287 0	226 22	+11 50	π.
	3292		480	0·192	349 29·4	298 33	236 55	+15 38	P_1.
	3293		480	0·212	343 12·3	298 41	237 3	+15 21	P_2.
	3294		480	0·192	353 1·4	296 16	234 48	+15 7	p_1.
	3295		480	0·179	347 12·4	295 50	234 12	+14 33	p_2.
	3296		480	0·162	358 47·7	292 25	230 47	+13 50	o_1.
	3297		480	0·156	352 10·3	292 53	231 15	+14 6	o_2.
	3298		480	0·146	357 18·8	290 57	229 19	+13 42	o_3.
	3299		480b	0·551	299 10·4	324 18	262 40	+ 7 6	y.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1863.									
Sept. 28.	3300	270·546	480c	0·553	271° 3′·7	320° 57′	259° 19′	− 7° 45′	w.
	3301	270·612	481	0·910	123 33·4	236 14	173 39	− 3 48	S.
	3302		481	0·940	122 29·1	221 41	159 6	− 3 25	s.
	3303		481	0·932	124 5·0	222 59	160 24	− 5 11	σ.
	3304		480	0·146	83 31·2	282 51	220 16	+11 28	Π_1.
	3305		480	0·140	79 16·1	284 29	221 54	+11 33	Π_2.
	3306		480	0·100	56 32·0	288 38	226 3	+11 45	π.
	3307		480	0·199	345 48·6	299 28	236 53	+15 35	P_1.
	3308		480	0·219	339 51·6	299 35	237 0	+15 20	P_2.
	3309		480	0·198	349 50·5	297 15	234 40	+15 2	p_1.
	3310		480	0·187	343 28·1	296 45	234 10	+14 39	p_2.
	3311		480	0·168	354 13·6	293 20	230 45	+13 40	o_1.
	3312		480	0·162	347 8·9	293 33	230 58	+14 7	o_2.
	3313		480	0·149	352 16·7	291 59	229 24	+13 45	o_3.
	3314		480b	0·562	298 49·1	325 16	262 41	+ 7 6	y.
	3315		480c	0·562	271 4·9	321 10	259 15	− 7 50	w.
	3316		480a	0·812	80 16·1	239 44	177 9	+32 34	x.
29.	3317	271·437	481	0·806	125 23·6	247 37	173 20	− 3 45	S.
	3318		481	0·868	123 13·3	233 10	158 53	− 3 28	s_1.
	3319		481	0·827	124 7·1	233 36	159 19	− 3 21	s_2.
	3320		481	0·851	126 10·9	234 14	159 57	− 5 5	σ_1.
	3321		481	0·846	125 57·5	234 20	160 3	− 5 19	σ_2.
	3322		481	0·875	126 27·5	233 38	159 21	− 6 31	t.
	3323		480	0·340	323 57·7	310 34	236 17	+15 37	P.
	3324		480	0·371	320 28·7	311 15	236 58	+15 19	P_2.
	3325		480	0·335	325 35·3	308 38	234 21	+15 5	p_1.
	3326		480	0·336	321 13·8	308 22	234 5	+14 41	p_2.
	3327		480	0·315	324 12·7	308 48	234 31	+14 58	o.
	3328		480	0·174	325 47·0	300 37	226 20	+11 35	π.
	3329		480	0·195	113 59·5	280 31	206 14	+ 7 0	x.
	3330		480	0·156	114 23·0	282 49	208 32	+ 6 55	z.
	3331		482	0·759	297 12·8	341 15	266 58	+ 5 17	y_1.
	3332		482	0·721	297 35·0	340 50	266 33	+ 5 14	y_2.
	3333		482	0·727	299 33·8	340 58	266 41	+ 5 29	y_3.
	3334		482	0·692	302 59·1	340 15	265 58	+ 5 19	y_4.
	3335	271·596	481	0·787	125 55·1	249 47	173 15	− 3 41	S.
	3336		481	0·855	123 34·4	235 16	158 44	− 3 32	s_1.
	3337		481	0·810	124 28·7	236 5	159 33	− 3 20	s_2.
	3338		481	0·835	126 36·2	236 15	159 43	− 5 7	σ_1.
	3339		481	0·818	126 2·7	236 22	159 50	− 5 20	σ_2.
	3340		481	0·859	127 0·4	235 44	159 12	− 6 28	t.
	3341		480	0·368	321 52·5	312 32	236 0	+15 33	P.
	3342		480	0·362	323 21·4	310 39	234 7	+14 59	P_1.
	3343		480	0·356	322 8·2	310 21	233 49	+14 38	P_2.
	3344		480	0·166	112 15·0	282 42	206 10	+ 6 57	x.
	3345		480	0·116	112 59·1	284 43	208 11	+ 6 59	z.
	3346		482	0·787	297 37·6	343 25	266 53	+ 5 19	y_1.
	3347		482	0·744	297 58·4	343 1	266 29	+ 5 15	y_2.
	3348		482	0·754	299 51·0	343 9	266 37	+ 5 31	y_3.
	3349		482	0·719	303 18·1	342 32	266 0	+ 5 17	y_4.
30.	3350	272·486	481	0·640	129 55·9	262 10	173 0	− 3 46	S.
	3351		481	0·741	126 5·0	247 19	158 9	− 3 27	s_1.
	3352		481	0·665	127 46·1	248 7	158 57	− 3 18	s_2.
	3353		481	0·712	130 23·5	248 27	159 17	− 5 11	σ_1.
	3354		481	0·679	129 31·6	248 58	159 48	− 5 19	σ_2.
	3355		481	0·730	130 13·5	247 57	158 47	− 6 20	t.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Heliographical Longitude.	Heliographical Latitude.	Spot.
1863.									
Sept. 30.	3356	272·486	480	0·023	297° 44′·9	294° 37′	205° 27′	+ 6° 47′	X.
	3357		480	0·534	315 16·5	325 1	235 51	+15 40	P.
	3358		480	0·529	316 12·5	322 51	233 41	+14 47	p_1.
	3359		480	0·524	315 11·4	322 2	232 52	+14 21	p_2.
	3360		480	0·511	314 43·9	321 27	232 17	+13 33	o.
	3361		480	0·498	317 35·6	319 54	230 44	+13 40	O.
	3362		482	0·916	299 6·9	355 9	265 59	+ 5 17	y_1.
	3363		482	0·864	299 15·3	355 14	266 4	+ 5 14	y_2.
	3364		482	0·887	301 3·9	354 38	265 28	+ 5 37	y_3.
	3365		482	0·831	295 51·9	354 23	265 13	+ 3 11	m.
	3366		482	0·857	295 3·8	353 57	264 47	+ 3 38	n.
Oct. 1.	3367	273·397	481	0·469	136 47·6	274 38	172 33	− 3 40	S.
	3368		481	0·585	130 7·6	259 55	157 50	− 3 29	s_1.
	3369		481	0·477	131 46·2	260 45	158 40	− 3 16	s_2.
	3370		481	0·543	135 42·4	261 26	159 21	− 5 15	σ_1.
	3371		481	0·511	136 4·7	262 4	159 59	− 5 24	σ_2.
	3372		481	0·566	136 17·2	261 8	159 3	− 6 25	t.
	3373		480	0·690	312 11·1	338 7	236 2	+15 37	P.
	3374		480	0·686	313 4·2	335 54	233 49	+14 50	p_1.
	3375		480	0·691	310 51·1	334 39	232 34	+14 19	p_2.
	3376		480	0·669	311 11·4	334 0	231 55	+13 28	o.
	3377		482	0·983	300 26·6	8 14	266 9	+ 5 23	y.
	3378		482	0·916	297 40·5	8 42	266 37	+ 4 29	m.
	3379		482	0·960	298 33·0	0 16	258 11	+ 4 9	n.
	3380	273·635	481	0·426	139 11·4	277 59	172 31	− 3 38	S.
	3381		481	0·545	131 44·4	263 24	157 56	− 3 34	s_1.
	3382		481	0·429	133 31·6	264 11	158 43	− 3 18	s_2.
	3383		481	0·501	137 42·5	264 45	159 17	− 5 16	σ_1.
	3384		481	0·470	138 20·8	265 32	160 4	− 5 25	σ_2.
	3385		481	0·528	138 17·1	265 43	160 15	− 6 17	t_2.
	3386		481	0·560	136 11·7	264 30	159 2	− 6 28	t.
	3387		480	0·728	311 52·7	341 40	236 12	+15 40	P.
	3388		480	0·724	312 40·2	339 3	233 35	+14 52	p_1.
	3389		480	0·729	310 40·1	338 7	232 39	+14 20	p_2.
	3390		480	0·708	311 2·0	337 18	231 50	+13 29	o.
	3391		482	0·995	300 53·5	11 22	265 54	+ 5 20	y.
	3392		482	0·979	299 23·9	12 17	266 49	+ 4 30	m.
	3393		482	0·939	298 7·0	3 53	258 25	+ 4 6	n.
	3394		482*a*	0·354	299 57·4	314 46	209 18	+ 7 32	x.
	3395		482*a*	0·338	299 37·5	313 45	208 17	+ 7 24	v.
5.	3396	277·423	481	0·472	275 12·2	323 48	164 37	− 4 1	S_2.
	3397		481	0·473	273 27·1	323 30	164 19	− 4 47	S.
	3398		481	0·445	272 6·5	321 36	162 25	− 4 37	u.
	3399		481	0·469	271 49·4	322 24	163 13	− 5 8	v.
	3400		481	0·358	269 8·9	316 14	157 3	− 3 21	S_1.
	3401		483	0·835	130 58·2	243 7	83 56	− 8 26	M.
6.	3402	278·422	481	0·648	282 58·2	337 51	164 29	− 3 39	S_2.
	3403		481	0·648	281 25·5	337 32	164 10	− 4 50	S.
	3404		481	0·624	280 38·1	335 39	162 17	− 4 41	u.
	3405		481	0·639	280 4·4	336 40	163 18	− 5 7	v.
	3406		481	0·550	280 29·4	330 43	157 21	− 3 19	S_1.
	3407		483	0·686	136 16·2	256 36	83 14	− 8 47	M.
	3408	278·498	481	0·668	283 6·5	338 51	164 25	− 4 1	S_2.
	3409		481	0·668	281 49·0	338 37	164 11	− 4 56	S.
	3410		481	0·645	281 6·5	336 38	162 12	− 4 40	u.
	3411		481	0·669	280 26·7	337 45	163 19	− 5 9	v.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1863.					° ′	° ′	° ′	° ′	
Oct. 6.	3412	278·498	481	0·570	280 48·6	331 34	157 8	− 3 22	S.
	3413		483	0·681	136 48·2	257 25	82 59	− 8 49	M.
13.	3414	285·587	483	0·829	280 16·0	359 36	84 31	− 9 52	M.
	3415		484	0·378	232 35·8	315 40	40 41	−13 59	N.
	3416		484	0·320	219 23·2	310 0	35 1	−12 15	O.
14.	3417	286·405	484	0·487	251 34·4	327 28	40 52	−14 1	N.
	3418		484	0·478	251 28·3	327 17	40 41	−13 54	N$_0$.
	3419		483	0·913	282 39·3	11 5	84 29	− 9 50	M.
	3420		484	0·429	245 2·7	322 36	36 0	−13 55	q.
	3421		484	0·443	248 42·0	324 24	37 48	−13 30	p.
	3422		484	0·406	246 13·8	321 59	35 23	−12 28	O.
	3423		484	0·413	245 31·3	321 45	35 9	−13 17	o.
16.	3424	288·412	485	0·931	196 26·6	242 31	287 27	− 7 13	W.
	3425		486	0·973	112 44·1	231 49	276 45	+ 4 50	V.
	3426	288·587	485	0·917	196 51·3	245 2	287 29	− 7 11	W.
	3427		486	0·964	112 57·0	234 12	276 39	+ 4 53	V.
17.	3428	289·412	485	0·821	198 59·4	256 17	287 2	− 7 16	W.
	3429		486	0·886	114 23·7	245 33	276 18	+ 4 57	V.
	3430		486	0·918	114 21·8	245 8	275 53	+ 4 50	V.
	3431	289·599	485	0·797	199 44·0	259 2	287 8	− 7 18	W.
	3432		486	0·865	114 39·4	248 9	276 15	+ 4 58	V.
22.	3433	294·633	487	0·339	256 49·0	330 8	286 50	− 7 20	W.
	3434		487	0·335	254 23·2	329 27	286 9	− 7 49	W$_0$.
	3435		488	0·312	292 40·4	332 57	289 39	+ 3 53	V.
	3436		488	0·326	290 11·1	333 40	290 22	+ 3 1	v.
	3437		488	0·238	294 28·3	328 33	285 15	+ 4 40	q.
	3438		488	0·258	294 4·6	329 46	286 28	+ 4 30	p.
	3439		488	0·240	297 26·6	328 43	285 25	+ 5 22	o.
23.	3440	295·428	487	0·480	270 40·4	341 15	286 40	− 7 18	W.
	3441		487	0·471	268 52·8	340 39	286 4	− 7 45	W$_0$.
	3442		488	0·510	293 40·4	343 52	289 17	+ 3 51	V.
	3443		488	0·421	296 10·6	339 46	285 11	+ 5 19	o.
	3444	295·591	487	0·514	272 15·9	343 31	286 37	− 7 17	W.
	3445		487	0·503	270 30·4	342 54	286 0	− 7 47	W$_0$.
	3446		488	0·549	293 51·2	346 7	289 13	+ 3 53	V.
	3447		488	0·457	296 12·1	342 1	285 7	+ 5 20	o.
31.	3448	303·414	489	0·817	271 9·1	14 40	206 48	−16 25	M.
	3449		489	0·761	268 19·7	8 44	200 52	−16 46	m.
	3450		489	0·779	268 56·8	10 28	202 36	−16 53	n.
	3451		489	0·759	267 12·4	8 16	200 24	−17 30	q.
	3452		490	0·178	43 46·1	320 11	152 19	+13 58	R.
	3453		490	0·183	48 38·7	319 14	151 22	+13 55	R$_0$.
	3454		490	0·228	73 55·7	313 29	145 37	+12 48	S.
	3455		490	0·231	77 35·8	312 47	144 55	+12 14	S$_0$.
	3456		491	0·961	127 32·8	251 1	83 9	−10 57	s.
	3457	303·591	489	0·837	271 58·0	17 25	207 3	−16 31	M.
	3458		489	0·799	269 45·7	10 36	200 14	−16 41	m.
	3459		489	0·781	269 12·9	12 41	202 19	−16 57	n.
	3460		489	0·779	268 12·2	10 22	200 0	−17 31	q.
	3461		490	0·170	29 43·4	322 29	152 7	+13 54	R.
	3462		490	0·168	35 16·8	321 41	151 19	+13 50	R$_0$.
	3463		490	0·198	70 58·6	315 33	145 11	+12 49	S.
	3464		490	0·200	67 7·2	315 21	144 59	+12 18	S$_0$.
	3465		491	0·952	127 50·7	253 12	82 50	−10 52	s.
Nov. 1.	3466	304·398	489	0·922	274 40·5	29 40	207 51	−16 40	M.
	3467		490	0·247	340 1·1	335 0	153 11	+13 58	R.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Heliographical Longitude.	Heliographical Latitude.	Spot.
1863.									
Nov. 1.	3468	304·398	490	0·148	10° 40·0	328° 6	146° 17	+12° 50	S.
	3469		490	0·177	2 51·0	329 4	147 15	+13 11	a.
	3470		490	0·196	356 55·0	330 52	149 3	+13 42	b.
	3471		490	0·222	347 13·2	332 13	150 24	+13 51	c.
	3472		491	0·867	129 51·6	263 40	81 51	−10 54	s.
2.	3473	305·452	489	0·982	277 53·7	44 46	208 0	−16 43	M.
	3474		490	0·444	319 55·9	350 52	154 6	+13 51	R.
	3475		490	0·418	321 17·9	349 48	153 2	+13 50	m.
	3476		490	0·301	323 49·3	342 43	145 57	+12 53	S.
	3477		490	0·289	325 19·7	340 17	143 31	+12 28	S_1.
	3478		490	0·276	327 36·0	338 58	142 12	+12 38	n.
6.	3479	309·427	490	0·974	307 41·9	46 41	153 32	+14 27	S_0.
9.	3480	312·510	492	0·521	91 35·4	302 53	6 0	+13 50	P_1.
	3481		492	0·515	91 35·4	303 13	6 20	+13 44	P_2.
12.	3482	315·572	494	0·972	102 10·1	259 12	278 53	+10 32	A.
	3483		493	0·943	124 12·3	266 19	286 0	−10 11	B.
13.	3484	316·492	494	0·916	101 9·8	271 43	278 21	+10 25	A.
	3485		493	0·864	125 50·4	278 59	285 37	−10 17	B.
19.	3486	322·549	495	0·300	323 17·7	357 36	278 19	+11 31	A_0.
	3487		495	0·310	322 37·8	358 11	278 54	+11 39	A_1.
	3488		493	0·466	265 32·7	8 1	288 44	− 9 12	B.
	3489		493	0·330	254 40·2	358 26	279 9	− 8 54	b_0.
	3490		493	0·339	256 9·4	359 10	279 53	− 8 47	b_1.
	3491		493	0·224	243 58·0	351 40	272 23	− 7 5	c_0.
	3492		493	0·232	238 6·3	350 57	271 40	− 8 18	c_1.
	3493		496	0·615	101 40·9	305 6	225 49	+ 7 2	D.
	3494		497	0·869	129 9·3	284 33	205 16	−15 6	E.
20.	3495	323·477	495	0·475	310 34·1	10 37	278 11	+11 29	A_0.
	3496		495	0·484	310 9·5	11 29	279 3	+11 42	A_1.
	3497		493	0·634	272 13·1	20 32	288 6	− 9 4	B.
	3498		493	0·496	267 32·2	10 55	278 29	− 8 59	b_0.
	3499		493	0·510	267 6·6	12 6	279 40	− 8 42	b_1.
	3500		498	0·395	266 52·1	4 25	271 59	− 7 6	c_0.
	3501		498	0·408	261 34·4	3 36	271 10	− 8 23	c_1.
	3502		496	0·431	97 45·1	318 30	226 4	+ 7 5	D.
	3503		497	0·462	125 52·3	317 16	224 50	− 5 23	E.
	3504		497	0·506	124 42·8	314 20	221 54	− 5 34	e.
	3505		497	0·403	209 46·3	347 52	255 26	−21 16	G.
	3506		499	0·872	98 2·1	283 27	191 1	+11 25	H.
	3507		499	0·901	95 42·6	279 40	187 14	+14 4	h.
21.	3508	324·477	495	0·664	304 32·2	25 28	278 51	+11 53	A_1.
	3509		495	0·658	306 58·7	24 17	277 40	+11 40	A_2.
	3510		493	0·686	274 17·3	25 42	279 5	− 8 40	b_1.
	3511		493	0·666	274 3·4	24 53	278 16	− 9 0	b_2.
	3512		498	0·487	238 20·7	2 51	256 14	−20 43	G_1.
	3513		498	0·463	233 36·7	1 8	254 31	−21 9	G_2.
	3514		497	0·267	138 39·1	331 56	225 19	− 5 28	E.
	3515		497	0·599	138 41·3	327 41	221 4	− 5 40	e.
	3516		497	0·267	163 20·8	335 54	229 17	−10 37	m.
	3517		497	0·243	173 14·4	339 17	232 40	−10 51	n.
	3518		496	0·214	86 40·7	332 10	225 33	+ 7 14	D.
	3519		499	0·723	96 2·2	297 25	190 48	+11 21	H.
22.	3520	325·430	495	0·810	301 37·8	38 49	278 41	+11 57	A_1.
	3521		495	0·805	302 18·9	38 2	277 54	+11 33	A_0.
	3522		493	0·826	277 20·5	38 57	278 49	− 8 41	b_1.
	3523		496	0·089	356 59·2	345 25	225 17	+ 7 18	D.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1863.					° ′	° ′	° ′	° ′	
Nov. 22.	3524	325·430	497	0·130	190 6·3	344 56	224 48	− 5 23	E.
	3525		497	0·224	215 13·9	349 11	629 3	−10 33	m.
	3526		497	0·247	228 29·1	352 37	232 29	−10 35	n.
23.	3527	326·420	495	0·922	300 19·9	52 46	278 35	+11 58	A_1'.
	3528		495	0·920	300 44·1	52 0	277 49	+11 29	A_0'.
	3529		493	0·933	278 50·1	52 30	278 19	− 8 49	b_1'.
	3530		496	0·301	305 56·5	3 24	229 13	+ 6 36	D.
	3531		496	0·266	308 36·4	1 11	227 0	+ 6 43	d.
	3532		497	0·247	255 55·4	359 12	225 1	− 5 26	E.
	3533		497	0·368	251 59·1	3 17	229 6	−10 40	m.
	3534		497	0·408	257 4·2	6 52	232 41	−10 44	n.
28.	3535	331·477	500	0·580	125 49·8	317 57	112 2	− 9 44	a.
	3536		500	0·530	128 45·7	321 47	115 52	−10 14	b.
	3537		506	0·493	133 36·9	325 0	119 5	−11 39	c.
	3538		500	0·565	129 10·6	319 32	113 37	−11 14	d.
Dec. 3.	3539	336·517	503	0·570	261 39·7	28 59	111 35	−12 49	a.
	3540		503	0·611	261 17·4	31 44	114 20	−14 1	b.
	3541		502	0·698	267 59·3	39 26	122 2	−11 39	c.
	3542		501	0·880	297 19·4	57 40	140 16	+10 43	F.
	3543		501	0·820	296 57·1	51 12	133 48	+ 9 44	f.
4.	3544	337·449	501	0·962	296 9·6	71 12	140 35	+10 39	F.
	3545		501	0·927	295 16·5	63 54	133 17	+ 9 48	f.
	3546		503	0·721	266 17·4	42 44	112 7	−12 55	a.
	3547		503	0·766	269 45·7	45 25	114 48	−14 9	b.
	3548		502	0·829	270 38·9	53 24	122 47	−11 33	c_0'.
	3549		502	0·836	270 34·4	53 48	123 11	−11 38	c_1'.
10.	3550	343·467	504	0·698	273 25·4	47 32	41 33	− 6 35	a'.
	3551		504	0·668	273 55·3	45 16	39 17	− 6 0	b'.
	3552		505	0·588	108 21·4	328 5	312 6	− 3 52	A.
	3553		506	0·712	108 11·5	318 42	302 43	− 4 26	B_1'.
	3554		506	0·723	109 4·6	317 50	301 51	− 5 8	B_2'.
	3555		506	0·614	109 50·9	326 17	310 18	− 4 56	m.
	3556		506	0·645	108 34·7	323 54	307 55	− 4 20	n.
	3557		506	0·655	109 28·3	323 14	307 15	− 4 58	o.
	3558		507	0·930	88 57·2	296 17	280 18	+12 18	C.
14.	3559	347·492	505	0·339	271 35·5	27 18	314 13	− 3 58	A.
	3560		506	0·234	265 31·1	20 47	307 42	− 4 27	a.
	3561		509	0·277	334 34·9	17 18	304 13	+11 54	S.
	3562		508	0·320	324 57·4	21 14	308 13	+11 55	t.
	3563		507	0·444	81 16·4	342 53	269 48	+ 7 30	C.
	3564		507	0·342	56 35·0	353 12	280 7	+12 44	c.
18.	3565	351·446	505	0·953	274 28·7	84 6	314 56	− 4 34	A.
	3566		510	0·700	79 5·4	329 35	200 25	+12 30	B.
	3567		510	0·750	78 32·7	325 35	196 25	+13 58	b.
	3568		510	0·767	78 26·9	324 6	194 56	+14 25	c.
	3569		510	0·815	78 8·4	319 38	190 28	+15 45	d.
	3570		510	0·866	80 23·0	313 45	184 35	+15 3	e.
	3571		510	0·768	82 17·4	313 16	184 6	+11 35	f.
	3572		510	0·753	81 9·7	324 51	195 41	+12 7	g.
	3573		510	0·730	80 43·5	326 51	197 41	+11 59	h.
19.	3574	352·531	505	0·997	274 4·6	99 42	315 9	− 4 38	A.
	3575		510	0·521	71 1·8	345 6	200 33	+12 36	B.
	3576		510	0·578	71 49·4	341 21	196 48	+14 3	b.
	3577		510	0·614	73 17·8	339 42	195 9	+14 31	c.
	3578		510	0·657	71 38·7	335 12	190 39	+15 46	d.
	3579		510	0·683	75 4·7	329 28	184 55	+15 9	e.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1863.					° ′	° ′	° ′	° ′	
Dec. 19.	3580	352·531	510	0·725	76 21·4	328 20	183 47	+15 4	s.
22.	3581	355·513	510	0·291	328 57·8	27 51	201 0	+12 43	B.
	3582		510	0·284	352 10·0	20 12	193 21	+13 53	C.
	3583		510	0·297	20 48·4	11 44	184 53	+14 41	s.
	3584		510	0·305	28 13·0	9 28	182 37	+14 26	e.
	3585		510	0·305	4 32·1	16 42	189 51	+15 38	d_1.
	3586		510	0·306	0 46·3	17 53	191 2	+15 39	d_2.
	3587		511	0·777	81 15·9	326 34	139 43	+10 43	M.
	3588		512	0·646	109 17·9	336 23	149 32	− 9 30	N_1.
	3589		512	0·642	110 11·8	336 44	149 53	−10 2	N_2.
	3590		512	0·655	110 35·6	335 51	149 0	−10 26	N_3.
	3591		512	0·711	109 27·7	331 16	144 25	−10 19	o.
	3592		512	0·737	109 50·8	329 9	142 18	−50 52	m.
	3593		513	0·883	107 20·7	314 18	127 27	−10 10	P.
	3594		513	0·901	105 44·4	311 56	125 5	− 8 51	p.
23.	3595	356·555	510	0·402	318 36·5	35 27	193 50	+13 57	C.
	3596		510	0·381	328 24·9	31 4	189 27	+15 39	d_1.
	3597		510	0·394	325 20·1	32 51	191 14	+15 43	d_2.
	3598		510	0·324	339 9·2	26 48	185 11	+14 47	s_1.
	3599		510	0·330	337 58·9	26 55	185 18	+14 39	s_2.
	3600		514	0·299	344 31·5	24 34	182 57	+14 28	e_1.
	3601		514	0·290	344 29·8	24 7	182 30	+14 27	e_2.
	3602		511	0·611	76 32·8	341 25	139 48	+10 12	M.
	3603		512	0·545	114 19·3	346 14	144 37	−11 26	N.
	3604		512	0·502	116 21·1	349 20	148 43	−11 43	o.
	3605		512	0·445	115 20·2	352 49	151 12	−10 13	m.
	3606		511*a*	0·633	71 51·4	341 50	140 13	+13 28	r.
	3607		511*a*	0·587	70 43·5	344 5	142 28	+12 4	S.
25.	3608	358·420	510	0·775	292 18·1	70 6	202 11	+12 46	B.
	3609		510	0·719	298 33·7	61 59	193 54	+13 57	C.
	3610		510	0·682	302 3·0	58 22	190 17	+15 40	d_1.
	3611		510	0·691	300 57·2	59 53	191 48	+15 45	d_2.
	3612		514	0·572	303 52·4	51 19	183 14	+14 29	e_1.
	3613		514	0·568	303 18·2	51 5	183 0	+14 23	e_2.
	3614		512	0·126	180 54·0	18 54	150 49	−10 33	N.
	3615		513	0·199	148 2·8	10 18	142 13	−10 49	m.
	3616		513	0·401	117 33·8	356 11	128 6	−10 19	P.
	3617		515	0·786	91 10·9	327 26	99 21	+ 1 54	O.
30.	3618	363·471	512	0·916	264 46·3	90 15	150 31	− 8 45	N.
	3619		513	0·675	258 13·9	65 42	125 58	−12 6	a.
	3620		513	0·720	261 32·2	69 39	129 55	−10 18	b.
	3621		513	0·763	261 59·3	73 23	133 39	−10 19	c.
	3622		516	0·231	301 39·4	35 42	95 58	+ 3 26	S.
	3623		515	0·323	288 51·8	42 3	102 19	+ 3 37	O.

Explanation of the Plates.

PLATE I.

A. Represents the absolute length, in inches, of six lines, photographed in different parts of the field of the object-glass. The abscissæ are the distances of the lines from the centre of the object-glass, the ordinates the corresponding lengths in inches.

B. RR′ represents the mean length of all observed lines in one direction, WW′ the mean length of the lines measured in a direction perpendicular to the former. MM′ is the mean length of all measured lines. The coordinates are the same as before.

PLATE II.

Represents the pinnacle of the Kew Pagoda as it appears on the photographs in different parts of the field. The measurement of the distances ab, $a'b'$, cd, $c'd'$ gave the length of the same lines in different parts of the field, while the distances aa', $a'c$, cc', bb', $b'd$, dd' supplied the material for measurements in a direction perpendicular to the former.

II. *A Third Memoir on Skew Surfaces, otherwise Scrolls.* By Professor CAYLEY, F.R.S.

Received May 30,—Read June 18, 1868.

THE present Memoir is supplementary to my "Second Memoir on Skew Surfaces, otherwise Scrolls," Phil. Trans. vol. cliv. (1864), pp. 559–577, and relates also to the theory of skew surfaces of the fourth order, or quartic scrolls. It was pointed out to me by Herr SCHWARZ*, in a letter dated Halle, June 1, 1867, that in the enumeration contained in my Second Memoir I have given only a particular case of the quartic scrolls which have a directrix skew cubic; viz. my eighth species, $S(1, 3^2)$, where there is also a directrix line. And this led me to observe that I had in like manner mentioned only a particular case of the quartic scrolls with a triple directrix line; viz. my third species, $S(1_3, 1, 4)$, where there is also a simple directrix line. The omitted species, say, *ninth species*, $S(1_3)$, *with a triple directrix line*, and *tenth species*, $S(3^2)$, *with a directrix skew* cubic, are considered in the present Memoir; and in reference to them I develope a theory of the reciprocal relations of these scrolls, which has some very interesting analytical features.

The paragraphs of the present Memoir are numbered consecutively with those of my Second Memoir above referred to.

Quartic Scroll, Ninth Species, $S(1_3)$, *with a triple directrix line.*

54. Consider a line the intersection of two planes, and let the equation of the one plane contain in the order 3, that of the second plane contain linearly, a variable parameter θ; the equations of the two planes may be taken to be

$$(p, q, r, s\,\text{X}\,\theta, 1)^3 = 0,\ (u, v\,\text{X}\,\theta, 1) = 0,$$

where (p, q, r, s, u, v) are any linear functions whatever of the coordinates (x, y, z, w). Hence eliminating θ we have as the equation of the scroll generated by the line in question

$$(p, q, r, s\,\text{X}\,v, -u)^3 = 0,$$

viz. this is a quartic scroll having the line $u=0$, $v=0$ for a triple line; that is, the line in question is a triple directrix line.

55. Taking $x=0$, $y=0$ for the equations of the directrix line, or writing $u=x$, $v=y$, and moreover expressing (p, q, r, s) as linear functions of the coordinates (x, y, z, w), the equation of the scroll takes the form

$$(*\,\text{X}\,x, y)^4 + z(*\,\text{X}\,x, y)^3 + w(*'\,\text{X}\,x, y)^3 = 0;$$

* I take the opportunity of referring to his paper on Quintic Scrolls, SCHWARZ, "Ueber die geradlinigen Flächen fünften Grades," Crelle, t. lxvii. (1867), pp. 23–57.

and we may, by changing the values of z and w, make the term in (x, y) to be

$$(*\nabla(x, y)^4+(\alpha x+\beta y)(*\nabla(x, y)^3+(\gamma x+\delta y)(*'\nabla(x, y)^3,$$

where the arbitrary constants α, β, γ, δ may be so determined as to reduce this to a monomial kx^4, kx^3y, or kx^2y^2.

56. The coefficient k may vanish, and the equation of the scroll then is

$$z(*\nabla(x, y)^3+w(*'\nabla(x, y)^3=0,$$

or, what is the same thing, it is

$$(*\nabla(x, y)^3(z, w)=0,$$

viz. the scroll has in this particular case the simple directrix line $z=0$, $w=0$, thus reducing itself to the *third species*, S(1$_3$, 1, 4), *with a triple directrix line and a single directrix line*. It is proper to exclude this, and consider the ninth species, S(1$_3$), as having a triple directrix line, but no simple directrix line.

57. The scroll S(1$_3$) may be considered as a scroll S(m, n, p) generated by a line which meets each of three given directrices; viz. these may be taken to be the directrix line, and any two plane sections of the scroll. The section by any plane is a quartic curve having a triple point at the intersection with the directrix line; moreover the sections by any two planes meet in four points the intersections of the scroll by the line of intersection of the two planes. Conversely, taking any line and two quartics related as above (that is, each quartic has a triple point at its intersection with the line, and the two quartics meet in four points lying in a line), the lines which meet the three curves generate a quartic scroll S(1$_3$). This appears from the formula

$$\mathrm{S}(m, n, p)=2mnp-\alpha m-\beta n-\gamma p \text{ (Second Memoir, No. 5)};$$

we have in the present case

$$m=1,\ n=4,\ p=4,\quad \alpha=4,\ \beta=3,\ \gamma=3,$$

and the order of the scroll is $32-4-12-12$, $=4$, that is, the scroll is a quartic scroll; there is no difficulty in seeing that through each point of the line there pass three generating lines, but through each point of either of the plane quartics only a single generating line; that is, that the line is a triple directrix line, but each of the plane quartics a simple directrix curve.

58. We may instead of the section by any plane, consider the section by a plane through a generating line, or by a plane through two of the three generating lines which meet at any point of the directrix line; if (to consider only the most simple case) each of the planes be thus a plane through two generating lines, the section by either of these planes is made up of the two generating lines, and of a conic passing through the directrix line; the directrices are thus the line and two conics each of them meeting the line; we have therefore in the foregoing formula

$$m=1,\ n=2,\ p=2,\quad \alpha=0,\ \beta=1,\ \gamma=1,$$

and the order of the scroll is $8-2-2$, $=4$ as before.

Quartic Scroll, Tenth Species, (3^2)*, with a directrix skew cubic met twice by each generating line**.

59. Consider a line the intersection of two planes; and let the equation of each plane contain in the order 2 a variable parameter θ; the equations of the two planes may be taken to be

$$(p, q, r)(\theta, 1)^2=0, \quad (p', q', r')(\theta, 1)^2=0,$$

where (p, q, r, p', q', r') are linear functions of the coordinates (x, y, z, w); hence eliminating θ, we have as the equation of the scroll generated by the line in question, $\square=0$, where $\square$ is the resultant of the two quadric functions. The equation may be written

$$4(pq'-p'q)(rq'-r'q)-(pr'-p'r)^2=0;$$

and the scroll has thus as a nodal (double) line the skew cubic determined by the equations

$$\left\| \begin{matrix} p, & q, & r \\ p', & q', & r' \end{matrix} \right\|=0.$$

It is easy to see (and indeed it will be shown presently) that this curve is met twice by each generating line of the scroll, and that the scroll is consequently a quartic scroll as described above.

60. The coordinates (x, y, z, w) may be fixed in such manner that the equations of the skew cubic shall be

$$\left\| \begin{matrix} x, & y, & z \\ y, & z, & w \end{matrix} \right\|=0,$$

or, what is the same thing,

$$yw-z^2=0, \quad zy-xw=0, \quad xz-y^2=0;$$

each of the equations $pq'-p'q=0$, $rq'-r'q=0$, $pr'-p'r=0$ is then the equation of a quadric surface passing through the skew cubic, or, what is the same thing, each of the functions $pq'-p'q$, $rq'-r'q$, $pr'-p'r$ is a linear function of $yw-z^2$, $zy-xw$, $xz-y^2$; and the equation of the scroll is given as a quadric equation in the last-mentioned quantities. It will be convenient to represent the equation in the form

$$(\mathrm{H}, \mathrm{F}, \mathrm{C}, \mathrm{B}, \mathrm{A}-\mathrm{F}, -\mathrm{G})(yw-z^2, zy-xw, xz-y^2)^2=0,$$

or, writing for shortness

$$yw-z^2, \ zy-xw, \ xz-y^2=p, q, r,$$

which letters (p, q, r) are used henceforward in this signification only, the equation

* I have worded this heading in accordance with that of the eighth species, Second Memoir, No. 47, but the two headings might be expressed more completely thus:

Eighth Species, $S(1, 3_2^2)$*, with a directrix line and a double directrix skew cubic met twice by each generating line;*

Tenth Species, $S(3_2^2)$*, with a double directrix skew cubic met twice by each generating line;*

viz. the subscript 2 would indicate that the skew cubic is a nodal (double) line on the scroll, the exponent 2 indicating that it is met twice by each generating line.

will be

$$(H, F, C, B, A-F, -G)(p, q, r)^2=0,$$

viz. this is a quadric equation in (p, q, r), with arbitrary coefficients.

61. Comparing with the result, Second Memoir, Nos. 47 to 50, we see that in the particular case where the coefficients (A, B, C, F, G, H) satisfy the relation $AF+BG+CH=0$, we have the *eighth species*, $S(1, 3^2)$, *with a directrix line and a directrix skew cubic met twice by each generating line*. We exclude this particular case, and in the tenth species consider the relation $AF+BG+CH=0$ as not satisfied, and therefore the scroll as not having a directrix line.

62. I consider how the scroll may be obtained as a scroll $S(m^2, n)$ generated by a line meeting a curve of the order m twice and a curve of the order n once. The first curve will be the skew cubic, that is $m=3$; the second curve may be any plane section of the scroll; such a section will be a quartic curve having three nodes, one at each intersection of its plane with the skew cubic. Conversely, if we have a skew cubic, and a plane quartic meeting the skew cubic in three points, each of them a node on the quartic, then the scroll generated by the lines which meet the skew cubic twice and the quartic once will be a quartic scroll. In fact (see First Memoir, No. 10, and Second Memoir, No. 5) the order of the scroll is given by the formula $S(m^2, n)=n([m]^2+M)-$reduction, $=16-$reduction. And in the present case the reduction arises (Second Memoir, No. 4) from the cones having their vertices at the intersections of the skew cubic and the quartic, and passing through the skew cubic. Each cone is of the order 2, and each intersection *quà* double point on the quartic gives a reduction $2\times$order of cone, $=4$; that is, the reduction arising from the three intersections is $=12$; or the order of the scroll is $16-12$, $=4$.

63. We may, instead of the section by a plane in general, consider the section by a plane through a generating line; the section is here made up of the generating line and of a plane cubic passing through each of the two points of intersection of the generating line with the skew cubic, and having a node at the remaining intersection of its plane with the skew cubic. Or we may consider the section by a plane through the two generating lines at any point of the skew cubic; the section is here made up of the two generating lines and of a conic passing through the second intersections of the two generating lines with the skew cubic; that is, meeting the skew cubic twice.

64. Conversely, consider a skew cubic, and a conic meeting it twice; the lines which meet the skew cubic twice, and also the conic, generate a quartic scroll; this appears by the before-mentioned formula $S(m^2, n)=n([m]^2+M)-$reduction; viz. we have $m=3$, $n=2$, and the order is $=8-$reduction; the reduction arises from the cones having their vertices at the intersections of the skew cubic and the conic. Each cone is of the order 2, and (*quà* simple point on the conic) each intersection gives a reduction$=$order of the cone; that is, the total reduction is $=4$, and the order of the scroll is $8-4$, $=4$ as above.

65. But a more elegant mode of generation of the scroll may be obtained by means of the skew cubic alone; viz. considering the system of lines which are in involution

with five given lines, or say simply the lines which belong to an involution*, I say that the locus of a line belonging to the involution, and meeting the skew cubic twice is the quartic scroll, *tenth species*, $S(3^2)$. In the particular case where the line (instead of belonging to a proper involution) meets a given line, the locus is a quartic scroll, *eighth species*, $S(1, 3^2)$.

66. The analysis is almost identical with that given (Second Memoir, Nos. 47 to 50) in regard to the scroll $S(1, 3^2)$. Considering a line defined by its "six coordinates" (a, b, c, f, g, h), the condition which expresses that the line shall belong to an involution is

$$(A, B, C, F, G, H)(a, b, c, f, g, h)=0,$$

where (A, B, C, F, G, H) are arbitrary coefficients; if they are the coordinates of a line, that is, if $AF+BG+CH=0$, then the condition expresses that the line (a, b, c, f, g, h), instead of belonging to a proper involution, meets the line (F, G, H, A, B, C).

We have to determine the locus of the line (a, b, c, f, g, h) the coordinates whereof satisfy the relation

$$(A, B, C, F, G, H)\}(a, b, c, f, g, h)=0,$$

and which besides meets the skew cubic $yw-z^2=0$, $yz-xw=0$, $xz-y^2=0$.

The equations of the skew cubic are satisfied by writing therein

$$x : y : z : w=1 : t : t^2 : t^3;$$

and hence taking θ, ϕ for the parameters of the points of intersection of the line (a, b, c, f, g, h) with the skew cubic, we have

$$1, \theta, \theta^2, \theta^3$$
$$1, \phi, \phi^2, \phi^3$$

as the coordinates of two points on the line in question; whence forming the expressions of the six coordinates of the line, and omitting the common factor $\phi-\theta$, these are

$$(a, b, c, f, g, h)=\theta\phi, \ -(\theta+\phi), \ 1, \ \theta^2+\theta\phi+\phi^2, \ \theta\phi(\theta+\phi), \ \theta^2\phi^2,$$

and hence the condition of involution gives between the parameters θ, ϕ the equation

$$(A, B, C, F, G, H)\}(\theta\phi, \ -\theta-\phi, \ 1, \ \theta^2+\theta\phi+\phi^2, \ \theta\phi(\theta+\phi), \ \theta^2\phi^2).$$

* The theory is explained in my memoir "On the Six Coordinates of a Line," Camb. Phil. Trans. t. xi. 1868. In explanation of the subsequent analytical investigations of the present memoir, it is convenient to remark that if on a given line we have the two points $(\alpha, \beta, \gamma, \delta)$ and $(\alpha', \beta', \gamma', \delta')$, and through the given line two planes $Ax+By+Cz+Dw=0$ and $A'x+B'y+C'z+D'w=0$; then we have

$$\beta\gamma'-\beta'\gamma : \gamma\alpha'-\gamma'\alpha : \alpha\beta'-\alpha'\beta : \alpha\delta'-\alpha'\delta : \beta\delta'-\beta'\delta : \gamma\delta'-\gamma'\delta$$
$$=AD'-A'D : BD'-B'D : CD'-C'D : BC'-B'C : CA'-C'A : AB'-A'B;$$

and denoting either of these sets of equal ratios by

$$a : b : c : f : g : h,$$

then (a, b, c, f, g, h) satisfy identically the relation $af+bg+ch=0$, and are said to be the six coordinates of the line.

Moreover the coordinates of any point on the line in question are given by

$$x : y : z : w = l+m : l\theta+m\varphi : l\theta^2+m\varphi^2 : l\theta^3+m\varphi^3;$$

and writing as above p, q, $r = yw-z^2$, $yz-xw$, $xz-y^2$, we thence find, omitting the common factor $(\theta-\varphi)^3$,

$$p : q : r = \theta\varphi : -(\theta+\varphi) : 1;$$

and eliminating $\theta\varphi$, $\theta+\varphi$, we at once obtain

$$(A, B, C, F, G, H)(pr, qr, r^2, q^2-pr, -pq, p^2) = 0,$$

or, what is the same thing,

$$(H, F, C, B, A-F, -G)(p, q, r)^2 = 0$$

as the equation of the scroll generated by the line in involution which meets the given skew cubic twice.

Reciprocal of the Quartic Scroll $S(3^2)$.

67. I propose to reciprocate in regard to the quadric surface $x^2+y^2+z^2+t^2=0$ the foregoing scroll

$$(H, F, C, B, A-F, -G)(p, q, r)^2 = 0.$$

If the coordinates (a, b, c, f, g, h) of a line satisfy the condition of involution

$$(A, B, C, F, G, H)(a, b, c, f, g, h) = 0,$$

then the coordinates (a, b, c, f, g, h) of the reciprocal curve will satisfy the condition of involution

$$(F, G, H, A, B, C)(a, b, c, f, g, h) = 0.$$

The reciprocal of the before-mentioned skew cubic $x : y : z : w = 1 : t : t^2 : t^3$ is the quartic torse having for its edge of regression the skew cubic $3XZ-Y^2=0$, $YZ-9XW=0$, $3YW-Z^2=0$; or, what is the same thing, the skew cubic $X : Y : Z : W = 1 : 3t : 3t^2 : t^3$; see my paper "On the Reciprocation of a Quartic Developable," Quart. Math. Journ. t. vii. (1866) pp. 87–92.

68. Hence the reciprocal of the quartic scroll is the scroll generated by a line (a, b, c, f, g, h) the coordinates of which satisfy the condition of involution

$$(F, G, H, A, B, C)(a, b, c, f, g, h) = 0,$$

and which is moreover the intersection of two osculating planes of the skew cubic $X : Y : Z : W = 1 : 3t : 3t^2 : t^3$. For the point the parameter whereof is t, the equation of the osculating plane is

$$\begin{vmatrix} X, & Y, & Z, & W \\ 1, & 3t, & 3t^2, & t^3 \\ & 1, & 2t, & t^2 \\ & & 1, & t \end{vmatrix} = 0,$$

or, what is the same thing, the equation is

$$(t^3, -t^2, t, -1)(X, Y, Z, W) = 0.$$

Hence for the line which is the intersection of the two osculating planes

$$(\theta^3, -\theta^2, \theta, -1)(X, Y, Z, W)=0,$$
$$(\varphi^3, -\varphi^2, \varphi, -1)(X, Y, Z, W)=0,$$

forming the expressions of the six coordinates, but omitting the common factor $\varphi-\theta$, these are

$$a, b, c, f, g, h=\theta^2+\theta\varphi+\varphi^2, -\theta-\varphi, 1, \theta\varphi, \theta\varphi(\theta+\varphi), \theta^2\varphi^2;$$

we have thus between the parameters θ, φ the relation

$$(F, G, H, A, B, C)(\theta^2+\theta\varphi+\varphi^2, -\theta-\varphi, 1, \theta\varphi, \theta\varphi(\theta+\varphi), \theta^2\varphi^2)=0;$$

and the equation of the scroll is obtained by eliminating θ, φ between this equation and the last-mentioned two equations satisfied by θ, φ respectively.

69. We see that θ, φ are two of the roots of the equation

$$(X, -Y, Z, -W)(u, 1)^3=0;$$

let ϱ be the third root, then we have

$$\theta+\varphi+\varrho = \frac{Y}{X},$$
$$\theta\varphi+\varrho(\theta+\varphi)=\frac{Z}{X},$$
$$\theta\varphi.\varrho = \frac{W}{X},$$

and thence

$$\theta+\varphi=\frac{1}{X}(Y-\varrho X), \quad \theta\varphi=\frac{1}{X}(Z-\varrho Y+\varrho^2 X)=\frac{1}{\varrho X}W,$$
$$(X, -Y, Z, -W)(\varrho, 1)^3=0.$$

Substituting for $\theta+\varphi$ and $\theta\varphi$ their values in terms of ϱ, we find

$$F\varrho\{Y^2-ZX-\varrho XY\}-G\varrho X(Y-\varrho X)+H\varrho X^2$$
$$+W\{(AX+BY+CZ)-\varrho(BX+CY)+\varrho^2 CX\}=0,$$

or, what is the same thing,

$$\varrho^2 X(GX-FY+CW)$$
$$-\varrho\{F(Y^2-ZX)-GXY+HX^2-BXW-CYW\}$$
$$+W(AX+BY+CZ)=0;$$

from which and the equation

$$(X, -Y, Z, -W)(\varrho, 1)^3=0,$$

we have to eliminate ϱ.

70. Writing for shortness

$$\begin{aligned}
(\ .,\ \ H, -G, A\mid X, Y, Z, W) &= \alpha,\\
(-H,\ .,\ \ F, B\mid\ \ ,, \ \ ) &= \beta,\\
(\ G, -F,\ .,\ C\mid\ \ ,, \ \ ) &= \gamma,\\
(-A, -B, -C,\ .\mid\ \ ,, \ \ ) &= \delta,
\end{aligned}$$

and therefore

$$\alpha X+\beta Y+\gamma Z+\delta W=0:$$

the two equations are

$$\begin{aligned}
\varrho^2 X\gamma+\varrho(-\gamma Y-\beta X)-\delta W&=0,\\
\varrho^2 X-\varrho Y+\varrho Z-W\ \ \ \ &=0.
\end{aligned}$$

Writing the first equation in the form

$$\gamma(\varrho^2 X-\varrho Y+Z)-\beta(\varrho X-Y)+\alpha X=0,$$

mutiplying by $-\varrho$, and reducing by the other equation,

$$\beta(\varrho^2 X-\varrho Y)-\varrho\alpha X-\gamma W=0,$$

or, as this may be written,

$$\beta(\varrho^2 X-\varrho Y+Z)-\alpha(\varrho X-Y)-\alpha Y-\beta Z-\gamma W=0.$$

From this and the preceding equation we deduce the values of $\varrho^2 X-\varrho Y+Z$ and $\varrho X-Y$; viz. writing for shortness

$$\beta\delta-\gamma^2,\ \beta\gamma-\alpha\delta,\ \alpha\gamma-\beta^2=p,\ q,\ r,$$

we find

$$\varrho^2 X-\varrho Y+Z : \varrho X-Y : 1=-rZ+qW : rY-pW : -r,$$

or, what is the same thing,

$$\begin{aligned}
\varrho^2 X-\varrho Y+Z&= \ Z-\tfrac{q}{r}W,\\
\varrho X-Y\ \ &=-Y+\tfrac{p}{r}W;
\end{aligned}$$

whence also

$$\begin{aligned}
\varrho^2 X-\varrho^2 Y+\varrho Z-W&=\ 0,\\
\varrho^2 X-\varrho Y\ \ \ \ &=-\tfrac{q}{r}W,\\
\varrho X\ \ \ \ &=\ \tfrac{p}{r}W,
\end{aligned}$$

and thence

$$\begin{aligned}
\varrho\left(Z-\tfrac{q}{r}W\right)&=\ W,\\
\varrho\left(Y-\tfrac{p}{r}W\right)&=\tfrac{q}{r}W,\\
\varrho\ X\ \ \ \ &=\tfrac{p}{r}W,
\end{aligned}$$

and we have therefore

$$\left(\frac{W}{\varrho}=\right)\quad Z-\frac{q}{r}W=\frac{r}{q}\left(Y-\frac{p}{r}W\right)=\frac{r}{p}X,$$

or omitting the first equation, we have (independent of ϱ) a system which is clear must be equivalent to a single equation.

71. I take any one of these equations, for instance the equation

$$Z-\frac{q}{r}W=\frac{r}{q}\left(Y-\frac{p}{r}W\right),$$

or, what is the same thing,

$$qrZ-r^2Y+(pr-q^2)W=0,$$

and I proceed to reduce it so as to obtain the result in a symmetrical form. For this purpose I observe that from the values of $\alpha, \beta, \gamma, \delta$, if only $AF+BG+CH\neq 0$, we have

$$\begin{aligned} X:Y:Z:W= &(\ .\ , -C,\ B, -F)(\alpha, \beta, \gamma, \delta)\\ :&(\ C,\ .\ , -A, -G)(\ \ ,,\ \ )\\ :&(-B,\ A,\ .\ , -H)(\ \ ,,\ \ )\\ :&(\ F,\ G,\ H,\ .\ )(\ \ ,,\ \ ); \end{aligned}$$

and substituting these values, the equation in question becomes

$$\begin{aligned} &qr(-B\alpha+A\beta-H\delta)\\ -&r^2(\ C\alpha-A\gamma-G\delta)\\ +&(pr-q^2)(\ F\alpha+G\beta+H\gamma)=0. \end{aligned}$$

This becomes

$$\begin{array}{lll} Ar(q\beta+r\gamma) & =Ar(-p\alpha) & =0\\ -Bqr\alpha & -Bqr\alpha & \\ -Cr^2\alpha & -Cr^2\alpha & \\ +F(pr-q^2)\alpha & +F(pr-q^2)\alpha & \\ +G\{r^2\delta+(pr-q^2)\beta & +Gpq\alpha & \\ +H\{-qr\delta+(pr-q^2)\gamma\} & -Hp^2\alpha, & \end{array}$$

viz. the whole equation divides by α; and, omitting this factor, the equation is

$$Apr+Bqr+Cr^2+F(q^2-pr)-Gpq+Hp^2=0,$$

or, what is the same thing, it is

$$(H, F, C, B, A-F, -G)(p, q, r)^2=0,$$

where I recall that we have

$$p, q, r=\beta\delta-\gamma^2,\ \beta\gamma-\alpha\delta,\ \alpha\gamma-\beta^2,$$

$\alpha, \beta, \gamma, \delta$ being linear functions of the current coordinates (X, Y, Z, W), viz. we have

$$\begin{aligned}\alpha &= (\ .\ ,\ \ H, -G, A\,\}(X, Y, Z, W),\\ \beta &= (-H,\ .\ ,\ \ F, B\,\}(\quad ,,\quad),\\ \gamma &= (\ G, -F,\ .\ , C\,\}(\quad ,,\quad),\\ \delta &= (-A, -B, -C,\ .\ )\}(\quad ,,\quad).\end{aligned}$$

72. It thus appears that when $AF+BG+CH$ is not $=0$, the reciprocal of the scroll

$$(H, F, C, B, A-F, -G)(p, q, r)^3=0$$

has an equation of the very same form,

$$(H, F, C, B, A-F, -G)(p, q, r)^3=0; \quad . \ . \ . \ . \quad \text{(Rec. I.)}$$

so that in fact the scroll, *tenth species*, $S(3^3)$, defined as the scroll generated by a line in involution which passes through two points of a skew cubic, may be reciprocally defined as the scroll generated by a line in involution which lies in two osculating planes of a skew cubic.

73. If for $(\alpha, \beta, \gamma, \delta)$ we substitute their values in terms of (X, Y, Z, W), the foregoing equation of the reciprocal scroll is obtained as an equation of the fourth order in the coordinates (X, Y, Z, W), and (in the first instance) of the fifth degree in the coefficients (A, B, C, F, G, H). It is a remarkable circumstance that the whole equation contains the constant factor $AF+BG+CH$, so that throwing this out, the reduced equation will be only of the third degree in the coefficients.

74. The transformation is a very troublesome one, but I will indicate the steps by which I succeeded in accomplishing it. Each of the functions (p, q, r) is a quadric function of (X, Y, Z, W), say,

$$\begin{aligned}p &= (a, b, c, d, f, g, h, l, m, n)(X, Y, Z, W)^2,\\ q &= (a', \ldots \quad)(\quad ,, \quad)^2,\\ r &= (a'', \ldots \quad)(\quad ,, \quad)^2;\end{aligned}$$

we have to form the value of

$$(H, F, C, B, A-F, -G)(p, q, r)^3,$$

viz. representing this for shortness by

$$(H, F, C, B, A-F, -G)\begin{pmatrix} a, b, c, d, f, g, h, l, m, n \\ a', \ldots \\ a'', \ldots \end{pmatrix}^2,$$

the coefficient of X^4 is

$$(H, F, C, B, A-F, -G)(a^2, a'^2, a''^2, a'a'', a''a, aa''),$$

that of X^3Y is

$$(H, F, C, B, A-F, -G)(2af, 2a'f', 2a''f'', a'f''+a''f', a''f+af'', af'+a'f),$$

and so on, the successive terms a^3, a'^3, &c., $2af$, $2af'$, &c. being derived by an obvious law from the first terms a^3, $2af$, &c.; and these first terms are merely the coefficients of the terms X^4, X^3, Y, &c. in the development of

$$p^3, = \{(a, b, c, d, f, g, h, l, m, n)(X, Y, Z, W)^3\}^3;$$

viz. this is

X^4 X^3Y X^3Z X^3W X^2Y^2 X^2YZ X^2YW X^2Z^2, X^2ZW, X^2W^2, XY^3, XY^2Z, XY^2W, XYZ^2

$$\begin{array}{cccccccccccccc} a^3 & 2af & 2ag & 2al & 2ab & 2af & 2am & 2ac & 2an & 2ad & 2bh & 2bg & 2bl & 2ch \\ & & & & +h^2 & +2gh & +2hl & +g^2 & +2gl & +l^2 & & +2fh & +2hm & +2fg \end{array}$$

$XYZW$, XYW^2, XZ^2, XZ^2W, XZW^2, XW^3, Y^4, Y^3Z, Y^3W, Y^2Z^2, Y^2ZW, Y^2W^2, YZ^3,

$$\begin{array}{cccccccccccccc} 2lf & 2dh & 2cg & 2cl & 2dg & 2dl, & b^2, & 2bf, & 2bm, & 2be, & 2bn, & 2bd, & 2cf \\ +2mg & +2lm & & +2gn & +2ln & & & & & +f^2 & +2fm & +m^2 & \\ +2nh & & & & & & & & & & & & \end{array}$$

YZ^2W, YZW^2, YW^3, Z^4, Z^3W, Z^2W^2, ZW^3, W^4

$$\begin{array}{cccccccc} 2cm & +2df & 2dm, & c^2, & 2cn, & 2cd, & 2dn, & d^2 \\ +2fn & +2mn & & & & +n^2 & & \end{array};$$

and the values of the coefficients $a, b, \ldots$ which enter into the formulæ are given by means of the following values of p, q, r; viz. these are

$$\begin{array}{cccccccccccc} & X^3 & Y^3 & Z^3 & W^3 & YZ & ZX & XY & XW & YW & ZW & \\ p = (& AH, & -F^3, & -CF, & -C^3, & -BF, & -AF, & BH, & -AB, & -B^3, & -BC &)(X, Y, Z, W)^3, \\ & -G^3 & & & & & +CH & +2FG & -2CG & +2CF & & \end{array}$$

$$\begin{array}{cccccccccccc} q = (& -GH, & BH, & -CG, & BC, & -BG, & -AG, & AH, & A^2, & AB, & AC &)^3, \\ & & & & & +CH & +FG & +FH & +BG & -BF & +CF & \\ & & & & & -F^2 & & & -CH & & & \end{array}$$

$$\begin{array}{cccccccccccc} r = (& -H^3, & -FH, & -F^2, & AC, & FG, & 2FH, & GH, & 2AG, & -AF, & -2BF &)^3. \\ & & & -B^2 & & -G^2 & & +BH & +CH & -CG & & \end{array}$$

75. As an instance of the calculation of a single term, the coefficient of X^4 is

$$(H, F, C, B, A-F, -G)(AH-G^2, -GH, -H^2)^2;$$

viz. this is

$$\begin{array}{lcl} H(AH-G^2)^2 & = & A^2H^3-2AG^2H^2+G^4H^2 \\ +FH^2G^2 & = & FG^2H^2 \\ +CH^4 & = & CH^4 \\ +BGH^3 & = & BGH^3 \\ +(A-F)(-AH^3+G^2H^2) & = & -A^2H^3+AG^2H^2 \\ & & +AFH^3-FG^2H^2 \\ -G(-AGH^2+G^2H) & = & AG^2H^2-G^4H^2 \\ \text{whole term is} & = & (AF+BG+CH)H^2, \end{array}$$

viz. there is the factor $AF+BG+CH$ as mentioned above.

76. Throwing out the factor in question, $AF+BG+CH$, the equation of the reciprocal scroll is found to be

$$
\begin{aligned}
0 = {} & X^4.H^3 \quad . \quad . \quad .

YW^3 is $AC^3-3FC^2+B^2C$, $=AC^2+B^2C-3C^2F$. Or, again, the coefficient of Y^2Z being $BFH-F^2G$, that of YZ^2 is $-GFC+F^2B$, $=BF^2-CFG$.

77. But the equation may be written in the much more simple form

$$\begin{aligned} &X(-\alpha^2\delta+3\alpha\beta\gamma-2\beta^3) \quad \text{(Rec. III.)}\\ &+Y(-\alpha\beta\delta+2\alpha\gamma^2-\beta^2\gamma)\\ &+Z(\alpha\gamma\delta-2\beta^2\delta+\beta\gamma^2)\\ &+W(\alpha\delta^2-3\beta\gamma\delta+2\gamma^3)=0, \end{aligned}$$

or, what is the same thing,

$$-\tfrac{1}{27}(3X\partial_z-Y\partial_y+Z\partial_\beta-3W\partial_\alpha)(\alpha^2\delta^2-6\alpha\beta\gamma\delta+4\alpha\gamma^3+4\beta^3\delta-3\beta^2\gamma^2)=0, \quad \text{(Rec. III.)}$$

as may be verified by actual substitution of the values of the coordinates.

78. By what precedes, substituting for p, q, r their values in terms of α, β, γ, δ, it appears that we have the remarkable identity

$$\begin{aligned} &(H, F, C, B, A-F, -G)(\beta\delta-\gamma^2, \beta\gamma-\alpha\delta, \alpha\gamma-\beta^2)^2\\ &=(AF+BG+CH)\times\\ &\left\{\begin{array}{l} X(-\alpha^2\delta+3\alpha\beta\gamma-2\beta^3)\\ +Y(-\alpha\beta\delta+2\alpha\gamma^2-\beta^2\gamma)\\ +Z(\alpha\gamma\delta-2\beta^2\delta+\beta\gamma^2)\\ +W(\alpha\delta^2-3\beta\gamma\delta+2\gamma^3) \end{array}\right\} \end{aligned}$$

79. In the case above considered of the tenth species, $S(3^2)$, for which $AF+BG+CH=0$, the three forms of the reciprocal equation are of course absolutely equivalent to each other. The first form has the advantage of putting in evidence the fact that the reciprocal scroll is also of the tenth species; the other two forms do not, at least obviously, put in evidence any special property of the reciprocal scroll.

Reciprocals of Eighth Species, $S(1, 3^2)$, *and Ninth Species*, $S(1^3)$.

80. If $AF+BG+CH=0$, then the equation

$$(H, F, C, B, A-F, -G)(p, q, r)^2=0$$

is a scroll of the eighth species, $S(1, 3^2)$. The first form of the reciprocal equation becomes identically $0=0$, on account of the evanescent factor $AF+BG+CH$, but the second and third forms continue to subsist, and either of them may be taken as the equation of the reciprocal scroll. Taking the third form, and calling to mind the significations of $(\alpha, \beta, \gamma, \delta)$, viz.

$$\begin{aligned} \alpha&=(\ .,\ H, -G, A)(X, Y, Z, W),\\ \beta&=(-H,\ .,\ F, B)(\ \ ,\ \ ,\ \ ,\ \ ),\\ \gamma&=(\ G, -F,\ .,\ C)(\ \ ,\ \ ,\ \ ,\ \ ),\\ \delta&=(-A, -B, -C,\ .)(\ \ ,\ \ ,\ \ ,\ \ ), \end{aligned}$$

it is to be observed that $\alpha=0, \beta=0, \gamma=0, \delta=0$ are the equations of four planes passing through a common line, viz. the line whose coordinates are (A, B, C, F, G, H), and the equation thus puts in evidence that this line is a triple line on the reciprocal scroll; that is, the reciprocal scroll is a scroll of the ninth species, $S(1^3)$. Or stating the theorem more completely: For the scroll, *eighth species*, $S(1, 3^3)$,

$$(H, F, C, B, A-F, -G)(p, q, r)^3=0,$$

generated by a line meeting the line (F, G, H, A, B, C), and the skew cubic $p=0$, $q=0$, $r=0$ twice, the reciprocal scroll is of the *ninth species*, $S(1^3)$,

$$\begin{aligned} &X(-\alpha^2\delta+3\alpha\beta\gamma-2\beta^3)\\ +&Y(-\alpha\beta\delta+2\alpha\gamma^2-\beta^2\gamma)\\ +&Z(\alpha\gamma\delta-2\beta^2\delta+\beta\gamma^2)\\ +&W(\alpha\delta^2-3\beta\gamma\delta+2\gamma^3), \end{aligned}$$

having for its triple line the reciprocal line (A, B, C, F, G, H).

81. It should of course be possible, starting from the equation

$$(*)(X, Y)^4+Z(*)(X, Y)^3+W(*)(X, Y)^3=0$$

of a scroll $S(1^3)$, to obtain the equation of the reciprocal scroll $S(1, 3^3)$. But I content myself with a very particular case. I consider the equation

$$Y^2Z^2-Y^3W-Z^3X=0,$$

which belongs to a scroll $S(1^3)$ having the line $Y=0$, $Z=0$ for its triple line. To find the equation of the reciprocal scroll, write

$$\begin{aligned} -Z^3+\lambda x&=0,\\ 2YZ^2-3Y^2W+\lambda y&=0,\\ 2Y^2Z-3Z^2X+\lambda z&=0,\\ -Y^3+\lambda w&, \end{aligned}$$

we find without difficulty, reducing by means of the equation of the scroll,

$$\begin{aligned} \lambda^3(yw-z^2)&=-3Z^2\{Y^4+3XZ(XZ-Y^2)\},\\ \lambda^3(xw-yz)&=\ \ 3Y^2Z^2\{YZ-3WX\},\\ \lambda^3(xz-y^2)&=-3Y^2\{Z^4+3YW(YW-Z^2)\}. \end{aligned}$$

Hence writing for a moment

$$\Omega=\{Y^4+3XZ(XZ-Y^2)\}\{Z^4+3YW(YW-Z^2)\}-Y^2Z^2.(YZ-3WX)^2,$$

we have

$$\begin{aligned} \Omega=&Y^4Z^4+3Y^2W(YW-Z^2)+3Z^2X(XZ-Y^2)+9XYZW(Y^2Z^2-Y^3W-Z^3X+XYZW)\\ &-Y^4Z^4+6Y^2Z^2PW-9Y^2Z^2X^2W^2, \end{aligned}$$

that is,

$$\begin{aligned}\tfrac{1}{3}\Omega = {} & Y^3W(YW-Z^2)+X^2X(XZ-Y^2)+2Y^2Z^2XW\\ = {} & -Y^2Z^2(XZ-Y^2)(YW-Z^2)-Y^2Z^2(YW-Z^2)(XZ-Y^2)+2Y^2Z^2XW\\ = {} & -2Y^2Z^2\{(XZ-Y^2)(YW-Z^2)-XYZW\}\\ = {} & -2Y^2Z^2\{Y^2Z^2-Y^2W-Z^2X\}\\ = {} & 0, \text{ by the equation of the scroll;}\end{aligned}$$

and we thus see that the equation of the reciprocal scroll is

$$(yw-z^2)(xz-y^2)-(yz-xw)^2=0,$$

or say

$$q^2-pr=0,$$

viz. it is a scroll S(1, 3^2) generated by a line meeting the line $x=0$, $w=0$, and the cubic curve $p=0$, $q=0$, $r=0$ twice. The equation is obviously included in the general equation

$$(H, F, C, B, A-F, -G)(p, q, r)^2=0,$$

where $AF+BG+CH=0$; viz. writing $A=B=C=G=H=0$, this becomes $F(q^2-pr)=0$

82. Returning to the general case of the scroll, *eighth species*, S(1, 3^2), it is proper to show geometrically how it is that the reciprocal is a scroll, *ninth species*, S(1^3). Consider in the scroll S(1, 3^2) any plane through the directrix line; this contains three generating lines of the scroll, viz. these are the sides of the triangle formed by the three points of intersection of the plane with the skew cubic: hence in the reciprocal figure we have a directrix line such that at each point of it there are three generating lines; that is, we have a scroll S(1^3) with a triple directrix line. Conversely, starting with the scroll S(1^3), each plane through the triple directrix line meets the scroll in this line three times, and in a single generating line; whence there is in the reciprocal scroll a simple directrix line; but in order to show that it is a scroll, S(1, 3^2), we have yet to show that there is, as a nodal directrix, a skew cubic met twice by each generating line; this implies that, reciprocally, in the scroll S(1^3) each generating line is the intersection of two osculating planes of a skew cubic (tangent planes of a quartic torse), each such plane containing two generating lines of the scroll—a geometrical property which is far from obvious; and similarly in the scroll, ninth species, S(3^2), where the reciprocal scroll is of the same form, the property that each generating line is a line joining two points of a skew cubic leads to the property that each line is also the intersection of two osculating planes of a skew cubic (or, what is the same thing, two tangent planes of a quartic torse).

Addition, May 18, 1869.

Since the foregoing Memoir was written I received from Professor CREMONA a letter dated Milan, November 20, 1868, in which (besides the ninth and tenth species considered above) he refers to two other species of quartic scrolls. He remarks that there is

a bitangent torse which should in the classification be considered along with the nodal curve; and he enumerates in all 12 species as follows:—

Deficiency.	No. of species.	Nodal curve.	Bitangent torse.	Corresponding to my species
$p=0$	1	Γ_3	Σ_3	10
	2	H_2+R_1	K_2+R_1	7
	3	R_1^3	K_2+R_1	— (say, 12)
	4	H_2+R_1	R_1^3	— (say, 11)
	5	$R_1+R_1'+S_1$	$R_1+R_1'+S_1$	2
	6	$R_1^2+S_1$	$R_1^2+S_1$	5
	7	Γ_3	R_1^3	8
	8	R_1^3	Σ_3	9
	9	R_1^3	R_1^3	3
	10	R_1^3	R_1^3	6
$p=1$	11	$R_1+R_1^2$	$R_1+R_1^2$	1
	12	R_1^3	R_1^3	4

where Γ_3 denotes a skew cubic, Σ_3 a torse of the 3rd class (or quartic torse), H_2 a conic, K_2 a quadric cone, R_1, R_1', S_1 different right lines, R_1^2, R_1^3 a line counted twice or three times &c. I have in the last column added the references to my species 9 and 10; Professor CREMONA notices (what I knew, but did not recollect) that the species 10 had been considered by M. CHASLES, Comptes Rendus, June 3, 1861.

I have not yet examined the two new species mentioned in this enumeration; viz. these are (CREMONA 3), say *twelfth species*, a scroll having a triple line, but a bitangent torse made up of a quadric cone and a line; and (CREMONA 4), say *eleventh species*, a scroll having a nodal conic and line, but for its bitangent torse a triple line: the two species are, it is clear, reciprocal to each other; although properly treated as distinct, species 11 may be considered as a subform of 8, and species 12 as a subform of 9.

III. *Description of the Great Melbourne Telescope.*

By T. R. Robinson, *D.D., F.R.S., &c., and* Thomas Grubb, *Esq., F.R.S., &c.*

Received June 11,—Read June 11, 1868.

In 1862 the authorities of the Colony of Victoria formed the design of adding to the Observatory, which they were then establishing at Melbourne, a powerful telescope which should be applied in reviewing the Nebulæ of the Southern Hemisphere. They applied through the Duke of Newcastle, then Foreign Secretary, to the President and Council of the Royal Society, for encouragement in this undertaking, and advice as to the best means of carrying it into execution. The subject was not new to that Body. They had been, conjointly with the British Association, engaged, though ineffectually, for several years in trying to induce the British Government to adopt a similar plan*. With this object they had appointed a Committee, including several of the brightest names of British Science†, to examine the subject thoroughly and recommend the plan which they considered most desirable to be adopted. Subsequent to their report, Mr. Lassell had actually constructed a 4-feet Newtonian, which he was using most successfully at Malta, M. L. Foucault, whose recent death all lament, had invented the silvered glass speculum which bears his name, and Mr. Warren De La Rue had changed Celestial Photography from a toy into a potent instrument of astronomical research. These new facts required new discussion, which was carefully made, but resulted in adopting the former report with little change.

In consequence the legislature of Victoria, acting on the recommendation of our President and Council, voted in 1865 the requisite sum; and Mr. Grubb undertook the construction of this gigantic equatorial, under the direction of a Committee consisting of the late Lord Rosse, Mr. Warren De La Rue, and myself. After Lord Rosse's death, his son, the present Earl, was nominated to succeed him by the President. The instrument has been very successfully completed; and we hope that a detailed account of its construction will be acceptable, both from the interest which belongs to the accomplishment of a great undertaking, and because, though the late Lord Rosse and Mr. Lassell have published their methods of making large specula, the subject is by no means exhausted, and anything which lessens the difficulties which still beset it, and makes the

* For some account of this see Reports of British Association, 1850, p. xvii; 1851, p. xxiv; 1853, p. xxv; and the printed correspondence of the Committee.

† It consisted of Lord Rosse and General Sabine, Presidents of the Royal Society and British Association, Lord Wrottesley, Sir D. Brewster, Sir J. Herschel, Sir J. Lubbock, the Dean of Ely (Peacock), Messrs. Adams, Airy, E. J. Cooper, Lassell, Nasmyth, Phillips, and myself. It is sad to think how many of these we have lost.

use of large reflectors more attainable, cannot but tend to the progress of some of the most interesting branches of astronomy.

Before entering on this description, it may be well to make some remarks on the most important recommendations of the Committee, which may be found in the printed "Melbourne Telescope Correspondence."

I. The Committee recommend a 4-feet reflector. No doubt a 6-feet would be preferable; but it would be five times as expensive, and the execution of it thrice as difficult: on the other hand, two of them were familiar with what Lord ROSSE's 3-feet performed, and were thus able to say with confidence that an instrument of nearly twice its power would be amply sufficient for the work proposed. Two of the Committee wished for a 5-feet, but some fears were entertained that it might be difficult to mount it on a thoroughly effective equatorial, and they decided on the safer course: it has, however, turned out that this caution was not necessary, for the actual mounting is strong enough to carry a 5-feet, should it ever be required.

II. They preferred the reflector to the achromatic; and with good reason. It is not probable that an achromatic can ever be made which shall have as much light as a 4-feet reflector; and if it could, the cost of it would be tremendous. The late M. MERZ, when consulted by one of the Committee about a 30-inch achromatic, expressed much doubt as to the possibility of making one; but added that if it were practicable, the cost of the object-glass alone would be from £8500 to £9000, and that the equatorial complete would not be less than £20,000. What would the equivalent of a 4-feet cost? Erroneous, I may even say absurd opinions are often expressed as to the relative power of these two sorts of telescopes. Thus FRAUNHOFER says specula reflect "an exceedingly small quantity of light." Even the elder STRUVE seems to think that the Dorpat achromatic, 9·58 inches diameter, "may rank with the most celebrated of all reflectors, namely, HERSCHEL's." He cannot mean the "most celebrated one," that known as the 40-feet; but if we even suppose him to speak of the 18-inch front view, the statement is preposterous*. A speculum reflects 0·64 of the incident light after being many years in use, and even the Newtonian with its double reflection gives 0·401, allowing for that intercepted by the small speculum. And the achromatic does not by any means transmit all the light that falls on it. Light is lost in it from two causes: first, from the reflection at the four surfaces of the lenses. This can be calculated *accurately* from FRESNEL's formula, which gives that for two surfaces of crown ($\mu=1{\cdot}521$) the transmitted light $\varrho^2=0{\cdot}9164$; for two of flint ($\mu=1{\cdot}662$) $\varrho'^2=0{\cdot}8842$; and for the four $\varrho^2.\varrho'^2=0{\cdot}8122$†. Hence it is easily inferred that even if its glasses were perfectly transparent, the aperture of an achromatic would be to that of its equivalent Newtonian as 1 : 1·42; or in other words, one equivalent to the 4-feet Newtonian cannot be less than 33·73 inches. But it must be much more; for, secondly, all glass absorbs a portion of the light which passes

* See on this HERSCHEL and SOUTH, Quarterly Journal of Science, vol. xx. pp. 286, 293.

† The loss must be more than this; for except at the very centre of the object-glass the incidence is a little oblique.

through it. The law of this transmission is

$$I = e^{-nt},$$

I being the intensity of the emergent light, t the thickness of the medium, and n a constant depending on the nature of the medium and the colour of the light. In optical glass it may be supposed the same for each ray. The form of the equation shows that the intensity diminishes very rapidly as the thickness increases; and since this last is as the diameter of the object-glass, we shall soon come to a size which will not have more light than a Newtonian of equal aperture. This size could be easily determined if we knew n; but I have found no information on the subject except what I have got from some measures made by the late Lord Rosse and Mr. Grubb, several years ago, for a different purpose. Lord Rosse's specimen was a reflecting prism of English crown; Mr. Grubb's were two London plate, one French plate; and the fourth he described as a reflecting prism, but with no note of the quality of its glass. I assume it to have been crown, as flint of such size was then rare in England. The intensity of the transmitted light was measured by Bunsen's photometer, and the loss by reflection is computed, assuming μ to be that given above for crown. The expression of n is

$$n = \frac{\log \varrho^2 - \log I}{t \times \text{modulus}}.$$

The data are—

1. Lord Rosse	$I = 0{\cdot}746$;	$t = 1{\cdot}125$;	hence $n = 0{\cdot}1829$
2. French	$0{\cdot}805$;	$0{\cdot}750$;	$0{\cdot}1728$
3. London	$0{\cdot}860$;	$0{\cdot}300$;	$0{\cdot}2142$
4. London	$0{\cdot}770$;	$0{\cdot}600$;	$0{\cdot}1446$
5. Reflecting prism	$0{\cdot}810$;	$2{\cdot}000$;	$0{\cdot}0617$
			$0{\cdot}1552$

This last is undoubtedly too small; and possibly some false light was present in the experiment. I, however, retain it not to overstate my case. The error in I should not exceed $\frac{1}{60}$ of the whole: ϱ^2 is in some degree doubtful, from μ not having been specially determined for each specimen. All but the last, however, are known to be very near what I have assumed, and the error of ϱ^2 is but $\frac{1}{4}$ of that of the assumed μ. We will take $0{\cdot}15$ for n. As to the t of an achromatic, it will be sufficient in this discussion to take its mean thickness. For this Mr. Grubb measured for me the mean thickness of a fine 12-inch object-glass of 18 feet focus, which he made some years ago. It is $1{\cdot}75$ inch, whence that of a similar lens of aperture A is $\frac{A \times 1{\cdot}75}{12}$, and the expression of its intensity

$$I = \log^{-1}(9{\cdot}90964) \times e^{-A \times \log -1(9{\cdot}33966)}.$$

If in this we put $A = 33{\cdot}73$ inches, we find that I will be only $0{\cdot}3883$, and that such an object-glass, instead of being equiluminous with the 4-feet reflector, will be equal to one of $37\frac{1}{4}$ inches. For an achromatic of 48 inches, if such a one could be made, I would be

only 0·2842, and therefore even this would not equal the reflector. These conclusions may seem startling to many; but they can only be avoided by supposing an error in n very much greater than can be admitted. It is, however, my purpose, in cooperation with Mr. GRUBB, to institute a series of experiments on the subject which will set the question at rest*.

If we seek any comparison of the light of the two kinds of telescope based on actual measures, little is to be found but vague conjectures. Thus:—1st. M. OTTO STRUVE *thinks* that Mr. LASSELL'S 2-feet Newtonian "is nearly equal in light to the Pulkova 16-inch object-glass:" this seems to refer to stars only. 2nd. I have examined faint objects with Mr. COOPER'S achromatic of 13·6, and with Sir J. HERSCHEL'S front view of 18, and have no doubt of the superiority of the latter: the ratio here is 1·32. 3rd. 72 circular inches of the central part of the Armagh Cassegrain showed the fifth star of Orion's trapezium as well as a good Cauchoix object-glass of 6·5 inch; this gives 1·31 for the ratio. The only two definite measures which I know are; 4th. Mr. POTTER found that a 4-inch object-glass by DOLLOND transmitted only 0·66 of the incident light†. Lastly, AMICI ascertained that an object-glass of 2·5 aperture was as bright as a Newtonian of $\frac{4}{3}$ the aperture; this would give 36 inches for the equivalent of the 4-feet reflector; but what has been already stated will show that it would be far short of this‡.

As to defining power, it is certain that specula can be finished to possess this in the highest degree, while their great angle of aperture with the absence of the secondary spectrum and of diffraction-rings adds greatly to their distinctness. The champions of the achromatic rely entirely on its power of showing small stars, which they assert are not visible in the reflector. This, however, is not proved; for it is no proof that because a star does not appear in the drawing of a nebula it was therefore invisible, the fact being that the observer's attention was directed to a different object. The effect of this is shown, among other examples, by the fifth star of the trapezium, and the dark ring of Saturn, which were long overlooked in telescopes which showed them easily when they were looked for. Besides, minute points of intrinsic brightness are brought out by high powers, but are quite invisible with the low ones which are used for nebulæ; but the case is very different with faint objects of sensible diameter. That any comparison may tell the truth, it should be made with the same power, at the same time (for many small stars are believed to be variable), and by the same eye. The only real advantage of the achromatic is that, from its smaller aperture, it is less disturbed by the air's unsteadiness. But long before it reached a size equivalent to the 4-feet this difference would disappear, and a part of it is removed by the arrangement described in the next section.

* The results of these are given in an Appendix.

† The focal length is not mentioned: I found that the old object-glass of the Armagh circle, 3·8 and 6·2-inch focus, transmitted only 0·60. It is a very good one, but the crown is rather strongly greenish. Many of the glasses of the elder DOLLOND have this tint.

‡ A 3-feet object-glass would have 54 feet focal length and would weigh about 5 cwt. As it could be supported only at the circumference, it would be difficult to prevent flexure in it.

III. The Committee recommended that the tube of the telescope should be of metal lattice-work, with a view to lessen the disturbing influence of currents in its interior. The speculum after sunset is in general warmer than the air which is in contact with it, and therefore heats it. It of course rises, and in an ordinary tube escapes along its upper side, while a cold stream descends along the lower side to replace the ascending one. The two form eddies at their common surface; and the result is a medium of irregular density very unfavourable to good vision. As the night advances the external air cools faster than the speculum, and the evil increases so as in extreme cases to destroy all sharp definition. In the proposed construction the warm air escapes through the openings almost as soon as it leaves the speculum, and the counter-current enters similarly, so that the disturbance is very much diminished. This plan was proposed by Sir J. HERSCHEL in the Preface to his Cape Observations, but was neglected till revived in 1852. Lord ROSSE then applied it to his 3-feet, and Mr. LASSELL and Mr. DE LA RUE to their telescopes—in every case with marked advantage. It is also strong, light, gives very little hold to the wind, and from the arrangement of its framing is unfavourable to the propagation of tremors.

IV. It was decided that the great speculum should be of metal, not silvered glass. The Committee were unwilling to risk the success of the noble work entrusted to them by venturing on an experiment whose success on so large a scale was very uncertain. All the telescopes which have been made on this latter plan are, they believe, with one exception, of which little is known, not larger than 12 or 15 inches. When it is a question of 48 inches, several difficulties present themselves which at present cannot be solved. Such a speculum must be of considerable thickness to keep its shape; it would be no easy matter to manufacture so large a block of glass, which must be homogeneous in structure and well annealed; the latter to prevent its breaking in working; the first because otherwise it will expand unequally and change its figure. Nor is it known whether a silver film of uniform thickness (which according to FOUCAULT is essential) can be deposited over so large a surface. And there are two still more decided objections. Glass, though rather less than half the specific gravity of speculum metal, somehow seems more liable to abnormal flexure. One of these silvered specula, apparently of uniform thickness and consistence, has been found to give a good image with one diameter vertical and not with another; and they all require the utmost precaution in supporting them. This doubt has recently been confirmed; for one of the small specula of the Melbourne Telescope, which will be described hereafter, is glass 8 inches in diameter; yet even though so small it gave bad images while it was mounted as the metal ones were which acted perfectly, until its edge pressure was uniformly distributed. Secondly, there is not the great increase of light which was expected from the high reflective power of polished silver. By JAMIN'S experiments, solid silver reflects at perpendicular incidence 0·93 of red light and 0·87 of indigo, while speculum metal gives 0·69 and 0·60. But the silver film which is deposited on the glass is much inferior. Lord ROSSE, who expected considerable gain from using it as the small speculum of his

6-feet, found that it only gave 0·67. Part of this deficiency may arise from its molecular condition, but more arises from its being partly transparent to the more refrangible rays of the spectrum, which it transmits so freely that FOUCAULT proposed and used this silvered glass as a shade in solar observations; for this reason also the images will be tinged with red. The silver film would tarnish faster than the speculum metal; and though on a small scale it is easily renewed, the manipulation of a speculum of such size, and probably of 5 cwt., would present considerable difficulty. Nor is it impossible that the film might break up under considerable changes of temperature, such as occur at Melbourne, or be spotted by rain or dew.

V. The last matter which requires explanation is the Committee's decision that the telescope should be of the Cassegrain construction. This was not made without considerable discussion; for very few have ever been made on this plan, and only two of the Committee had any practical acquaintance with them. Lord ROSSE made one of 18 inches aperture; but the specula were not properly proportioned, and he used it but little. I had a good deal of experience with a 15-inch one, and was perfectly satisfied with its performance on such stars as ε Arietes, η Coronæ, and ζ Herculis when closest; but still the step from 15 to 48 inches was an adventurous one. The hostility of NEWTON to this telescope has probably created a prejudice against it; and it *has* objectionable points, which, however, are, I think, more than compensated by certain advantages which belong to it. Its greatest defect is the difficulty of obtaining a low magnifying-power. As the image formed by the great speculum is magnified by the small one, from five to six times, the eyepiece must be as much weaker than in the Newtonian; and as the lowest power must be such, to obtain the whole effect, that the eye can take in the whole pencil, which according to Sir W. HERSCHEL may not exceed two-tenths of an inch diameter, we come to rather formidable dimensions for it. With a 4-feet speculum this lowest power $=240$, and the Huyghenian eyepiece which gives it is nearly 9 inches diameter and 12 long. The glass for its lenses, though it need not be so faultless as for an object-glass, must be of good quality, and therefore costly. This is not of so much importance as the thickness of the glass in the lenses, which, as I have already shown, diminishes the light, and this thickness cannot be diminished without a corresponding decrease of the field of view*. In a telescope of this size it might perhaps be desirable to employ a triple eyepiece, possessing the Huyghenian properties of achromatism and equal flexure of the pencils; such a one, in which the distance of the second and third lenses $=\frac{3}{4}$ the focal length of the third, would have only 0·6 the thickness of glass, and this difference might probably be more than equivalent to the extra pair of reflections. This eyepiece has a much flatter field and is as sharp as the Huyghenian.

Secondly, the small mirror is something larger than its rival's, and therefore intercepts more light; the difference is only a fiftieth of the whole, and it must be remembered that the central rays are not those which give the best vision.

Thirdly, the rays traverse the tube thrice, but in the other a little more than twice;

* But as to this, see Appendix.

as, however, the Cassegrain is shorter, the space actually traversed, and therefore the chance of disturbance from this cause are as 1·12:1. Even this difference is nearly obviated by the lattice-tube.

Fourthly, NEWTON advanced an objection which is often urged. When light is reflected very obliquely from a body even of feeble reflecting power like glass, the reflection is almost total, its intensity decreases with the incidence, and when that is perpendicular, becomes very feeble. Now in the Cassegrain the incidence is nearly perpendicular, in the Newtonian at 45°; and hence he inferred that the latter has most light. But we now know that metals reflect differently from glass, and that the intensity of their reflected light follows a more complicated law. It *decreases* down from the perpendicular incidence to one depending on the incidence of maximum polarization (which is different for different rays), and then increases till at 90° the reflection is total. In the researches to which I have already referred, M. JAMIN has given the intensity of red light from speculum metal, which when perpendicular =0·692, but for 45° is only 0·646; for the other rays the intensities are less, but follow the same law; so that the advantage is nearly $\frac{1}{10}$ *in favour of the Cassegrain.*

The other advantages are, first, the tube is shorter, therefore lighter, and less acted on by the wind; second, the magnitude of the second image gives facility for micrometer measures; it is also flatter than in any other telescope; third, the errors of the small speculum tend to correct those of the large; and this is of some importance when they have to be repolished by persons not so skilful as the original maker. It is easier to figure the small mirror properly than the plane one of the Newtonian. Fourth, the greatest of all is the facility which it offers to the observer. The eyepiece is near the ground, and travels in a spherical surface of some 7-feet radius while the telescope sweeps the whole sky; the observer has to move but little, and the observing chair is light and easily managed. But with the Newtonian he must be at the upper end of the tube, which with a 4-feet looking to the zenith would be at least 37 feet above the ground; and he must have an adequate apparatus to support him, the use of which requires much labour and is not altogether exempt from danger. This had most weight in deciding the Committee in favour of the Cassegrain, and the trials which have been made of the instrument since its completion fully justify their preference.

In determining the relative powers of the specula and the lowest eyepiece, special formulæ were in this case required, and those found in ordinary treatises of optics have the defect of assuming the distance of distinct vision infinite in respect of the focal lengths of the lenses. The conditions of the problem are three. 1. The opening in the large speculum should be as large as the aperture of the small one: no light is lost by this. 2. The opening of the field-lens of the lowest power should be of the same size to obtain the largest possible field of view. 3. The lowest power must be such that the eye can take in the whole emergent pencil. Call F, f, f', and f'' the focal lengths of the specula and the lenses of the Huyghenian, A and a the apertures of the specula, d and d' the distances of the first and second images from the small speculum, φ and u

those of the second and third images from the first lens, and z that of the fourth (a virtual one) from the second lens, then the fourth image is seen directly by the eye placed at the eyestop: this stop must be at the image of the small speculum formed by the second lens, and of the same diameter as that image, to exclude all light except what comes from the specula. The distance of the stop from the lens $=\frac{1}{2}f''+\tau$, τ being a small quantity, this distance $+z=\mathrm{V}$ the distance of distinct vision. Hence we obtain in succession

$$z=\frac{2\mathrm{V}-2\tau-f''}{2};\quad u=\frac{f''(2\mathrm{V}-2\tau+3f'')}{2\mathrm{V}-2\tau+f''};\quad \varphi=\frac{3}{4}\frac{f''(2\mathrm{V}-2\tau+3f'')}{\mathrm{V}-\tau},$$

and hence

$$\text{magnifying-power } \mathrm{M}=\frac{\mathrm{I}_{IV}}{\mathrm{V}}\times\frac{\mathrm{F}}{\mathrm{I}_1}=\frac{2\mathrm{F}d'}{f'a}\left(1-\frac{\tau}{\mathrm{V}}\right).$$

As the small speculum should receive all the light from the large one, $d=\frac{\mathrm{F}a}{\mathrm{A}}$, and as in these eyepieces f' bears a given ratio to $a'=a$, we have $a=nf'$; and substituting these in M,

$$\mathrm{M}=\frac{2\mathrm{A}d'}{nf'^2}\left(1-\frac{\tau}{\mathrm{V}}\right);\quad d'=\frac{\mathrm{M}nf'^2}{2\mathrm{A}\left(1-\frac{\tau}{\mathrm{V}}\right)}.$$

But we have a second value of d',

$$d'=\mathrm{F}-d+b+\varphi,$$

b being the distance of the great speculum from the first lens; and equating the two,

$$f'^2\times\left(\frac{\mathrm{M}n}{2\mathrm{A}\left(1-\frac{\tau}{\mathrm{V}}\right)}-\frac{1}{4\mathrm{V}\left(1-\frac{\tau}{\mathrm{V}}\right)}\right)+f'\left(\frac{n\mathrm{F}}{\mathrm{A}}-\frac{1}{2}\right)=\mathrm{F}+b,$$

which, when A, F, n and V are known, gives f', and thence the other elements of the telescope. In this case F=366 inches, b=11, A=48; opticians generally make n=0·5. About V there is doubt; it has been estimated from 10 to 5 inches, but I will take 8 inches, that adopted by Sir W. Herschel, and several others. M=240*, hence

$$f'^2\left(\frac{1\cdot21875}{1-\frac{\tau}{\mathrm{V}}}\right)+f'\times3\cdot3125=377.$$

As τ is small, omitting it, we get an approximate value of f', and thence of d', from

* M is nearly inversely as f'; for though $\frac{d'}{d}$ changes with the adjustment of focus, the change is trifling. It may also be remarked that the common mode of getting M, dividing A by x, the diameter of the image of the large speculum at the stop is not correct unless f' is small; the expression is

$$\mathrm{M}=\frac{\mathrm{A}}{x}-2\varphi\left(\frac{2\mathrm{A}}{f'^2}+\frac{\mathrm{F}}{d'}\left[\frac{1}{2\mathrm{A}}-\frac{1}{f'}\right]\right).$$

which τ is found $=0{\cdot}2051$, and thence finally $f'=16{\cdot}09477$. Omitting the effect of τ, the field of view is in general $\theta=\frac{\text{cotang}\,1'}{M}$; hence the constants of the telescope are

Large speculum	F=366 inches
	A= 48 „
Small speculum	f= 74·71
	a= 8·05
	d= 61
	d'=332·31
	$\frac{d'}{d}$= 5·4477
Lowest power	f'= 16·10
Distance of eyestop from 2nd lens	2·89
	M=240
	θ= 14′·32
Equivalent focus	1994

As soon as these were decided on, the contract (which, beside the telescope and a duplicate large speculum, included an apparatus for polishing and a steam-engine for working it) was signed at the close of February 1866, and the preparations for casting the specula were urged rapidly forward. All had been so carefully considered by Mr. GRUBB, that scarcely in any instance was there any necessity to change what he had planned. While these were going on, much progress was made with the more massive parts of the equatorial, and the polishing-machine was got ready. The alloy for the specula was also prepared. It is that of Lord ROSSE, four equivalents of copper to one of tin; it possesses more power of resisting tarnish than those which deviate a little on either side of this proportion, and is probably as reflective as any. The liability of good speculum metal to tarnish has been much exaggerated. I have elsewhere given some proofs of this, and may add that there is at the Armagh Observatory a six-inch Gregorian by SHORT, bearing the date of 1745, which is nearly as bright as at first. As to this Mr. LASSELL and Mr. W. DE LA RUE are good witnesses, especially the latter; for his 13-inch showed no signs of tarnish, though it was for several years exposed to the influence of photographic chemicals, some of which are very bad neighbours for polished metals. Like most other brilliant metals, not excepting silver, it reflects the less refrangible rays in greater proportion than the others, which gives stars a tendency towards red or orange in reflecting-telescopes. This, perhaps, may not be the case with an alloy proposed by Rev. WILLIAM T. KINGSLEY, who adds to the above compound one-fourth of an equivalent of zinc. A small piece of this, with which that gentleman

favoured me, is so white* that, though in uncertainty as to its permanence, I did not venture to propose casting a large speculum of it, Mr. GRUBB thought so well of it that he has tried it in a duplicate small speculum of 8 inches, which will test its endurance.

From the great difference between the melting-points of copper and tin, and the great liability of the latter to oxidation, the alloy is generally formed by pouring the tin into the melted copper, stirring them, and rapidly casting into ingots. The alloy fuses at a far less heat than the copper, and the castings made from the second melting are supposed to be less porous. Pores, I believe, are never totally absent; it seems that the alloy absorbs gases at a high temperature, as is notably the case with silver and copper, and as has lately been shown in some remarkable instances by GRAHAM for some other metals. Copper when so charged with oxygen is brittle, and it is made marketable by "poling," stirring it while fluid with a pole of dry wood, the carbon and hydrogen of which combine with this oxygen as they bubble through it; and the same process has been found to lessen considerably the porousness of speculum metal. All these precautions were taken here.

†[Plate III. fig. 1 is a plan of the casting- and annealing-room, 36 feet long by 16½ wide. A is the hot hearth for heating the bed of hoops which (as described by Lord ROSSE) forms the bottom of the mould. It consists of four low pillars of brick with intermediate grates for coke fires. B is the melting-furnace (which replaced two smaller ones used in mixing the alloy); it is similar to the common brass-founder's one. Its internal dimensions are 42 inches square by 69 from top to the fire bars, with a central pillar rising 15 inches above them to support the crucible. C is the mould as laid for a casting; and at D is a crane which commands A, B, and C. It also commands E, the open cradle, seen in section (Plate IV. fig. 8), in which is placed the crucible full of fused metal previous to pouring. F is an inclined causeway leading to the mouth of the annealing-

* The experiments of JAMIN, already referred to, give the coefficients of perpendicular reflection for each ray from several metals. I give those for speculum metal and zinc.

	Speculum.	Zinc.
Red	0·692	0·576
Orange	0·654	0·594
Yellow	0·632	0·602
Green	0·625	0·616
Blue	0·606	0·628
Indigo	0·599	0·635
Violet	0·593	0·636

It appears from this that while the reflective power of the speculum metal decreases as the refrangibility increases, that of zinc increases. It is therefore possible that a proper mixture of the two might act equally on all the rays.

† The matter enclosed in brackets is contributed by Mr. GRUBB; and I am sure that the scientific world will duly appreciate the valuable information which he has so liberally communicated.

oven G. This oven was constructed with great care; it is circular, to ensure uniformity in the cooling of the enclosed speculum, and is strongly bound with a massive band of iron to prevent cracking. Upon a thick foundation of rubble work were laid several courses of brick with a layer of sheet iron interposed to prevent moisture rising from the ground, and on this were constructed the fire-place and flues for heating the bed of the oven. This bed is formed, first, of several courses of fire-bricks, on which are laid fire-tiles 12 inches square previously ground separately to the required curve, and then set so as to form a continuous bed of the radius required for the under side of the speculum. The circular wall of the oven is 27 inches thick, and the details of its construction are fully shown in Plates III. & IV. figs. 2 to 8. The system of flues below the bed, of which one or more can be readily stopped if required, gives great facility for heating it equably; and the wall and arched roof of the oven (which was covered with ten tons of sand) are heated by a mixed fire of coke and turf, burning in the oven itself, the combustion being regulated by its independent air-hole, damper, and chimney.

For the quick opening and closing of the mouth of the oven, a series of cast-iron stoppers were provided, coated internally with loam, and with removable handles outside. These, when *in situ*, complete the circle of the wall, and give great facilities for opening and closing the oven when the speculum is introduced.

A single crucible of cast iron holds enough of metal for a casting; it was cast with the precautions indicated by Lord Rosse; its internal dimensions are 25 inches diameter, 32 inches deep; the bottom is rounded, and its thickness is from 2 inches there to 1½ at top. It easily holds 30 cwt. of the alloy, 27 being required for a 4-feet speculum.

The operations of casting and annealing were conducted essentially as described by Lord Rosse, with such modifications as the case required, or were indicated by experience. In constructing that part of the mould which forms the edge of the speculum, a different plan was followed from the beginning, for two reasons. First, Lord Rosse had experienced a difficulty in "timing" the operation, because the wet sand-ring which was the edge of the mould was quickly dried when placed in contact with the hot bed of hoops, and consequently was liable to crumble down. Second, the plan for lateral support of the speculum proposed by Mr. Grubb (G. M. T. Correspondence, p. 25) makes it desirable that the speculum should be cast with a central band about ¼ inch thick, and one-third of the speculum thickness in breadth; and to have effected this with a mould of damp sand would have been nearly impracticable. The edge of the mould was therefore formed as follows. A strong sheet-iron hoop, 8 inches deep, and of the same diameter as the bed of hoops, strengthened internally with rings of angle-iron, and its lower edge turned true, was lined with loam moulded to the required shape. This being fully dried, the ring was attached by clamps to the bed of hoops previously heated, where it remained while other matters connected with the casting were preparing. It may be remarked that this ring was not withdrawn from the speculum when it was removed to the oven; this was convenient for the manipulation, and Mr. Grubb thought it would probably

assist in the annealing. It has been used thus three times, and has sustained no injury, nor even oxidation of its surface.

To provide the hole required in the centre of the Cassegrain speculum, a core prepared in the usual manner, 7 inches diameter, was attached to a stout bar of iron extending across the hoop-ring, and clamped to it before pouring.]

On July 2, 1866, the casting of the first speculum was commenced. At 1 P.M. the crucible was set in its place, and the furnace filled with fuel was lighted at the top, and cautiously heated. At 2 A.M. on the 3rd the crucible was at a full red heat, and the charging of it commenced. The alloy, broken into small pieces, was supplied slowly, so as not to risk cracking the cast iron by sudden cooling of it. By noon about 7 cwt. was fused; but the furnace, which up to this time was partly fed with compressed peat, became choked, and the heat was much lowered. By energetic poking, feeding with large coke, and adding an extempore couple of feet to the chimney-shaft, the whole 27 cwt. was fused by $8^h\ 30^m$ P.M. During the last hour the mould, which had been heated on the hearth A till blue, was transported by the crane to its place and carefully levelled; its surface was washed with kaolin to prevent the alloy from adhering to it.

The oven, which had been for three weeks fired with a mixture of coke and peat both in its flues and chamber, so that its interior was at a full red heat, was raked out in the chamber, and the flues were closed.

All being ready, and the mould covered with a disk of sheet iron to keep out any dirt that might fall into it, the crucible was lifted by the crane from the furnace and transferred to the cradle. The tackle of the crane was now shifted from the crucible to the cradle, and the former tilted until the metal reached its lip; the metal was once more stirred with a birch pole and skimmed, the cover, which during these operations had protected the mould, was removed, and by rapidly turning the winches of the crane the pouring was effected in six seconds—too quickly as it seemed, for from 15 to 20 lbs. of the metal splashed over the opposite wall of the mould. When sufficiently cool (which was tested in Lord ROSSE'S manner by tapping with an iron rod), the central core was removed, and the speculum still on the bed of hoops was drawn to the oven's mouth. This was effected by a powerful crab, placed outside the building, acting on a chain and bar passed through the opposite side of the oven, and hooked into a large stirrup attached to the trunnions of the bed of hoops. The latter was now close to a step formed at the oven's mouth, and the bottom of the speculum was on a level with the oven's bed. The stirrup being now removed from the trunnions of the bed to those of the loam-ring, this, with the included speculum still red-hot, was drawn to the centre of the oven, leaving the bed of hoops behind. There was great difficulty in effecting this; and all the appliances provided failed at first to separate them, though a pull of at least two tons was exerted, aided by blows given to the mould with a large piece of wood. At last they were started by one of the party jumping on the chain while it was so tightly strained. The cause of this difficulty, which had well nigh prevented the speculum from being drawn into the oven in time, was soon discovered. The loam-lining of the hoop

was not in sufficiently close contact with the bed of hoops, and a film of fluid metal found its way here and there, and sunk partially into the small hollows where the ends of the hoop iron abutted on the ring which confined it. The remedy was obvious—to lute the joint with a little wet loam, which dries instantly; and it proved quite successful in subsequent casting. The iron stoppers of the oven, already described, were set in their places, hot bricks were placed outside them, and the whole luted up; all other apertures, including the entries to the chimneys, were carefully stopped. In doing this a thermo-couple of platinum and iron wire was inserted, so that its joint was nearly over the centre of the speculum; and its elements were connected by copper wires with a galvanometer in Mr. HOWARD GRUBB'S office, where it kept watch over the cooling. Next morning the galvanometer deflected 69°, which in twenty-four days decreased to 0°, showing that the cooling was complete. See Appendix No. I.

The whole was effected without any accident, though the workmen in some parts of the room suffered a good deal from the heat. Gas had not yet been introduced, and the candles melted so quickly that they were of little use.

On removing the casting from the oven it proved to be sound, but its surface was "in winding" about $\frac{1}{4}$ inch; beside these were some of the imperfections which Lord ROSSE has called "crowsfeet." Had these been at the highest part of the surface, they would have disappeared in the grinding; but this was not the case, and Mr. GRUBB preferred recasting it.

[Advantage was taken of breaking up the cast to try its strength. It was placed horizontally on four short blocks of wood under an iron ram weighing 70 lbs., which could fall on it from a height of 4 feet. A strong wooden bar, about 30 inches long, was laid across the speculum and received the blows of the ram without effect; but when a metal bar was substituted for the wooden one, the disk broke into four very equal portions. These, on being brought together, showed no sign of unequal tension, which, along with the considerable force required for fracture, shows how perfect the annealing had been.

The breaking up of this casting revealed an unexpected defect—a cavity about 9 inches long, its cross section being a flattened oval large enough to admit three fingers; it lay in the middle thickness of the metal, and was lined with oxide studded with brilliant metallic crystals. Probably it was caused by air entangled in the rapid pouring, and had the speculum been polished, its figure might have changed with any considerable variation of temperature.

Before a second trial the following changes were made:—

1. To guard against flexure of the bed of hoops, it was strengthened by additional underlying bars, and also placed for the casting on a strong cast-iron frame*.

2. Mr. GRUBB observed in the first pouring that the fluid metal was separated by the central core into two waves meeting at the opposite side of the mould, returning and

* The importance of this is obvious; for as the bed of hoops becomes red-hot when the fluid metal is poured on it, it is liable to "sag" under the great pressure, and thus warp the surface of the casting.

again meeting where the metal first touched the mould, which part till then was kept exposed by the impetus of the pour; and he remarked that the surface-lines of imperfect union ("crowsfeet") occurred chiefly where these waves met. To provide for a more regular diffusion of the melted metal over the mould, the cast-iron frame supporting it was made to turn at the end next the crucible, on an axis parallel to that of the cradle. The other end was raised or lowered by a wedge, driven by a rack and pinion under the floor, which again was actuated by a vertical shaft worked at a convenient distance from the casting.

3. As the central core interfered with the free flow of the metal, it was raised $1\frac{1}{2}$ inch above the surface of the mould, and in part supported there by a pin of the same diameter.

4. Instead of kaolin, the heated bed of hoops was washed with two parts animal charcoal and one of plumbago in fine powder, mixed with ale.

These preparations being made, the second speculum was cast September 22, 1866. The oven had been fired for eight days and nights previous, the melting-furnace was lighted the evening before, and the charging the crucible commenced at 2 A.M. and was finished at 11. Up to this time coke was mostly used, and the melting took place earlier than was expected. Peat now was used and the draught greatly reduced; the pouring took place at $12^h\ 20^m$.

The mould was set by the apparatus already described at an angle of 18° with the horizon, so that when a third of the metal was poured it would cover only half the mould on one side, and on the other rise to the edge of the loam-ring at its lowest part. At this instant the mould was rapidly lowered to be horizontal, the pouring being continued without interruption, and the fluid covering the entire surface with a nearly unbroken wave. The time was sixteen seconds. Instead of the bar and stirrup used to draw the cast off the bed, a simple bight of chain was found sufficient, and it was removed into the oven without any difficulty. The thermoscope marked 73*.

The third casting was made November 24, at $12^h\ 10^m$, without any change of the plan which has been described, and both these disks turned out exceedingly perfect.

In these the centre was closed by a piece of metal $1\frac{1}{2}$ inch thick. This piece was removed by grinding with emery and a ring of soft iron (like a trepan saw); it was attached to a double handle and worked backwards and forwards by two men, its entry being at first guided by a board with a hole of the same size. The cutting each disk required three days.]

Difficult as it is to obtain a sound disk of good speculum metal, it is even more so to give it the figure required to form a perfect image combined with a fine polish. The accuracy of the finest cutting-tool that ever was devised falls almost infinitely short of what is wanted here; which can only be obtained by the mutual abrasion of two surfaces working in contact. Yet it is not at first evident how such abrasion can produce any-

* The details of the pouring, and the removal of the speculum into the oven are shown in Plate IV. fig. 8. The dotted lines show the position of the mould at the commencement of the pouring.

thing but a spherical figure, which is quite unfit for a speculum; for one might expect that the acting surfaces would wear into uniform contact, and therefore uniform curvature. But both the speculum and its polisher are elastic, and allow the contact to continue, notwithstanding a minute difference of curvature; how minute appears from this, that even at the edge of one of these 4-feet specula the distance of its parabola from the circle is only 0·000106.

This being possible, we can increase the abrasion at various parts of the surface till the desired figure is attained. For a long time it was believed that this could only be successfully done by the hand *feeling* the action, but Lord Rosse found that it could be performed as well, and with more certainty, by machinery. In this he has been followed by others, and though their methods differ, the general principle is the same. The speculum revolves slowly on its axis, while the polisher traverses it more rapidly, describing a track which is some continuous curve, and crosses it in every possible direction, not returning to the same place till after a great number of revolutions. In Lord Rosse's machine the motion is the resultant of two nearly rectilinear and at right angles, one less and of much slower period than the other. Latterly he reverted to an earlier plan of his own, and used a motion which is nearly elliptic. He also surrounded the speculum with water of a given temperature. Mr. Lassell and Mr. W. De La Rue use an epicycloidal motion, given by a mechanism like that of Suardi's pen, which is very effective. Mr. Grubb many years ago made one which combines the power of both these*, but gave it up for the simpler one described here, which is remarkable for the precision and smoothness of its action.

[Plate IV. fig. 9 is an isometrical view of the grinding- and polishing-machine. A, A are strong A-shaped castings connected by three collared stay-bolts and nuts, and having V-shaped bearings at top, in which turn the circular ends, or trunnions, of the hollow prism-shaped casting B. This casting is bored through its centre, and a circle on its upper side truly faced; to it is fitted a spindle with a large face-plate, whose under surface applies to the trued surface of the hollow beam for steadiness, and which serves to carry the speculum. The lower end of the spindle carries the wheel C, by which circular motion is given to the speculum, C being driven by the wheel and pulley D, by means of a belt driven from the shaft.

To the principal framing are attached the brackets E, E, which carry the plate F F; this plate can be raised or lowered to suit different heights, and it carries the horizontal shaft G, which, by means of two pairs of bevil wheels of equal numbers, drives the two vertical shafts with their cranks H, H.

These cranks are adjustable to any desired length of stroke from zero, and give motion to the connecting-rods I, I, which rods are adjustable as to length.

The rods I, I act conjointly upon the lower end of the vertical bar K, and this bar entering a central hole in the grinder (or polisher), produces the requisite horizontal movements in either process. It may be seen that by the adjustment of the strokes of

* See for descriptions of these the Art. "Speculum" in Nichol's Cyclopædia.

the cranks H, and of the length of the rods I, a motion can be produced varying from nearly a straight line through a series of ovals to a circle of any extent, and either concentric with, or excentric to, the underlying speculum.

The vertical bar K passes at its upper end through a hole in a bar attached to the roof of the building, and besides producing the horizontal motion mentioned, performs another important office.

To the lower end of the bar attaches, by a cross-key, a flanged socket which supports the triangular piece I, and at its upper end the bar is attached to a lever with adjustable weights, so that a pull upwards in the direction of the bar to any desired amount is provided, and thus any portion of the weight of the grinding- or polishing-tool in excess of that desired is relieved.

The figure shows the details of the system of support as applied to the grinding-tool. The central piece L being upheld by the bar K, supports the three straight bars M, and these again support the six triangular pieces N.

Thus any portion of the entire weight of the tool is supported equally from eighteen points. The following arrangement was provided for the quick and convenient trial of the speculum while undergoing the process of polishing. To the hollow beam B is attached the apparatus O, P, Q, O being a massive toothed sector, P an endless screw working in same, and Q a wheel and pinion, with a hand-winch. This part of the apparatus serves either to retain the speculum in an horizontal position for polishing &c., or for quickly bringing it to the vertical, when by the opening of those doors, purposely provided in the building, the speculum can be tested upon a day object sufficiently distant; and again by a reverse motion of the winch the speculum is as quickly restored to the horizontal position for continuing the process if required: this facility of testing the figure is of the highest importance to the ease and certainty of the polishing; without it one would be working in the dark.

As it was deemed necessary to grind both back and face of each speculum, two grinders were provided; the form of the back of these is shown in the figure; both grinders were cast with grooved surfaces, forming projecting squares of 3-inch with $\frac{1}{2}$-inch spaces, and prepared in the lathe, one of them flat for grinding the back, the other to the required curvature by a slide-rest actuated by a guide to the intended radius; each speculum was treated as follows.

First, the face was ground to a good surface and approximately to the required curvature, then reversed and ground flat on the back; in both cases the speculum rested in the face-plate of the grinding-machine with felt interposed, and thus free access was obtained to the edge of the speculum for the accurate trueing by grinders of the projecting band. Next, the speculum was once more reversed and placed in its box upon its ultimate supports, from whence it was not disturbed during the subsequent operations of fine grinding and polishing. Before commencing the fine grinding of the face, the grinding-tool was, by cross-cutting the squares on the face, formed into squares of about $1\frac{3}{8}$ inch each, and by a little contrivance and some care the system of supporting-levers,

balls, &c., was preserved from the intrusion of either grinding matter or water during the subsequent processes of both fine grinding and polishing*.

The polisher is constructed similarly to that previously used by Mr. GRUBB for smaller specula. It is formed of a great number of strips of fine deal, $1\frac{1}{2}$ inch wide and $1\frac{1}{2}$ inch apart, laid in layers and crossed, each layer being firmly glued and nailed to the next. The pieces, in the middle of its thickness, are thicker in the centre than at the ends, being formed as if they were cut out of a lenticular disk of 4 feet diameter, $1\frac{1}{4}$ inch thick at centre, and $\frac{3}{8}$ inch at edge; on either side of their centre were built, by crossing, parallel slips in layers, the outer course at both sides being without intervals. The apertures occurring round the edge were stopped up, and the whole turned to the required curvature on the face, and strongly varnished. The entire presents the qualities of great stiffness, lightness, and freedom from change of figure from either heat or moisture.

The pitch used for coating the polisher, after being adjusted to the desired hardness, was rolled into parallel thickness, cut by pressure into squares while still soft, and these applied to the surface of the polishers, using a spirit- or gas-lamp for momentarily softening one side of the patch before laying it; the patches were $\frac{7}{8}$-inch squares with $\frac{3}{8}$-inch intervals, and only one hardness of pitch was used.

This method of coating a polisher is (for a large-sized one) necessarily tedious, but can be done by inexperienced hands, and assuming the proper degree of hardness to have been obtained, involves no other uncertainty; it will also serve for several operations. The focus of the first speculum polished was purposely kept within that specified; it turned out to be $30\frac{1}{2}$ feet instead of 32 feet. It was desirable that both large specula should be approximately of the same focal length, and this has been accomplished by the use of a large spherometer, the difference of focus being about 1·1 inch only, or $\frac{1}{340}$ part of the entire.]

In the process of grinding the pressure is considerable, on an average 112 lbs., and the strokes 32 in the minute and of considerable extent; but for polishing all these are much diminished; the pressure is from 30 to 20 lbs., the strokes are 24 in the minute, and a fifth of their former length. The speculum revolves once for 14 strokes; the polisher more slowly, according to the nature of the action. In Mr. W. DE LA RUE'S machine and mine this last motion is effected mechanically, but Mr. GRUBB has not found this to be essential.

The small speculum is polished on a similar machine but much smaller; it is rather too large for hand-work, and requires to be specially figured to match the great one, as that must be parabolic to be available for photography; but in this there is no difficulty, as the tendency to error is in the direction which the small speculum requires.

The uncertainty of the polishing process arises from the pitch not being of a proper hardness (which depends on the temperature), and the hygrometric state of the air. It must always be a very delicate operation, but Mr. GRUBB seems to have reduced the

* For some numerical data respecting these operations, see Appendix No. II.

difficulties to a minimum; and it is quite manageable by any one who has a good head and a good eye, with a little training.

[In addition to the ordinary (metallic) small speculum of the Cassegrain there is supplied one of silvered glass, of a construction originally proposed and described by Mr. GRUBB in 1857 (see Report of British Association for that year).

It consists essentially of an achromatized combination of crown and flint glass, the outer surface, or that which receives the rays coming from the large speculum, being of a curvature which will sufficiently disperse any light *reflected* from same, its fourth surface being made highly reflective by a *thick* deposit of silver. The inner surfaces of the compound are made coincident in curvature so as to admit of being cemented, and the refractive and reflective powers of the compound are so adjusted that their sums (taking into account that the rays are twice refracted) shall be equal to that of an ordinary or metallic speculum of the required focus.

In the supporting of this silvered glass speculum the increased difficulty experienced by others of supporting glass as compared with speculum metal has presented itself. It has not, however, been found that any particular diameter of the lenses being vertical caused a difference in this respect; perhaps this may be attributed to the glass used, having been prepared in the form of disks as generally used in the constructing of object-glasses, whereas it has probably occurred in other cases that, as the quality of the glass suited to the reflectors of M. FOUCAULT is understood to be by no means material, *rolled* glass has been used, and that therefrom arises a difference in different directions to the power of resisting flexure.]

But it is not enough to have a fine speculum unless it is supported in the telescope so that it may be exposed to no stress which can change its figure. A local pressure of a very few pounds at the back or even the edge of one of these massive 4-feet specula would entirely deform its image; and such pressure is not easily avoided in the varying positions of the instrument. In the first reflecting telescopes, the specula were held by three front stops, against which they were kept by three or more screws behind. The screws were afterwards replaced by three springs, and this defective system remained till the HERSCHELS substituted one unobjectionable in principle. They placed the speculum in a strong metal box resting on several folds of soft and elastic cloth. This diffuses the pressure uniformly over the back of the speculum, while the box can rest on adjusting screws; if there were as good a lateral support, which could be easily applied, this system is (except for very large instruments) excellent. M. FOUCAULT supported his specula on air-cushions which were inflated as required; this would also give very uniform support, but would be affected by changes of temperature, and would, it seems to me, be inconsistent with that permanence of index corrections which is essential to the purpose of this telescope. It has not only to show nebulæ and faint stars, but to determine their places to the accuracy of a very few seconds; and therefore at every hour-angle and polar distance, the position of the optic axis of the great speculum with

respect to the tube must be almost invariable. This should also be the case with the small mirror; but there the difficulty is trifling. For the other, nothing has been devised more efficient than the arrangement of equilibrated lever and hoop lateral support.

[The first is illustrated by Plate V. figs. 10, 11, & 12. The speculum-box consists mainly of three parts, A, B, and C, fig. 10. A being a strong-ribbed casting, the left-hand side of the figure showing a section through one of the three main ribs; B is a wrought-iron hoop fitting to A, the contact parts being trued in the lathe, and connected by a number of screws; and C is a trued casting fitting the upper part of B, and attached to it also by screws. This construction has the advantages (in addition to that of great stiffness), 1st, of admitting of the grinding and polishing processes being performed without disturbing the speculum, or raising it in its box, the removal of the upper part C being sufficient; 2nd, by the removal of the hoop B, of permitting free access to be obtained to the system of levers &c., forming the vertical support of the speculum.

This system of vertical support is in principle the same as that applied by Mr. GRUBB in 1834 to a 15-inch speculum constructed for the Armagh Observatory, and subsequently used by Lord ROSSE, Mr. LASSELL, &c., for larger specula; but in the present instance it has probably been carried out more perfectly in its details, and these therefore appear to deserve a particular description. In applying this system to 6-feet specula by Lord ROSSE, and to 4-feet specula by Mr. LASSELL, it was considered necessary, partly from the considerable weight of the levers, and partly from the distance between the plane of the upper ends of the three main adjusting screws, and that of the back of the speculum (this distance being in the case of the 6-feet specula 14 inches), to apply a secondary system of supports to the levers themselves*; and whatever amount of inconvenience arises from this cause, where one diameter of the speculum is always retained nearly vertical, a much greater amount would arise where that condition is no longer to be retained.

The whole lever-apparatus has therefore in the present instance been constructed of steel, by which its necessary weight is reduced to a minimum; and secondly, by an ingenious construction which at first presented much difficulty, and which was ultimately worked out by Mr. HOWARD GRUBB; the secondary levers of the system, instead of being as usual placed between the primary and tertiary, are placed above the latter, or nearly touching the under side of the speculum, the balls of the tertiary levers acting (where necessary) through holes in the secondary. The result of this is that the distance between the back of the speculum and the bottom of the box is only $3\frac{1}{2}$ inches, and any necessity for counterpoising the weight of the supporting levers is removed.

In carrying out in detail this system of support no sacrifice of principle has been admitted. The speculum has been supposed to be divided vertically into forty-eight portions of equal weights contained in three annuli, viz. two outer of eighteen pieces each and one inner of twelve, each single portion being supported under its own centre of gravity. Plate V. fig. 11 is a diagram of this supposed division, and fig. 12 shows

* Mr. LASSELL's elegant arrangement is described in British Association Reports, 1850, p. 180.

in that third marked A its primary lever, in that marked B its two secondaries, and in that of C its six tertiary, in which the four triangular are triple and the two straight are double. These tertiary levers support the speculum by intervening balls, as originally adopted by Lord ROSSE; but it has not been attempted to keep these balls in their place by a wire and spring, which has been found occasionally to fail, but by being surrounded by a metallic ring allowing of a slight rolling movement, but affording no chance of displacement. The balls, $1\frac{1}{4}$ inch diameter, are of hard cast iron, made approximately true by grinding.

In Plate V. fig. 10, 1 is a vertical section of a primary lever, 2 a secondary, and 3 a tertiary with its balls and rings. Figs. 13 & 14, taken from photographs, further illustrate the system of vertical support; fig. 13 shows the supports as described; fig. 14 the same, but with the addition of two rings of sheet iron which are necessary for keeping the levers in their proper position laterally.

The lateral support of the specula is that originally described in a paper by Mr. GRUBB already referred to. It is shown in section in Plate V. fig. 10, where D is a ring of wrought iron trued after being attached to the inner side of the speculum-box, and E is another ring of same material which fits with a slight looseness on D, and also fits loosely the trued edge of the speculum. It may be seen that, except when the speculum is quite horizontal, its weight pressing against the lowest part of the ring E must bring its opposite or highest part into close contact with the ring D, while that part of the ring E, which the speculum is *pro tempore* bearing against, is thereby removed out of contact with the ring D, and consequently free to move up and down a small but sufficient quantity, which virtually places the speculum under equal advantages as respects lateral supports as if it were held in a flexible band, of the form of the letter U, supported at its upper ends.

This method of lateral support was applied long since by Mr. GRUBB to smaller specula, including one of 20 inches diameter at the Glasgow Observatory, and with entire success; but as grave doubts were expressed recently as to its efficacy in the case of a 4-feet speculum, it is desirable to mention that its efficiency is as decided in this latter case as in that of the 20-inch; no appearance of flexure being perceptible in either instance on placing the telescope in various positions, by which different parts of the speculum, including opposite sides of same, were brought successively into the lowest position.]

The tube consists of two parts; the lower, which is made of strong sheet iron, is firmly secured above in the cradle of the declination-axis, and below to the speculum-box by three strong bolts. Between two of these is a slit, through which, by means of guides passing through two openings above it, a cover which fits closely in the ring C (Plate V. fig. 10) can be introduced to protect the speculum when not in use. Then also the large eyepiece is replaced by a vessel containing quicklime to absorb moisture. This part of the tube is 7 feet long, of which only 17 inches are below the cradle; so the speculum is not quite 40 inches below the centre of the declination-axis. The upper

part is lattice-work made of slips of steel $\frac{1}{8}$ inch thick, 3 inches broad at bottom, and tapering to $1\frac{1}{2}$ at top. These were bent round a cylinder of the proper diameter, as shown in Plate XII. fig. 15, and each crossing was secured by a rivet, forming a series of lozenges about 9 by 17 inches. Strong iron rings were rivetted to each end, the lower of which was secured to the close part; and four strong diaphragms were similarly attached at equal distances in its interior. The open part is 21 feet long. As a proof of its exceeding stiffness, may be mentioned that a weight of 112 lbs. hung at its extremity only caused a deflection of $\frac{1}{100}$ inch. The small speculum is carried by a steel arm of considerable depth attached to the second diaphragm, which is of steel, and supported by struts from the first and third. Provision was made for transverse support, but this has not been found necessary.

[Plate VI. fig. 16 is an end elevation, and Plate V. fig. 17 a side elevation of the small speculum in its box, and apparatus for carrying and adjusting it; scale $1\frac{1}{2}$ inch to the foot.

A cast-iron plate *a*, 18 inches long, is attached to the inside of the tube, at one end by a small cross girder *b*, and at the other by a lunette steel ring *c* (fig. 16). To this is attached, by two thumb-screws, the plate *d* (fig. 17), forming the base of the hollow arm A, which is made of steel plate $\frac{1}{16}$ inch thick, and filled with wood to deaden the vibration. This carries at its upper end a V-shaped gun-metal slide B, in which works the arm *e*, to which the speculum-box is attached by three screws, *f*, *f*. Between the plate which forms the end of the arm and the speculum-box are two rings, *g*, *g*, of a greater thickness at one side than the other, by the relative revolutions of which the speculum is adjusted at right angles to the axis of the large speculum. For focusing the arm *e* is acted upon from behind by a screw, to which motion is given by the wire cords *h*. These cords are carried over the pulleys *b* and down the side of the tube to its lower end, where they are wound round a hand-wheel concentric with the eyepiece, to which motion can be given by the observer for the purpose of focusing (see next fig.).

Plate VI. fig. 18 is a plan of the bottom of the speculum-box, which forms the lower end of the telescope, showing the arrangements for hand motions, &c.; scale $1\frac{1}{2}$ inch to the foot.

a, *a* are two of the three bolts by which the speculum-box is attached to the end of the tube; *b*, *b*, *b* the three screws for levelling the speculum: these screws form the support of the three primary levers already described; their heads are countersunk in the bosses, and are covered when not in use by brass caps. A is the lowest eyepiece, power 220; field-lens 8 inches diameter; *c* is the hand-wheel for focusing; the cords *h*, *h* from this are carried up the side of the tube to the apparatus already described (Plate VI. fig. 16 & Plate V. fig. 17). B is the hand-wheel for quick motion in declination. The motion is obtained by a pinion on the upper end of the shaft of the wheel *d*, gearing into a toothed wheel 4 feet 6 inches diameter, attached to the cube of the polar axis, as seen at S in fig. 15. C is the wheel for slow motion in declination. This is obtained by a clamp (S, fig. 15) 4 feet 6 inches diameter, with a tangent screw and parallel motion similar to that afterwards to be described for R, and D is the Hook's-joint handles for the apparatus as

there described for slow motion in R.A. E is the finder, 4 inches aperture, fitted with eyepieces of large field.]

The equatorial which carries this huge telescope is not less remarkable. It will be seen in Plate XII. fig. 15 that the polar axis is inverted, so that the declination-axis, and of course the centre of gravity of all the moving parts, is near the ground. So also are the circles; which gives extreme facility in setting or reading the instrument. The axes are exceedingly massive; but from the beautiful arrangement of counterpoises which is adapted to them, their motion is very easy. One of these is seen at K (fig. 15); it relieves the upper pivot of the polar axis from its lateral friction to any required amount. Another, F, does the same thing for the lower pivot, and the peculiar one, L*, takes off the end pressure on the lower pivot. The result of their combined action is, that the instrument is turned round this axis by a force of 5 lbs. at a leverage of 20 feet.

[Plate VI. fig. 19 is an end elevation, and fig. 20 is a cross section of the upper end of the polar axis resting on its bearing; scale 1½ inch to the foot.

The axis in this part, 12 inches diameter, rests on two blocks of gun-metal inserted in cast-iron wedge-shaped pieces, b, b', sliding in a horizontal groove, and acted upon by the screws a, a', by the combined action of which the axis is adjusted in the direction of the plane of the meridian, or at right angles to it. At c (Plate VI. fig. 19) is seen the roller, 8 inches diameter, by which the Y supports are relieved of any desired portion of the weight of the axis, &c. This roller is acted upon by a pair of steel levers of the proportion of 3 to 1, to which weights are attached hanging down the eastern side of the polar pier. The fulcra of these levers are capable of adjustment, so that the roller may be brought exactly under the centre of the axis, as it is adjusted by the screws and wedges $a\,a'$, $b\,b'$. At d (Plate VI. fig. 19) is seen a sector acted upon in like manner as the roller by a lever and weights hanging down the western side of pier. This exerts an upward pressure on the bar e, which is connected with the relief of the friction of the declination-axis (see fig. 25, where this apparatus is described).

Plate VI. fig. 21 is a side elevation of the lower bearing of the polar axis, showing the plummer block in section; scale 1½ inch to the foot.

a is the lower pivot of the polar axis terminated by a piece of chilled cast iron, b, polished flat on its lower face. This under face revolves in contact with a piece of bell-metal c, flat on its upper side, and partially spherical on its lower, bearing in a correspondingly shaped spherical annulus formed to receive it in the bottom of the plummer block. This arrangement enables the bell-metal cushion to take its own position, as the axis is adjusted by the screws and wedges in the upper bearing (see $a\,a'$, $b\,b'$, Plate VI. figs. 19 & 20), and so ensures an equable bearing throughout the whole of the surfaces in contact of the pieces b and c. Although the component of the weight of the telescope

* The chain connected with the crank arm L is peculiar. It is made of circular iron disks connected by semicircular links of steel. Each link can twist a little on its partner with very little friction. They are hardened at their point of action; their ends pass through holes in the disks where they are secured by nuts, and can easily be removed for repair. No chain of the common construction would answer here.

acting upon this face is about five tons, any portion, or all of this, is relieved by the lever-apparatus on polar pier as seen in general drawing (Plate XII. fig. 15). The sector for relieving the lateral pressure of this bearing with its slide-box, springs, and adjusting-screw is seen at A. This sector is 27 inches radius, and works on a 1-inch hardened steel pin held in a cast-iron frame sliding in an outer box. This frame is forced up by the screw *d*, acting through the lamina of steel springs *e*. This sector relieves any desired position of the six-tons pressure in this direction, the friction due to this being reduced in the proportion of 54 to 1. At H are seen the fixed and differential hour-circles, 34 inches diameter, the lower (fixed one) reading sidereal time—$\mathcal{R}$ by its two fixed verniers, and the upper being set to sidereal time by its fixed verniers, two differential verniers read true $\mathcal{R}$s. They are divided on an alloy of palladium and silver to 1^m, and the verniers read to 1^s. For the purpose of setting this differential circle, pins *f*, *f*, *f* are inserted in it, by which it may be turned to its approximate position when the final adjustment is made by a delicate tangent motion.

Plate VII. fig. 22 is a plane of the $\mathcal{R}$ clamp and clock sector removed from the axis; scale $1\frac{1}{2}$ inch to 1 foot.

The sector driven from the clock by an endless screw is strung on a portion of the polar axis $7\frac{1}{2}$ inches diameter, immediately above the hour-circles by the cast-iron piece A bored accurately to fit it; to this are attached two steel tubular arms which carry on their extremity the gun-metal sector B, 5 feet radius; these are strongly braced with cast-iron lattice trussings, which carry at 3-feet radius a square-threaded tangent screw *a*, communicating with the clamp C by the nut-and-link motion *b*, shown in larger scale and described in Plate VII. figs. 23 and 24. The clamp C, 20 inches diameter, is clamped or unclamped to the axis by half a turn of either of the screws *c* or *c'*. When unclamped the instrument is free for motion in $\mathcal{R}$ by the quick-motion gearing, driven by the hand-wheel G (Plate XII. fig. 15). When clamped, the clockwork being in motion, the telescope follows the object. The final adjustment in $\mathcal{R}$ for bringing the object into the centre of the field of the telescope is effected by the differential screw *a*, actuated through the shafts *e*, by Hook's-joint handles attached to either *d* or *d'*.

Plate VII. fig. 23 is a plan, and fig. 24 a vertical section of the parallel-motion arrangement for the tangent screw referred to above; scale $\frac{1}{2}$ size.

Owing to the great sizes and weights of the masses to be moved, the ordinary constructions extant were considered insufficient. In the arrangement as adopted the screw is supported in bearings at *both* ends. This involves the necessity of the nut moving in a straight line, while the arm which carries it describes an arc of a circle. This is provided for by the arrangement shown in the figures. *a a* is a portion of the screw (which is purposely made of considerable length) working in the nut *b b*. This nut has a pair of trunnions towards one end, A A, working in the inner frame *c c*, which has itself a pair of trunnions, B B, working in the outer frame *d d*, which is attached to the arm of the clamp. At *e* is seen a counter-nut to take away any loss that may be occasioned by the wearing of the screw. This nut is cut into teeth on its edge, and is kept in its place by

a small brass cap *f*, with a pair of feathers on the inside which slide into two of the teeth of the nut.]

The declination-axis has also its three sets of counterpoises; but their function is of a more complicated character. The weight of itself, with the telescope and counterpoises, acts in the vertical passing through their centre of gravity, and may be conceived as the resultant of three rectangular components, one parallel to the polar axis, one parallel to the declination-axis, and one perpendicular to both. The first of these is constant in magnitude and direction, the others vary from 0 to a maximum at an inverse rate. The two first tend to make the axis shift in its Y's transversely and leave its bearings, the third to jam it against the cube of the polar axis.

[The second and third components are relieved by the mechanism shown in Plates VII. & VIII. figs. 25, 26, 27, and 28. It consists of two nearly semicircular cast-steel rings of a strong section. These surround the declination-axis at the point where its axis intersects that of the polar axis, and where it is shaped as in Plate VIII. fig. 26. Between the jaws of these rings are bolted the gun-metal blocks *c*, carrying each three rollers, two of which, *d*, *d*, roll on the trued rings *f*, *f* of the declination-axis, while the third, *e*, acts at right angles to the other two in a groove formed for the purpose in the declination-axis. The lower semiring has a steel stud *g*, which enters into a cavity prepared for it at the bottom of the polar axis, and forms the fulcrum of the lever. The upper ring is prolonged into a steel bar stiffened with a cast-iron jacket. This bar passes up through the polar axis, and projects at the upper end, where it is acted upon by a small sector and lever which produce a sufficient upward thrust (see Plate VI. fig. 19 *e*). By this means any desirable portion of the resultant of the second and third component can be relieved.

Plate VII. fig. 25 is an end view of this apparatus in its position embracing the declination-axis.

N.B. A portion of the axis is broken away to show the roller *e* working in the groove.

Plate VIII. fig. 26 is a plan of the rollers and declination-axis.

Plate VII. fig. 27 is a vertical section of the steel ring and rollers.

Plate VII. fig. 28 a side elevation of the same. It should be mentioned that, for the efficient working of this apparatus, it is necessary that both polar and declination-axis should be balanced "inter se" round their point of intersection. This is not the case in some instruments which have the counterpoise of the telescope hanging from the opposite side of the polar and not the declination-axis.

When this adjustment has been completed, then, and *not till then*, the first component may be relieved by the apparatus which is partly seen at O, Plate XII. fig. 15, and more fully in Plate VIII. figs. 29 & 30. Fig. 29 is an elevation of the larger, and fig. 30 half elevation, half section of the lesser roller-frames, scale 1·5 inch to 1 foot. That represented in fig. 29 is applied to the larger or telescope-end of the axis, immediately outside the larger bearing, between it and the cradle. The smaller, fig. 30, is applied immediately outside the smaller bearing, and can be seen at P in Plate XII. fig. 15. At *a*, *a'* are seen the ends of the two steel levers which bear up the roller-frames. The

fulcra of these levers are attached to the sides of the cube of the polar axis; one end of each bears up the larger frame (Plate VIII. fig. 29), as at *a*, *a'*; the other is attached to the end of a cross lever dropped on the stud *b* (Plate VIII. fig. 30), and fastened by a nut which is tightened till the elasticity of the levers acts with sufficient power to give the required relief of the friction. This is seen in Plate XII. fig. 15.]

As the size of the rollers is limited by the dimensions of the cube C (Plate XII. fig. 15), they are not quite as effectual as the antifriction apparatus of the polar axis; for it requires 12½ lbs. acting at the leverage of 20 feet to turn the telescope round this axis. Still one man can raise the telescope from the horizon to the zenith in twenty seconds. In reversing it from the east of the pier to west, or *vice versâ*, two men are necessary for quick work, as it must be moved in P.D. as well as R; they do it in forty-five seconds. The tangent movement in declination is so smooth that a star can be nicely bisected by it. The polar-distance circle P (Plate XII. fig 15), divided like the others on the alloy of palladium and silver, is 30 inches diameter, and has two verniers which read to 10″.

[The clock is lodged very compactly in a hollow in the pier; a shaft passes from it gearing with a vertical one which drives the screw of the sector. It is regulated by a conical governor, shown in Plate VIII. fig. 31 (scale 3 inches to 1 foot). *a* is a double fork-shaped frame of gun-metal, in which hang the two T-shaped pieces *b'*, *b'* carrying the balls A, A', 5 inches diameter, of hollow brass filled with lead. These, when not in action, are retained near their working angle 45° by the piece *c*. Attached to the steel bars are small brackets, *d*, *d'*, carrying screws with divided heads, *e*, *e'*, the extremities of which terminate in small cups to contain pieces of hard leather. These, when the balls attaining their speed fly out, rub on the disk B, the plane of which is set at right angles to the governor's axis of rotation, and by their friction prevent any increase of speed. The lower bearing of the governor's spindle is attached to a sliding-piece C: this slide is worked by a cam, *f*, actuated by the wheel and sector, *g*. The spindle of this sector projects in front of the clock-frame, where it carries an arm playing round a graduated arc, and kept in any required position by a pin and a circle of holes. The initial adjustment of this clock is made by the nuts *h*, *h'*, which alter the working length of the conical pendulum, and the screws *e*, *e'*, which alter its working angle. When the clock is adjusted to sidereal time, these require no alteration; for small differences, as in the case of planets, are corrected by the cam apparatus without stopping the clock. For the moon, the change to mean lunar time is made by a set of differential wheels, which are brought into play by the simple movement of a lever in an instant, and the final adjustment for the lunar rate at the time of observation is made by the cam.]

Mr. Grubb had intended to supplement the action of this governor by an elegant contrivance controlled by an ordinary seconds' pendulum, which was to add or subtract motive power as the clock lost or gained, but on trial we did not consider it necessary. The clock has a great excess of power over any probable resistance that may

come into play, and we found that the doubling its actual driving-weight (200 pounds on a single line) accelerated its rate only $\frac{1}{360}$. It rings seconds.

But the excellent action of the clock would be marred unless it were matched by a corresponding accuracy of the screw and sector through which it drives the telescope. These were finished with extreme care, and are believed to be as exact as many dividing engines. The means of obtaining an accurate screw are well known; but it may be useful to describe the process of cutting the sector.

[The arc of this, which is of gun-metal and 5-feet radius, after being duly fitted to the frame of the sector, had several faint arcs described near its edge concentric with the polar axis. It was then detached and transferred to a large wheel-cutting engine, where it was carefully centred by means of the faint arcs just mentioned, using a micrometer-microscope. The proposed run of the sector was two hours, or sixty teeth (the screw being intended to revolve once in two minutes). It was, however, purposely made long enough for seventy teeth, and so many radial lines were drawn on it, using the wheel-cutting engine as a dividing one. Subject to its errors, these divisions represent two minutes of time.

The second step in the work was to take sixty-four of these divisions, and to determine their *relative* errors by the process of continual bisection, using a pair of micrometer-microscopes. The third step was to cut from the erroneous divisions, using the table of errors and a micrometer-microscope, a set of corrected divisions.

The fourth step was to cut the teeth from these last divisions; the cutting part of the engine being provided with a clamp and tangent-screw, and also a micrometer-microscope; the last cutter bringing the teeth as nearly as possible to the shape required for the application of the endless screw. The gun-metal arc was then restored to its place in the instrument, and a racked screw of the calculated pitch being passed a few times through the whole extent of the teeth, they were brought to the precise shape required for working with the screw of the clock. The quantity of metal removed in this last process was barely perceptible.

The telescope has nine Huyghenian eyepieces, ranging from 220 up to 1000. It has also a micrometer, whose wires are illuminated in a dark field, and which is also provided with fine steel bars as more fit for faint nebulæ than wires. It is similar to a pair made many years ago by Mr. Grubb for the Armagh Cassegrain, and for the late Lord Rosse. Plate VIII. fig. 32 is a down view of it; fig. 33 is mostly sectional, and fig. 34 shows the sliding-pieces or forks which slide in V-grooves planed in the under side of the micrometer-box. The figures are half size. The micrometer-screws have twenty-five threads to the inch, and the number of their revolutions is read off from scales through A, A, glazed openings in the covering plate, fig. 32.

In fig. 33, C is a section of the part which screws into the telescope; to this the micrometer is attached by the ring D, so as to admit of being turned in any direction. E and F are rings connected by the bars G; outside and around the lantern part formed by these bars, revolves a short tube H, to which is attached a conical tube I carrying the

illuminating lamp; and inside the lantern, but without touching its bars, stands the concave illuminating reflector K, supported by two studs, L, which are attached to the ring M. This is on its upper edge cut into contrate teeth, which are acted on by the pinion and milled head O.

The nozzle of the lamp being made to turn in the conical tube I, and this latter being (by means of the tube H) revolvable on the lantern part, it follows that the lamp can be set vertical in any position of the telescope, while the reflector K can be revolved by the milled head O till it throws the lamp's light on the micrometer-wires. Thus one lamp is, by very simple management, in every case sufficient. In the Cassegrain telescope there is an eyehole so adjusted that it excludes all light except what comes from the object-speculum: now if the central opening of K be no larger than what will pass that light, it is evident the field will be dark, while the rays of the lamp intersecting in the focus of K will illuminate any object placed there, and of course the wires in whose plane that focus is. The same can be arranged, though not so easily, for a Newtonian. N in fig. 32 is a large milled head, by turning which, Q, the box of the micrometer is made to revolve. It works a pinion which acts on the wheel R, fig. 33. S in both figures is the position-circle reading to a minute by the verniers T, T'. The optical part is peculiar, and such as had been long since applied by Dr. ROBINSON to the micrometer of the Armagh Cassegrain to obtain a larger field than was given by the usual positive eye-pieces. A field glass P, common to all the eyepieces, is placed at a proper distance before the wires, and the eyepieces are single lenses placed behind them. This greatly enlarges the field of view without producing any sensible distortion of the image in a Cassegrain*. The micrometer is provided with powers from 300 to 600.]

In using it the small speculum must be fixed in the position which brings the third image into the plane of the wires, and focusing must be effected by sliding the eye-lens.

What we have described comprises all which was included in the contract, but it seemed to the Committee that the instrument would be incomplete without a photographic apparatus and a spectroscope, and they ventured to add these on their own responsibility, sanctioned by the opinion of the President and Council. Some photograms of the moon and stars taken with a temporary apparatus, were considered by the highest authority in this matter, Mr. WARREN DE LA RUE, to be of such good promise that they directed a very complete one to be provided similar to the one used by that astronomer, with only such modifications as the great bulk of the telescope makes necessary.

[The most important details are shown in figures 35 to 38, of which the first two are on a scale of 1 inch to the foot. Fig. 35 is a plan, and fig. 36 half section, half elevation of it. A A A is the angle-iron forming the upper end of the telescope, to which is attached the steel tripod frame B B B, by the thumb-screws x, x, x. This tripod supports the pair of brass tubes e, f; the steel arms being connected with the outer tube

* In the Armagh instrument the values of a revolution of the screws, deduced from ten, twenty, and thirty revolutions of them by transits of circumpolar stars, differ from the mean, $-0''{\cdot}028$; $+0''{\cdot}054$; and $-0''{\cdot}026$, which are quite unimportant.—R.

e by the gun-metal rings C and D (fig. 36). The frame carrying the prepared plate is attached by a convenient arrangement to the upper end of the tube *f*, and the exposing-shutter at the lower end, D, is worked by the lever and shaft, K, shown more fully in the two next figures (figs. 37 & 38). The adjustment for focus is made by the milled head *g* (fig. 26), working the rack and pinion *h*, while a scale of divisions, *s*, is used to register the exact position of the focus. Fig. 37 is a plan, and fig. 38 an elevation of the exposing-shutter, each one-fourth of the full size. B, B, B (fig. 37) are, as before, the three arms of the tripod frame. C is the lower of the two gun-metal rings to which these are bolted. To this ring is attached, by two milled-headed screws E, E, the gun-metal frame D (shown hatched in the figure) which carries the shaft *c* in a pair of bearings *f*, *f*, to which is attached the thin sheet brass shutter F; *a* is the shaft lettered K in fig. 36, which is acted upon from below by the lever K′, fig. 36, and a pair of parallel cords passing down the side of the tube. This shaft, in turning through 90°, communicates a similar motion to the shaft *c*, which carries the shutter by the pair of cranks *b*, *b* and the connecting-rod *d*. As it is desirable to be enabled to open the shutter from either the right or left side to give a greater or less exposure to a particular side of the moon, this shutter is so arranged that, by taking out the two screws E, E, the frame D, which carries the shutter F, can be turned round 180° and fixed in that position; while the connecting-rod *d*, being disconnected by taking out the stud *x*, is thrown to the other side and attached to the spare crank *b*′ at the other end of the shaft *c*. At the same time the edge of the shutter can be adjusted parallel to the terminator of the moon by the rocking-piece G. The necessity, or at least the advantage of this arrangement, was shown in the experimental trials already mentioned; for it was found that to obtain a tolerably uniform picture of the whole moon, many times more exposure was required for the parts near the terminator than for the bright edge. The apparatus includes a photographic micrometer similar to that described by Mr. W. De La Rue in his "Account of the Solar Eclipse of July 18, 1860," Philosophical Transactions, vol. clii. page 373, for tabulating the Photograms.

The spectroscope has some modifications of the common construction, which are intended to make its adjustments more permanent. This is specially important in the present instance, because, as the telescope's equivalent-focal length is very great, a minute shift of the collimator, the prisms, or the observing-telescope would make it extremely difficult to find the object's spectrum. It is also desirable that the reading off the places of lines should not be embarrassed by continual changes of index error or prism adjustment, and that the instrument should be able to bear free handling, especially in the dark. Figures 39, 40, and 41 explain the construction, especially those parts which are new. Fig. 39 is half size, figs. 40 and 41 full size. In fig. 39, the part from A to B (shown in section), being of the usual construction, requires no description: the cylindric lens, of 6-inch focus, is of course achromatic and aplanatic. C is a strong cylindric box which holds the prism, and also serves to connect firmly the collimator and observing parts of the instrument. It effectually excludes false light, and protects the prism from

dust or moisture. D is one of the prisms *in situ*. E is the observing-telescope, 1·1 inch aperture and 4·5 inches focus (the collimator is the same). The manner in which it is mounted is more fully shown in figs. 40 & 41. The first of these shows in section the prism-box C; L is a steel centre firmly fitting the bottom of C; on it turn the arms F and G; F being outside the box and capable of being turned into any position, G turning only through a limited arc, equal to the dispersion of the most powerful prism, by a slotting aperture in the side of the box, and carrying the telescope E. K is a circular plate, serving to retain the arm G in its place, and also to support the prism. The manner in which G and F are connected is shown in fig. 41. A portion of G is ratched, in which works the endless screw H, having a micrometer-head to measure fractions of a revolution. The teeth were cut upon an accurate engine, each being fifteen minutes of arc, and the micrometer-head is divided into sixty parts, each of which is therefore fifteen seconds. The revolutions are read on the divided arc seen in fig. 39. It will be obvious from the firmness with which these parts are put together that very little variation of the readings is to be expected. Three prisms are provided, of which two are constructed as shown in fig. 39. The central prisms are of CHANCE'S extra dense flint, to which are cemented outer prisms of light crown. The first of these has the angle of the flint $=100°$, those of the crown 28°; its deviation for D=48° 3′, and its dispersion from B to G′=5° 5′. The angles of the second are 90° and 29° 15′, its deviation for D=34° 40′, and its dispersion =3° 23′.

The third prism is an ordinary one of dense flint, its angle being 59° 50′, its deviation =48° 15′, and its dispersion=2° 59′. Each prism is permanently fixed to a disk of brass, which by means of steady pins can always be attached to the table K in the same position, and retained there by the milled-head screw F, fig. 39. This method of mounting permits the prisms once adjusted (to minimum deviation for the line D), to be readily placed, even in the dark, at a definite position with respect to the line of collimation. The compound prisms have nearly the same deviation as the simple one, which is very convenient for the observer: they might have been made exactly the same, but it was not thought necessary to fulfil this condition, as the difference is easily provided for. As F and G (fig. 40) turn on the same centre, a removeable steady pin, with as many holes in the former as there are required positions, connecting it with a hole in C, will be quite sufficient.]

As to the performance of this great telescope, it is unnecessary to add anything to the report which the Committee has presented to the Royal Society*, and Mr. LASSELL'S letter to the President. Its light is fully proportioned to its aperture; and in definition on a fine night it is not surpassed by any telescope with which I am acquainted. The movements on its axes are effected with a facility and precision which, when one considers its enormous weight†, seem almost incredible; and it appears to keep its adjustments with admirable steadiness. If, as I trust, it arrives safe at its destination, it will fully repay the magnificent liberality and public spirit of those who ordered its

* Proceedings of the Royal Society, vol. xvi. page 313. † See Appendix, No. III.

construction. It is impossible to contemplate without enthusiasm the treasures of discovery which are opening before the accomplished astronomer who is about to sweep, with an instrument that has but one superior in the world, a sky whose wonders only one has yet explored, in a climate such as British astronomers can only dream of. What strange nebular forms it may reveal, what new configurations of stars, what new cosmical laws it may unfold, what new elements it may disclose, we soon shall know; and the fulfilment of these high anticipations will I am sure be the best compensation to the maker of this noble telescope for all the thought and toil which were required to make it what it is; and also, may I be permitted to say, to the three whom the Society associated with him in this great work, and who, though in a far inferior measure, shared with him the intellectual labour and the anxieties connected with the task. Even the practical experience which has been gained in its construction is a precious addition to our knowledge: and in this belief we have thought it right to put on record its leading facts. They who, as I hope, will hereafter try to equal or surpass the Great Melbourne Telescope will not think these details tedious. Even they who have no special interest in optical or mechanical science may be reminded that every well-established fact, everything which extends man's control over the inert masses or the energetic forces which surround him, is an element of power, is an agent whose potency may extend far beyond his ken, is an everlasting possession of the human race.

Appendix No. I.

Rate of Cooling of the Specula.

[Fig. 42. A represents the temperature-curve of the first speculum, B of the second, and C the mean of the two. The abscissæ are the number of days from the casting to the removal of the specula from the oven; the ordinates are the degrees of the galvanometer. The temperature of A when removed was 70°, about 5° above the air, that of B was 72°, about 20° above the air. The oven, when they were introduced, was believed to be between 900° and 1000°.]

The irregularities in these curves are probably due to changes in the masses of iron which were in the vicinity of the galvanometer. The equatorial was about 200 feet distant, and underwent little change at this time; but other parts were in process of construction at about 60 or 70 feet distance.

No. II.

[Some facts as to the time of grinding the speculum A may be useful. It was commenced October 27. In four hours two-thirds of the surface was in action; in fourteen the whole surface was working, a proof how truly it had been cast. On October 29 a small hollow, 0·25 inch diameter and 0·043 inch deep, was made by a copper drill and emery. When measured next morning the amount ground off was known. This was repeated daily, and it was found that

The quantity ground off in twelve hours	= 0·0193
Total number of hours at this surface	250
Total quantities ground off	0·401 inch.
The total number of hours grinding was—	
Rough grinding face	250
Ditto back and edge	400
	650
Fine grinding	200
Removing a scratch	120
Altering focus 9 inches	200
	520
Total rough and fine	1170

This is equivalent to 2050000 strokes of the machine at thirty-three per minute for rough, and twenty-four for fine grinding.]

No. III.

Approximate weight of the principal moving parts of the Telescope.

	lbs.
[Speculum and box	3500
Tube boiler-plate	1300
Lattice	1370
	2670
Polar axis	3200
Declination-axis	1500
Cradle	1100
Counterpoises	4700
Smaller portions	1500
Total	18170

This is very near the original estimate.]

Appendix *to the Description of the Great Melbourne Telescope.* No. IV.

By T. R. Robinson, *D.D., F.R.S. &c.*

Received February 10, 1869.

Since this Paper was read I have made several observations of the quantity of light transmitted by object-glasses, and determined the index of absorption in various specimens of glass. The results of some of these are in accordance with the opinion which I expressed there; but others present a difference which is very satisfactory as indicating a surprising progress in the manufacture of optical glass.

The photometer which I used is that of Zöllner, respecting the arrangement of which a few words may be useful. On looking into it, the observer sees a luminous

circle divided by a vertical diameter into two semicircles, one of which is illuminated by plain, the other by polarized light. Supposing them equally bright, if a piece of glass be interposed in the plain beam, its semicircle becomes fainter, and the brightness of the other must be reduced so as to equal it by turning the eyepiece, in which is a Nicol's prism. The ratio of the intensities is known from the position (θ) into which it is turned. Or the equalization may be effected by changing the distance of one of the lights; and the intensities are as the inverse squares of the distances. I used this occasionally as a check, but do not consider it as accurate as the other. The circle of the eyepiece has two verniers reading to five minutes: this is sufficient; for an error of $2'{\cdot}5$ in the angle will in its maximum effect produce one in the measure of intensity of only 7 in the fourth place of decimals, while the uncertainty of the eye in estimating the equality of the illuminations is thirty-five times as great. I shall return to this immediately.

The photometer is fixed on a board furnished with parallel guides in which the base of the moveable light slides. A block for supporting glasses is also fitted to these guides, and its upper surface is ruled with lines parallel to them, so that any object can be surely placed on it in a given position. The lights were petroleum lamps with the edge of the flame turned towards the photometer: when the wick is not too high, and they have been burning for about twenty minutes, they give an intense and tolerably constant light. The verniers are adjusted to read 0 when the polarized semicircle disappears entirely, and are set to 90° when the lights are adjusted to equality. This equality is verified for each observation; then for any azimuth (θ) the intensity of the light transmitted by the glass $= \sin^2\theta$. If the lights be of the same colour, the bounding diameter disappears completely and the process is comparatively easy; but if there be any notable difference of tint, it is much less satisfactory. In such cases I found, as Fraunhofer had done long since, that one must make this diameter as faint as possible —a fact which implies that the two beams, whether equal in intensity or not, make nearly the same impression on the eye.

I must add that within the last few months my eyes have ceased to be fit for such delicate work, and that most of these comparisons have been made under my inspection by my Assistant, Mr. Charles Faris, and occasionally by Mr. Howard Grubb, both practised and good observers.

In examining object-glasses, they were placed in contact with the emeried disk of the photometer, so that no correction is required for any convergence of the light due to them. The results for them are given in the following Table I.

Description.	Aperture.	Focus.	Intensity.	Obs.
	in.	in.		
a. Triple object-glass	2·75	48	0·5497	15
b. Double	3·80	63	0·5962	5
c. Double	3·25	48	0·6567	
d. Double	6·50	96	0·6772	12
e. Double	5·50	...	0·7928	24
f. Double, inner surface cemented	5·0	...	0·8739	24
g. Double, cemented	12·0	222	0·8408	24

I regret that I had not access to any object-glasses by FRAUNHOFER* or MERZ. Of the above, *a* belongs to the Armagh Observatory; it is by one of the DOLLONDS, older than 1790, and from its small proportional aperture, it is probably one of their first attempts at a triple combination. It is deficient in light, but defines very sharply. *b*. is the original object-glass of the Armagh circle, extremely sharp; but as it could not in this climate be depended on for observing stars below the 8-inch magnitude, it was replaced in 1861 by a remarkable one made by Mr. GRUBB, of the same focus, but 7 inches aperture; it was made by TULLEY about 1828; the crown is greenish, and I suppose English; the flint was, I believe, from Daguet. I was much surprised at the great absorption of this glass, and therefore took several sets of measures; but they all told the same story. *c* was made for me by TULLEY in 1838; its glass is French, the crown is greenish. *d* is by CAUCHOIX. It came into my possession in 1837, but I have no information as to the maker of its glass; the crown is greenish, and has probably a high n, but its mean thickness is only only 0·39. *e* is by Messrs. COOKE; it had belonged to the late Judge BERWICK, an excellent and accomplished man, who perished in the Abergele collision; I do not know its date, but believe it recent; the glass is CHANCE'S. *f* is by GRUBB, the glass CHANCE'S: the very high transmission of this lens is in part due to the cementing of the adjacent surfaces, which, while it makes more difficult the correction of spherical aberration, removes almost entirely the reflection at a surface of crown and one of flint: the factor for this $=0{\cdot}9036$; and if the I be multiplied by this, we obtain 0·7896, nearly that of *e*, the difference being due to the reflection at the film of cement. *g* is also by GRUBB, and cemented; its glass is by CHANCE.

On examining this Table, the progressive increase in the light of the object-glasses is evident. The first two, which may be considered good specimens of the early achromatics, have less illuminating-power than the Herschelian reflector. A great advance was made by GUINAND and those who followed in his steps; and a still greater one by CHANCE, whose glass is nearly perfect as to colour and transparency.

The same inference follows from my measures of the index of absorption n, which, though not as numerous as I wish, are sufficiently significant. The specimens which I examined were, with two exceptions, prisms; and this form is very convenient. If a ray is incident on an isosceles prism parallel to its base, it emerges parallel to itself after reflection at the base. If A be the angle of the prism and R the refraction in this case, t the course of the ray in the prism $=$ base $\times \frac{\cos \frac{1}{2} A}{\cos R}$, μ can be found by one of the angles, and from it φ^s computed for an incidence $=\frac{1}{2}$ A. I assume as the mean reflection that due to the μ of E; it really is near *b*, but the greater illuminating-power of the yellow rays compensates for this. The due position of the prism is ensured by adjusting its base to the lines on the block already described.

I give the results in Table II., in which are introduced those given in the paper that

* March 9.—The President sent me his Fraunhofer, 3·2 aperture and about 45 focus. By 16 observations agreeing well, its coefficient $=0{\cdot}7393$.

they may be referred to at once; and I add to them one which I found in BOUGUER's "Traité d'Optique," which seems trustworthy.

Description.	ρ^2.	t.	I.	n.	Obs.
1. Prism, originally Captain Kater's...	0·9164	1·125	0·746	0·1829	5
2. French plate, Mr. Grubb		0·75	0·805	0·1728	
3. London plate, Mr. Grubb		0·30	0·86	0·2140	
4. Two of same, Mr. Grubb		0·60	0·77	0·1446	
5. Prism, Mr. Grubb.....		2·00	0·81	0·0617	
6. Bouguer's glass	0·9109	3·200	0·500	0·1895	
7. Gassiot's prisms.....	0·8380 0·9763	1·915	0·2490	0·6209	5
8. Prism by Dubosq, flint.....	0·8878	1·730	0·6844	0·1504	12
9. Prism by Merz, flint.....	0·8647	1·580	0·7549	0·1089	5
10. Prism by Merz, crown	0·8935	2·431	0·7953	0·0858	5
11. Prism by Merz, flint.....	0·8854	1·500	0·7550	0·1065	24
12. Prism by Grubb	0·8696	2·721	0·8196	0·0218	6
13. Cylinder of crown.....	0·9166	4·300	0·8155	0·0272	12
14. Cylinder of flint.....	0·8907	4·400	0·8563	0·0090	6

No. 1 was shown to me in 1830 by Captain KATER, as the *chef-d'œuvre* of the Glass-Committee; he used it as the small speculum of his Newtonian. Afterwards it came into the possession of the late Lord ROSSE, who made the above measures with BUNSEN's photometer in 1848. It is English plate, greenish. It is curious that its n should be so near No. 6, a century before.

Nos. 2, 3, 4, 5 were measured by Mr. GRUBB in 1857. No. 5 was a prism of 90°. He does not remember its history; but evidently it was of CHANCE's glass.

No. 6 is described by BOUGUER as "glace," 3 Paris inches thick. It was probably that of St. GOBAIN, which I suppose has not varied in composition, and I have used *its* μ in computing ρ^2.

No. 7 is two prisms of 60°, which Mr. GASSIOT, with his wonted kindness, entrusted to me for some inquiries about the improvement of the spectroscope. They are by MERZ, of glass which seems nearly identical with FARADAY's dense glass, having a specific gravity of 5·1, and a mean μ=1·7664. It is very pellucid, but, like its prototype, has a yellowish tinge, which I suppose is given by the large proportion of lead. As MERZ does not polish the base or ends of his prisms, I could not use my usual method, but I put them together with the angles opposed, and a drop of olive-oil between *. The incidence could not be perpendicular, as there was a partial reflection at the oil, but a full beam passed at an incidence of 30°. The great absorption is remarkable, and cannot, I think, be explained by the colour of the glass. The side of the largest of these prisms=2·60.

No. 8 is of 60°; its side=1·90, and its μ for E=1·6200. It is free from colour, and an evident improvement on the earlier ones.

No. 9, a prism of 90°, was given to me by Dr. LLOYD for a small mirror in the Newtonian form of the Armagh 15-inch reflector. I was surprised that it had so little superiority over the metal one; but the I explains this. A prism of No. 14 would have told a different story†. Its base=2·01, and its μ for E=1·6188.

* The second ρ^2 given is that of the film of oil.

† With such a prism the coefficient of the Newtonian would be 0·548.

No. 10, of 90°, was obtained by the late Lord Rosse to be similarly used in his 3-feet Newtonian. Like me, however, he was disappointed. Its base=3·05, and its μ for E=1·5321.

No. 11, of 60°, obtained by me at Munich in 1837. For these measures I had the ends polished flat; its μ for E=1·6405.

These three show considerable progress, and an object-glass made of such materials would have a great power of transmission, though much behind the following.

No. 12 is of 90°. Its glass is from Chance; its base=2·65, and its μ for E=1·6216.

No. 13 is a cylinder 2·2 inches in diameter and 4·3 long, which Mr. Grubb obtained from Messrs. Chance for these measures; its μ for E=1·5200.

No. 14 is a cylinder got at the same time, 2·1 inches in diameter and 4·4 long; its μ for E=1·6126; the ends of both are polished flat, and they are of wonderful transparency.

I was nearly as much surprised at the very low value of n in these three last specimens, as I was at the great absorption of the object-glasses a and b in Table I. But the following considerations will show that these measures, especially in the two last, must be very near the truth. I find from 110 observations, in which θ ranged from 29° 23′ to 75° 27′, that the probable error of a single determination of I by the Zöllner=$\pm$0·0251. As $dI=d\theta\times\sin 2\theta$, it may be inferred that the probable error of the single observations for the I of No. 14=0·0157. Again, $dn=\frac{dI}{1\times t}$; and the probable error of this $n=\pm$0·0019, only a fifth of its actual value, so that its correctness has a high probability. This is confirmed by the values of I in the object-glasses e, f, and g, which would not be possible if n was much larger. For instance, if it were=0·1, the I of g would be 0·7585 instead of 0·8408.

If, as I see good ground for hoping, Messrs. Chance shall succeed in manufacturing large disks of the same perfection as these two cylinders, my comparison of the achromatic and the reflector must be considerably modified. I think we may assume $n=0{\cdot}02$ as the highest excellence likely to be attained on such a scale; and if this be introduced into the expression given in the paper for the intensity of the achromatic, it becomes

$$I=\log^{-1}(9{\cdot}90964)\times e^{-A\times\log^{-1}(7{\cdot}65482)}.$$

If this be multiplied by A^2 and equated to $0{\cdot}401\times 48^2$, the quantity of light transmitted by a 4-feet Newtonian, we have an equation which when solved gives 35·435 for the aperture of an equivalent achromatic. This aperture would be diminished if the process of cementing were found applicable to lenses of such magnitude; but if such an object-glass were ever attempted, its focus would probably be much shorter than eighteen times its aperture, and therefore its increased thickness would produce a contrary effect.

I shall conclude with suggesting that, as very slight variations in the manufacture of glass seem to make great changes in its absorptive power, it would be prudent to examine the value of n in the disks intended for lenses of any importance. This could be done by polishing a couple of facets on their edges, and need not involve the sacrifice of many minutes.

IV. *The Formation and Early Growth of the Bones of the Human Face.* By George W. Callender, *Lecturer on Anatomy at St. Bartholomew's Hospital. Communicated by* J. Paget, *F.R.S.*

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I propose in the following notes to relate some observations respecting the formation and growth of the bones of the human face. It is not my purpose to consider minutely the process of ossification, but simply to mention such of my experiences as differ from the commonly received statements, or which seem to supply information regarding some few points with which we are as yet imperfectly acquainted, such as the growth of the maxillæ, and the formation and eventual obliteration of the intermaxillary bones.

Of the cartilaginous and membranous structures of the cranium, before ossification has commenced, two processes of cartilage, trabeculæ cranii of Rathke, extend into the anterior division of the skull, forming by their union the frontal nasal process, and from between which the internasal cartilage is derived. Lateral masses, inflected lateral cartilages, represent the future ethmoid and inferior turbinate bones.

The internasal cartilage bending or arching over on either side forms plates on which the nasal bones are developed, and is connected in front with the cartilages of the nose. In a fœtus nine-tenths of an inch long* it is joined to the lateral cartilages by

* This and all the specimens subsequently referred to are in the Museum of St. Bartholomew's Hospital. The figures, 3·5 for example, are used to indicate the length of a fœtus in inches and tenths of an inch. It seems best to distinguish each fœtus by its length; the following Table gives, as nearly as it is possible to determine it, the corresponding age in days and weeks in twenty cases.

Length of fœtus, in inches.	Age, in weeks.	Age, in days.
·12	3	7 to 14
·4	6	35 to 42
·5	7	42 to 49
·7	8	49 to 56
·9, 1·1	9	56 to 63
1·5	10	63 to 70
2·3, 3, 3·5, 3·7, 3·8, 4	12 to 16	84 to 112
4·3, 4·7, 6·5, 6·7, 7	16 to 20	112 to 140
9, 10	22 to 24	154 to 168

membrane, which extends from the lower edge of the internasal plate, with the perichondrium of which it is continuous, to the inner side of either lateral mass over which it is prolonged. By such membrane the passage of either nostril is completed, the anterior orifice being bounded by the cartilage of the nose above, by the extremity of the internasal cartilage in the middle line, and between these structures by the connecting membrane. Where the latter passes off from the septum, there is an elevation of tissue on either side of a central depression.

In a human fœtus four-tenths of an inch long the frontal-nasal process is still distinct; the lateral-nasal and the maxillary lobe, united, approach it on either side, most nearly above, and the maxillary lobe can be traced back as a slight elevation, the palate, towards the middle line (Plate XIII. fig. 1). On raising the frontal-nasal process the central part of its under surface is found to form two thickened masses, and represents the nasal cartilages, and from either side of this central part a narrow ridge curves downwards, the cartilage for the future ethmoid and turbinate bones (Plate XIII. fig. 2). These central parts growing forward are joined, beginning from above, to the maxillary lobes in a fœtus nine-tenths of an inch long, the union of opposite sides to form the palate taking place gradually from before backwards. In a fœtus 1·5 the union of the maxillary lobes is not yet complete, a fissure still extending through the soft palate.

If in a fœtus nine-tenths of an inch long the superficial parts are removed, that portion which forms the superior maxilla is seen to occupy the position, and to have the shape of the future bone, the orbital, palatal, and alveolar parts being defined, whilst two considerable projections indicate the position of the nasal and of the incisor processes, the last named lying immediately above the elevation, or thickening, at the anterior extremity of the internasal cartilage. The mass of the superior maxilla consists of cells crowded together, having well-defined outlines, and being filled with granular matter. After the development of the bone its subsequent growth is effected in the membrane which covers its various surfaces, and in the membranous part of the nasal wall.

Ossification and Growth of the Superior Maxilla.

No ossification has taken place in the superior maxilla of a fœtus nine-tenths of an inch long, but in a fœtus 1·1 numerous bony deposits are visible. These independent bone formations appear in that portion of the maxillary lobe which has been described as moulded to the shape of the upper jaw. The most conspicuous are ranged below the infraorbital plate, and stud the alveolar border and the nasal process; but although ossification commences at many distinct points in these parts of the bone, the rapidity with which the separate ossifications are fused makes it undesirable to name each as a distinct centre. In a fœtal pig 2 inches long (Plate XIII. fig. 3) the superior maxilla, besides a well-formed palatal portion, consists of osseous deposits which seem to radiate from a line corresponding with the base of the alveoli, and from a centre situated immediately below the infraorbital notch. A somewhat similar condition is observable in the mammary fœtus, nine-tenths of an inch long, of a Kangaroo.

There is no evidence in the human fœtus that the palatal-alveolar is ever a distinct process, although its ossification occurs a little less early than that of the other portions. In a fœtus 1·5 bone-granules are seen ranged along the edges of the alveolar plate, which they surrounded like a ring, and extend into the back part of the palatal process; but whilst the nasal and orbital portions of the bone are comparatively well shaped, the palate is but a ridge standing in towards the middle line, and is separated by a wide angular notch, the base of which is towards the middle line, from the incisor process.

In a fœtus 2·3 the bone presents the following appearances. Taking a point below the infraorbital notch as a centre, three processes are traced from it:—(1) The nasal forms a thin plate having the permanent shape of the process, but the ridge for the turbinate bone, the vertical thickening of its lower nasal surface, and the nasal groove are wanting. From its base a smooth plate of bone passes forward, the centrebit, as it were, of the incisor portion, chief part of the roof of which it ultimately forms, and to this hitherto, I believe, unrecognized portion the name of incisor process may be rightly given (Plate XIV. figs. 5 & 7). A channel or groove ascends from, and is continuous with, the fissure between the incisor process and the front edge of the palatal, and ends about halfway up the inner surface of the nasal process. (2) The palatal-alveolar extends with a slight slope inwards and backwards. Its nasal surface has no distinguishing mark, but on that of the palate a slight ridge indicates the situation of the base of the inner alveolar wall. (3) The orbital, divided on the upper surface from the palatal-alveolar process by a deep groove (narrow in front, where it forms the infraorbital fissure, and comparatively wide posteriorly), and continued backwards above the alveolar plate to end in an articular surface for the malar bone.

Below and anterior to the infraorbital fissure the maxilla is deeply notched to form the roof of the canine socket; but there is no such notch, the lower border being uniformly level in the maxilla of a fœtus 1·5.

A fœtus 4·7, a second 6·7, and a third 9 inches long may be selected as illustrating the changes which occur during the early growth of the bone (Plate XIV. figs. 5 to 11).

The outer surface of the nasal process becomes thickened and strengthened by bone deposits, which spread out above and laterally, forming in a fœtus 4·3 the anterior wall of the sockets for the incisors, covering the canine notch, and reaching posteriorly beyond the margins of the infraorbital fissure. In a fœtus 6·7 they constitute a ridge below the facial portion of the orbital plate for the outer wall of the alveoli of the molars, which gradually deepens but only reaches the posterior part of the alveolar plate in a fœtus 9 inches long. In the same order the inner wall of all the sockets is formed, but the inner and outer walls have not quite united posteriorly in a fœtus 9 inches long. These plates of bone, excepting the inner and median walls of the incisor sockets, are produced in membrane continuous with that in which the gums are formed.

The canine socket is isolated by lateral partitions of bone in a fœtus 4·3, the incisors begin to be separated in a fœtus 6·7, but the division between the lateral and middle

sockets is far from perfect in a fœtus 9 inches long, although the plate forming the inner wall of the middle incisor is complete in a fœtus 6·7. In the latter, too, the division between the bicuspids and molars begins to be apparent.

The canine socket is formed in the body of the bone, the incisor sockets grow down below the incisor process, those of the bicuspid and molar teeth are alone connected with the distinct aveolar plate.

The palatal-alveolar process in a fœtus 4·3 has lost its uniform appearance on the nasal side. Plates of bone in waving lines have thickened the upper surface of the palate, and ascend the posterior margin of the groove on the inner side of the nasal process, and assist in forming the nasal duct. By the turning up of some of these bone plates, which are formed in the membrane extending from the ethmoid and inferior turbinate cartilage to the septum, that portion of the maxilla which is the inner wall of the antrum is now formed. A second plate, constituting the outer wall of the antrum and part of the floor of the orbit, rises with an inward slope from the inner edge of the orbital process, whilst deposits of bone are accumulating on the orbit, chiefly on either side of the infraorbital fissure. The growing down of the alveolar walls alone marks the increase of this part of the bone on its palatal surface. The infraorbital fissure is still open in its entire length in a fœtus 9 inches long, but in the anterior half of the orbital plate it forms a deep canal in consequence of the deposits of bone on its either side.

The antrum is at first of large comparative size. In a fœtus 4·3 its greatest width is nearly one-tenth of an inch, the palate being at this period just over one-tenth of an inch wide. In the adult, owing to the rising of the orbital plate and the hollowing out of the zygomatic process, the antrum is 1·2 inch wide to ·7 for the palate. In a fœtus nine inches long, in consequence of the rapid increase in the width of the palate, the antrum is only as ·1 to ·2, but a deep fissure (Plate XIV. fig. 8) shows where the orbital plate is to be lifted up to allow of the lateral and vertical expansion of the cavity of the antrum.

Of the Vomer and Intermaxillary Bones.

These bones are developed in the membrane which covers the internasal cartilage, and forms part of the boundaries of the sides of the nostrils, Ossification is commonly said to commence in the posterior part of the vomer. This is not the case. In a fœtus 1·5 this bone consists of a keel from which spicula bud like ribs on either side of the cartilage of the septum. Those in front are most advanced in their development, those behind diverge from each other as they pass backward, as is shown in fig. 4, Plate XIII., which represents the vomer of a fœtus 2·3.

At the anterior extremity of the vomer the process of ossification extends to form the intermaxillary bones.

These bones, though their appearances vary in different mammals, have amongst these animals certain common features. (1) The line of articulation with the superior

maxilla is in front of the canine socket when the latter is present, when absent the relation of the suture to the maxilla virtually remains unaltered. (2) The præmaxilla, as it is more correctly named for these animals, is prolonged superiorly to a point which ascends to a varying distance, or is cut short abruptly. In any case, whatever be the length of the posterior margin, a groove is found in the anterior edge of the superior maxilla; and into this the wedge-shaped margin, broad or narrow (sometimes almost incisive, as in the sheep), of the præmaxilla is inserted, and is thus embraced, as it were, by the front border of the upper jaw. (3) The præmaxillæ carry the incisor teeth, when present. (4) They articulate in the middle line, sometimes leaving a notch in front for a continuation forward of the septum. (5) They form more or less of the front of the palate, of the boundaries of the incisive foramina, and of the complete or partial septum which separates them (see Plate XIII. figs. 9 & 10, Plate XIV. figs. 1, 2, 3, 4).

On the superior maxilla during fœtal life there may be seen in many animals, in front of the canine socket, a slight projection which appears to be analogous to the incisor process of the corresponding human bone. This may be observed in a fœtal Pig (Plate XIII. fig. 3). In the mammary fœtus of a Kangaroo also there is a short thick process which projects considerably forwards and downwards in remarkable contrast with the corresponding but elongated portion of the adult bone (Plate XIII. fig. 5).

Passing to an examination of the human fœtus, it is evident, from the shape and direction of the incisor process (which is best examined in a fœtus 4·3, being then easily detached from the intermaxilla), that it passes across the anterior boundary of the nostril as the latter is continued forward to the middle of the lip (Plate XIV. figs. 5 & 7). This boundary above is partly covered by the nasal process, below the palatal portion of the superior maxilla ends abruptly behind it, and between the two it is that the incisor process crosses, and indents the orifice of the nostril.

The membrane inflected by the incisor process extends from the middle line (where it is continued forward to be connected with the process of the maxillary lobe which joins it on either side to complete the upper lip), partly below and then behind the incisor process, fills the fissure between that and the palatal, and is continued up behind the nasal process; the groove on the inner surface of this process corresponds with the edge formed by the bending in of the anterior membranous boundary of the nostril, and in the membrane continued back from the angle thus formed there is developed the plate of bone which constitutes the anterior inner or nasal wall of the antrum and the channel for the nasal duct, whilst in the membrane which lies anterior to the groove the intermaxilla of either side originates.

In a fœtus 2·3 the intermaxilla consists of deposits of bone about the posterior edge of the incisor process, which subsequently grow down to form the plate of bone on the inner side of the middle incisor socket, and the posterior wall of the incisor sockets below and internal to the course of the incisor branches of the dental nerve. The front

wall of the middle and lateral incisor sockets is continuous, as previously stated, with the plate of bone which covers in the canine notch.

In a fœtus 4·3 the intermaxilla is completely formed and may be traced as a distinct bone (Plate XIV. fig. 6). It consists of the parts already described, which are continued backwards to form the front of the palate, filling up the notch between the incisor and palatal processes (Plate XIV. figs. 5 & 7), and of a narrow portion which ascends and fits by a convex surface into the groove of the nasal process, ending above at the ridge for the turbinate bone, part of which ridge it forms. In a fœtus 2·3, and also in one 3 inches long, two sets of bone-deposits are seen immediately in front of the vomer, one consisting of a fine process which projects in the middle line, the other formed of blocks, one above the other, which lie over the incisor process and add to its depth and strength. In a fœtus 9 inches long these bone-formations have made but little progress, and may easily escape detection.

In a fœtus 9 inches long, either intermaxillary bone is in great part fused with its corresponding upper maxilla. The anterior edge of its nasal process has united to the nasal process of the maxilla, and its horizontal portion is inseparably attached to the upper surface of the incisor process, but the outline of the bone is preserved by a fissure (Plate XIV. figs. 9 & 10), which is traced along its posterior margin from the middle of the nasal process of the maxilla, and which deepens as it reaches the palate through which it extends, often permanently. At its upper extremity this line, widening into a groove, contains the remains of the bent edge of the anterior boundary of the nostril, which can be traced downwards for some distance along a kind of canal, and which in a fœtus 6·5 retains its connexion with the soft structures lying beneath the nasal process, reaching up towards the nasal bone.

Lastly, in a fœtus 9 inches long, the boundary of the intermaxilla is still distinctly marked on the under surface of the palate, extending outwards to the posterior half of the outer wall of the lateral incisor, which it forms as far as the margin of the foramen for the nerve, thence it extends, as a fissure occupied by the nerve for the middle incisor, and crossing the septum between the incisors is lost in the plates of bone which have joined the incisor process.

The preceding observations account for the absence of all trace of the human intermaxillary bone on the facial aspect of the upper jaw in the adult, whilst the permanent fissure through the palate and on the inner side of the nasal process are equally explained. The bone, in fact, is shut off from the face by the nasal and incisor processes of the superior maxilla. Distinctly outlined (Plate XIV. fig. 6) at the close of the fourth month, it is joined to the superior maxilla during the latter part of the fifth or beginning of the sixth month; its nasal process, as it may conveniently be termed, is buried in that of the upper jaw, the apex assisting to form a permanent ridge for articulation with the inferior turbinate bone. In a fœtus 1·5 I notice a slight cleft in the nasal process of the superior maxilla corresponding with the top of the groove on its inner surface, but

I have never seen anything like an approach to the formation of a distinct anterior plate of bone, such as forms part of the præmaxillæ of mammals.

Yet, despite the peculiarity which results from the formation of the incisor process and the shutting in of the intermaxillaries in Man, these bones closely resemble those corresponding with them in mammalia. They have a similar wedge-shaped articular surface, fitting a groove in either superior maxilla; they extend towards the middle line and articulate there; they form the anterior extremity of the palate; they bound and divide the incisive foramina; they assist in forming no inconsiderable portion of the sockets for the incisor teeth; that they do not completely form them is a fact occasionally confirmed by the imperfect character of the sockets which lodge these teeth in those cases of cleft palate which have the intermaxillary bones isolated from the superior maxillæ.

Another interesting point may be noticed. In some mammals having a slight upper jaw, a Sheep for example, the outer wall of the nostril is smooth, there is no vertical ridge, such as that which in Man rises to the turbinate bone. In other mammals with strong upper jaws the præmaxillæ thicken towards their articulation with the superior maxillæ, and here is the most prominent vertical line on the nasal surface; beyond this the superior maxillæ recede somewhat. There is no such distinct ridge on the upper jaw itself. In Man the corresponding ridge is placed about the middle of the inner surface of the ascending process of the upper maxilla, but it results from, and marks none the less the junction of the nasal process of the intermaxilla with that groove on the inner surface of the upper jaw with which it originally articulated.

A few words may dispose of several of the bones of the face, as there is little that is new to be said respecting them. In a fœtus 1·5 ossification has commenced in the nasal bones; in a fœtus 2·3, and still better in one 3·7, the outer lower angle is seen to be prolonged downwards, and is in connexion with the fold of membrane continued up from the apex of the intermaxilla. The lachrymal bones are incompletely ossified in a fœtus 2·3 They are formed in membrane, covering a prolongation of cartilage from that which eventually forms the ethmoidal plate of the orbit. One of these bones is seen in position in a specimen from a fœtus 4·7. The ossification of the malar and of the palate-bones progresses with that of the upper jaw. That of the palate-bones takes place in the inner and back part of the maxillary lobe. In a fœtus 1·5 the palatal portion is represented by a slightly incurved ridge, the vertical is proportionately much larger. The bones are in the earliest stage remarkable for the large size of the posterior nasal foramen, around which ossification appears to commence. The remarkable size of the opening for the malar nerve and the early bone-deposits around it, the formation of the nasal bones over branches from the ophthalmic nerves, the building-up of the superior maxilla about the infraorbital and its branches, and of the inferior maxilla about the dental nerve, are points worth noticing with reference to the formation of these bones of the face. In a fœtus 9 inches long the palate-bone is completely outlined, with the exception of the surfaces which mark its upper processes.

The Inferior Maxilla.

This bone is developed partly in MECKEL'S cartilages, partly in membrane which covers them. In the human fœtus the cartilages are prolonged to the middle line in the first visceral arch, where the two are loosely connected by membrane. These outgrowths of cartilage taper as they pass forward, but expand behind the incisor portion of the jaw into a lobed extremity (Plate XIII. fig. 6 (2)), the front lower surface of which, unlike in this respect the remainder of the cartilage, is not covered by bone in a fœtus 1·5 (Plate XIII. fig. 6 (1)). Ossification commences in a fœtus ·9 by deposits along the lower middle border of the maxilla on either side, and extends thence in all directions in the membrane which covers MECKEL'S cartilage, so that the bone is well defined in a fœtus 1·1, and is then one-tenth of an inch long.

Each half of the maxilla may be said to grow from four centres, formed (1) by the cartilage which tips the condyloid extremity, (2) by the layer of membrane in front of MECKEL'S cartilage, (3) by the ossification of the anterior extremity of MECKEL'S cartilage, (4) by deposits of bone in the perichondrium of the anterior and middle thirds of the same cartilage from which is derived the plate of bone which forms the base of the dental canal.

In a fœtus 2·3 the outer surface of the bone has a uniformly smooth appearance, marked only by the opening for the mental nerve. The ascending ramus rises from the horizontal at an obtuse angle. The angle formed by the anterior (coronoid) border measures 169° in a fœtus 2·3, 156° in a fœtus 3·5, 129° in a fœtus 8 inches long. The angle formed by the posterior (condyloid) border measures 165° in a fœtus 2·3, 153° in one 3·5, and 144° in a fœtus 8 inches long. In a fœtus 1·5 the angles formed between the rami are scarcely appreciable (Plate XIII. fig. 6 (1)).

The alveoli are deepened by deposits in the membrane from which the gums are also formed; the canine socket, which is open on its outer side in a fœtus 9 inches long, is separated by a posterior septum in a fœtus 2·3, and from the lateral incisor in one 3·5, and the last-named from the middle incisor in a fœtus 3·7, in which also a partition is beginning to form between the sets of teeth behind the canine socket.

Whilst little is to be observed in the growth of the outer surface of either half of the inferior maxilla, the inner surface presents for examination, (1) the formation of the floor of the dental canal, (2) the changes in the anterior extremity of MECKEL'S cartilage.

In the immediate covering of the anterior half, or thereabouts, of the cartilage of MECKEL a plate of bone begins to be formed at a very early period. It stands out, in a fœtus 1·5, from the inner surface of the horizontal ramus, and at first, as it grows, it follows the curve of the cartilage beneath, and is consequently convex on its upper surface, upon which the dental nerve is situated. This plate, or ridge, extends from the future position of the dental foramen, and, in front, turns up to between the canine and incisor sockets. Gradually this ridge becomes concave on its outer surface, grows up to constitute the boundary of the dental canal and part of the posterior or inner wall of the sockets for the teeth, and in a fœtus 8 inches long its lower border is recog-

nized as the mylo-hyoidean ridge, the posterior extremity of which is turned upwards and outwards to join the inner surface of the ascending ramus, and thus forms the inferior dental foramen, whilst the shallow groove beneath this ridge is occupied by the remains of the cartilage of MECKEL (Plate XIII. fig. 8, *d* and *b*).

The extremity of MECKEL's cartilage is ossifying in a fœtus 2·3 to form the inner triangular block of bone (below the anterior part of the ridge) which determines the size and shape of the lower anterior portion of each half of the jaw, beneath, that is to say, the incisor sockets (Plate XIII. figs. 6 (3), 7, & 8). To the twist acquired in this portion of the bone the prominence of the front of the maxilla, known as the mentum, or chin, appears to be due. In a fœtus 3·5 the cartilage is ossified as far back as the junction of the middle with the anterior third of the body of the bone; behind this point it gradually shrinks during the subsequent growth of the ramus. In a fœtus 4·7 the block of bone formed in the anterior extremity of the cartilage of MECKEL is still clearly defined.

EXPLANATION OF THE PLATES.

PLATE XIII.

Fig. 1. Part of human fœtus (four-tenths of an inch long): *a*, plates for ethmoid and turbinate cartilages; *b*, maxillary lobe prolonged towards the middle line to form the palate (3 linear).

Fig. 2. The same as the preceding: *a*, *a*, plates for ethmoid and turbinate cartilages; *b*, maxillary lobe showing its irregular inner surface; *c*, frontal-nasal process turned back to expose the nodules on its under surface and the continuation of the ethmoid plates.

Fig. 3. Part of superior maxilla from fœtal Pig (2 inches long): *a*, process in the situation of the human incisor portion (5 linear).

Fig. 4. Vomer of human fœtus (2·3 inches long): *a*, anterior extremity of the bone (15 linear).

Fig. 5. Part of skull of mammary fœtus of Kangaroo (2 inches long): *a*, process in the situation of the human incisor portion (3·5 linear).

Fig. 6. (1) Outer surface of left inferior maxilla from human fœtus (1 inch and five-tenths long): the mental extremity is sharply bevelled off below; the situation of the mental foramen is indicated. (2) Anterior extremity of MECKEL's cartilage and a layer of membrane lying beneath it, from the same fœtus. (3) Right inferior maxilla, inner surface, from human fœtus (2·3 inches long): *a*, ossified anterior extremity of MECKEL's cartilage; *b*, plate of bone overlying the groove for the cartilage and separating it from that for the dental nerve (3·5 linear).

Fig. 7. Left inferior maxilla, inner surface, from human fœtus (3·5 inches long): *a* and *b* refer to the same structures as in the last figure; *c* points to the convex upper surface of the plate of bone bounding the dental canal (3·5 linear).

Fig. 8. Right inferior maxilla from human fœtus (8 inches long), inner surface: *a* and *c* as before; *b*, groove for the remains of MECKEL'S cartilage; *d*, mylohyoidean ridge (3·5 linear).

Fig. 9. Front of palate from the skull of a Monkey, showing the relations of the præmaxillary bones (natural size).

Fig. 10. The same parts of the skull of a Dog (half natural size).

PLATE XIV.

Fig. 1. Front of palate from the skull of a Hare, showing the relations of the præmaxillary bones (natural size).

Fig. 2. The same parts of the skull of a Sow (one-quarter natural size).

Fig. 3. The same parts of the skull of a Sheep (one-third natural size).

Fig. 4. The same parts of the skull of a Kangaroo (natural size).

Fig. 5. Right superior maxilla from human fœtus (4·7 inches long): *a*, inner wall of antrum; *b*, incisor process; *c*, fissure for intermaxillary bone (3·5 linear).

Fig. 6. Left superior maxilla from human fœtus (4·7 inches long): *a*, inner wall of antrum, continuous with the palate; *b*, incisor process; *c*, intermaxillary bone (3·5 linear).

Fig. 7. Incisor process (*b*) and fissure for intermaxillary from the same specimen as fig. 1, viewed from the front; *e*, infraorbital notch (3·5 linear).

Fig. 8. Right superior maxilla from human fœtus (9 inches long): *d*, fissure extending into the orbital portion of the bone, and indicating the line of extension of the cavity of the antrum (3·5 linear).

Fig. 9. Left superior maxilla from human fœtus (6·7 inches long): *c*, intermaxillary bone joined, with the exception of its apex, by its front border to the maxilla the posterior margin is free (3·5 linear).

Fig. 10. Left superior maxilla from human fœtus (9 inches long): *c*, intermaxillary bone completely united in front to maxilla; *g*, canal which extends for a short distance beneath the intermaxilla; *h*, plates of bone belonging to the intermaxilla, which assist in bounding the incisor sockets (3·5 linear).

Fig. 11. Front view of right superior maxilla of human fœtus (9 inches long), showing the plates of bone which form the front wall of the incisor and canine sockets and which extend to the alveolar plate: *f*, canine notch; *e*, infraorbital notch or fissure (3·5 linear).

V. *On Hydrofluoric Acid.* By G. Gore, *F.R.S.*

Received November 14, 1868,—Read January 28, 1869.

A. *Anhydrous Hydrofluoric Acid.*

The method I have adopted to obtain this compound is the same as that employed by Fremy*, viz. to heat to redness dry double fluoride of hydrogen and potassium in a suitable platinum apparatus.

As the results I have obtained differ somewhat from those described by Fremy, and as the anhydrous acid has been considered to be a *gas* by him and by several other able investigators†, it will be necessary for me to state with a sufficient degree of minuteness the conditions of my experiments.

Pure aqueous hydrofluoric acid, prepared by the method described in this paper (page 196), and free from sulphuric, sulphurous, hydrosulphuric, hydrochloric, and hydrofluosilicic acid, also from sulphur, arsenic (except minute traces), and all fixed substances, was added to carbonate of potassium (free from chlorides, sulphates, and silica) in a large platinum dish, until the liquid on evaporation evolved a distinct acid odour, and the vapour reddened litmus-paper. The clear solution was evaporated to perfect dryness, and the residue gently heated until it fused to a clear thin liquid, free from films upon its surface; it evolved a trace of acid vapour during the process, and did not corrode the platinum vessel. The salt when cooled formed a rather hard, white, non-deliquescent mass; but if the acid was in much excess the fused salt had tough films upon its surface, and formed on cooling a less hard, less white, and more transparent mass.

Fig. 1.

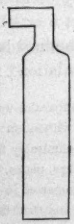
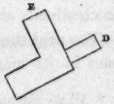


Fig. 2.



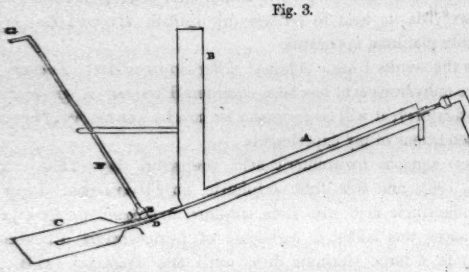
To obtain the acid from this salt, about 2800 grains weight of fragments of the substance was put into a platinum retort of the annexed form (fig. 1), about 6 inches (=15·3 centimetres) deep and 1½ inch (=3·8 centimetres) diameter, and gently heated to complete fusion to expel any traces of water. The vessel was then fitted air-tight (by means of melted sulphur mixed with lampblack) to a closely fitting platinum tube, about 36 inches (=91·5 centimetres) long, of a Liebig's condenser, A (fig. 3), the condenser being supplied with a freezing-mixture, composed of 1 part of pounded ice

* Ann. de Chimie (3), vol. xlvii. p. 5; Comptes Rendus, No. 9, Feb. 27th, 1854.

† *Vide* Fremy, Comptes Rendus, No. 9, Feb. 27th, 1854; Louyet, Comptes Rendus, vol. xxii. p. 960; also Miller, Elements of Chemistry, part 2, 4th edition, p. 161.

and 1½ part of crystallized chloride of calcium, through the open tube B. A plati bottle, C, immersed in a similar freezing-mixture, was employed to receive the d late. The bottle was provided with a three-necked angle-tube of platinum (see fig through the narrow opening, D, of which the condensing-tube passed into the b and the angle-tube had a long exit-tube of platinum, F, fixed upon its larger ing, E. The exit-tube had upon its upper end a short angle-tube of platinum, G, tu downwards to prevent attracted moisture running down into the bottle (see fig. The tubes all fitted tightly, and the *cold* junctions were made gas-tight by means mixture of melted paraffin and lampblack†.

Fig. 3.



Heat was now gradually applied and the acid rapidly distilled until (in about twenty minutes) the bottom of the retort became red-hot, indicating a completion of the process. The materials bumped considerably towards the end of the process (this may be diminished by applying the heat to the *side* of the retort), and a small quantity of the salt was carried mechanically over. The freezing-mixture is indispensable, and careful ma ment of the heat is required. Care must be taken to have all parts of the app quite free from moisture, and also from traces of fluoride of potassium, wh highly deliquescent, and is acted upon with dangerous violence by the acid. Th was redistilled without exposing it to the air, the retort being immersed in a b slightly warm water, kept at a temperature not exceeding 100° Fahr.; under circumstances any traces of water contained by the acid was retained in the ret the small quantity of saline matter. 2800 grains of the acid usually carried mechanically about 5 or 10 grains of saline matter in the first distillation.

The acid thus obtained is a highly dangerous substance, and requires the most ex care in its manipulation‡: it left no fixed residue on evaporation; it was a pe

* All the platinum apparatus used in this investigation was constructed for me with great care by Matthey and Co., and without such apparatus the experiments could not possibly have been made.

† Paraffin answers admirably for resisting hydrofluoric acid where heat is not applied; it may formed into stoppers, cups, plates, rods, and almost any desired shape; it is, however, liable to becom cracks when suddenly immersed in a freezing-mixture. Temporary stoppers were conveniently made by paraffin in a test-tube, and then breaking the tube.

‡ The highly dangerous nature of the anhydrous acid has received further confirmation in the recen death of Professor Nicklès of Nancy, which, according to the 'Moniteur Belge' of April 14th, 1869, v sioned by accidentally inhaling the vapour of the concentrated acid whilst engaged in making experi isolate fluorine.

colourless and transparent liquid at 60° FAHR., very thin and mobile, extremely volatile, densely fuming in the air at ordinary temperatures, and attracted water greedily from the atmosphere. It was perfectly retained in platinum bottles, the bottles having a flanged mouth, a platinum plate coated on its lower side with the paraffin mixture (see page 174) being tightly secured to the flanged mouth by eight clamp screws (see figure 4). The acid must be kept in a cool place, not above a temperature of 60° FAHR., otherwise it is very likely to burst the bottle; and a freezing-mixture should always be at hand whilst experimenting with it.

Fig. 4.

According to LOUYET* anhydrous hydrofluoric acid, obtained by distilling oil of vitriol and fluor-spar, and redistilling the condensed product with anhydrous phosphoric acid, is a *gaseous* substance and does not liquefy at −12° C. (=+10°·4 FAHR.) under ordinary atmospheric pressure; it fumes very strongly in the air, is rapidly condensed by water, has scarcely any action upon glass, and might probably be collected in glass vessels over mercury. According to FREMY† (who states that he assisted at the performance of M. LOUYET's experiments, and did not consider the results satisfactory), the dehydration effected by means of anhydrous phosphoric acid is not complete. He states that anhydrous hydrofluoric acid obtained by the distillation of dry double fluoride of hydrogen and potassium is *gaseous* at the ordinary temperature, but may be condensed at −20° C. by a mixture of ice and salt; it is then a very fluid liquid, volatilizing when removed from the cooling mixture, acting very powerfully upon water, diffusing white fumes in the air with an intensity like that of fluoride of boron, and, contrary to M. LOUYET's assertion, attacks glass rapidly‡.

To ascertain, therefore, whether the liquid I had thus obtained was really anhydrous or not, I made the following analytical experiments:—1st. I took 100 grains of the recently and well-dried double fluoride (which soon attracts a little moisture from the air), and heated it gently with constant motion in a platinum dish until it was all fused; it evolved an acid odour, and lost 0·8 grain in weight. I now again slowly fused it, gradually increasing the heat until at an incipient red heat it was quite solid and evolved no more acid odour, then cooled it under a cover and weighed quickly; its loss was now increased to 25·57 grains, the calculated theoretic loss of anhydrous hydrofluoric acid being 25·59 grains, assuming the salt to be KF, HF.

K	=39·1
F	=19·0
HF	=20·0
KF, HF	=78·1

* Comptes Rendus, vol. xxii. p. 960; also GMELIN's Handbook of Chemistry, vol. ii. p. 361.

† Comptes Rendus, No. 9, Feb. 27th, 1854; also 'The Chemist,' June 1854.

‡ FREMY also obtained the anhydrous acid by heating fluoride of lead (contained in a charcoal boat within a platinum tube) in a current of dry hydrogen, and DEVILLE obtained the dry acid by passing hydrochloric-acid gas over highly heated fluor-spar (Comptes Rendus; also Chemist, New Series, vol. iv. p. 330, and vol. ix. p. 556).

Berzelius found 25·1 per cent. of volatile matter in this salt, and also obtained 11·6 per cent. of water by heating the salt with six times its weight of protoxide of lead=1·288 per cent. of hydrogen in it; theory requires 1·280 per cent.* In a second trial 800 grains of the salt, which had been previously fused, was gradually heated to redness; it lost 204·6 grains (=25·575 per cent.), the theoretic quantity of HF being 204·78 grains. In a third experiment 2063·6 grains of the salt, which had not been sufficiently heated, lost 547 grains (including a little saline matter) in the process of distillation (it should have lost only 528 grains); for the analysis of the acid obtained from it see page 179.

2nd. I took another 100 grains of the same salt, dissolved it in water in a capacious platinum dish, added 1½ ounce of pure and strong hydrochloric acid, and evaporated nearly to dryness, then added more of the acid and slowly heated to perfect dryness, and finally to low redness; the weight of the residue was 95·45 grains, theory requiring 95·47 grains of chloride of potassium.

3rd. I now made several attempts to determine the amount of fluorine in the salt by means of silica. I took 78·2 grains of the perfectly dry salt, reduced it to fine powder, added 50 grains of pure precipitated silica (which was previously found to lose no weight by being boiled in strong sulphuric acid), and mixed them together in a platinum bottle; then added in small portions at a time a quarter of an ounce by measure of strong sulphuric acid, and after awhile applied heat gradually until the sulphuric acid was considerably expelled. The residue was then thoroughly washed in a filter, dried, heated to redness, and weighed; it amounted to 19·3 grains=30·7 grains dissolved=38·56 grains of fluorine, theory requiring 38·05 grains. In a second similar experiment 29·8 grains of silica was dissolved; in a third, with 40·7 grains of silica, 29·6 grains was dissolved; in a fourth experiment, with 100 grains of silica and half an ounce of the acid, 30·1 grains was dissolved; and in a fifth experiment, with 50 grains of silica, and a quarter of an ounce of water added before addition of the acid, 31·3 grains was dissolved: this addition of water appeared to facilitate the action by rendering the mixture more fluid, and also by retaining the liberated hydrofluoric-acid gas in solution until it had time to act upon the silica. The average quantity of silica dissolved in these five experiments was 30·30 grains, theory requiring 30·277 grains. The silica being highly hygroscopic was the chief cause of the variation in the numbers obtained; it was (in some of the experiments) cooled and weighed in a stoppered bottle, or in a platinum box (with closely fitting lid) in which it had been heated. In similar experiments, with 100 and 200 grains of purified fine white sand substituted for the precipitated silica, only 6·05 and 8·5 grains respectively were dissolved; these results, compared with those obtained with the precipitated silica, show that unless the silica is very finely divided, much hydrofluoric acid is apt to escape without acting upon it.

From the results obtained with the precipitated silica I conclude (1st) that the whole of the fluorine contained in the salt is evolved as tetrafluoride of silicon, SiF_4, and not any as hydrofluosilicic acid, notwithstanding the presence of water formed by the reaction,

* Gmelin's Handbook of Chemistry, vol. iii. p. 65.

or even of a limited amount of water added, as in the fifth experiment, and (2nd) that an equivalent, or 78·13 parts, of the dry salt contains two equivalents, or 38 parts, of fluorine; and therefore from these results, combined with the amount of potassium found as chloride in the foregoing determination, I conclude that the salt cannot contain more than a very minute amount, if any, of oxygen or water.

I also checked the foregoing results by neutralizing a known weight of the distilled acid with carbonate of calcium. The first approximate and reliable result was obtained as follows. Into a weighed platinum bottle was poured 33·18 grains of the acid; the stopper was reinserted, and the vessel and contents reweighed. The stopper was now taken out and replaced by a tightly fitting and open vertical tube of platinum about 22 inches (=55·9 centimetres) high, with a stopper at its upper end, the junction of the tube with the bottle being made gas-tight by means of melted paraffin. The bottle was then immersed in ice, and about one-eighth of an ounce of distilled water added drop by drop through the tube. After awhile 102·7 grains of white marble in very fine powder was very gradually added; the mixture stood about fifteen hours with occasional shaking, and was then heated. The mixture was now washed out into a large platinum dish (it was neutral to test-paper), and evaporated to dryness and weighed. The weight found was 84·92 grains=a loss of 17·78 grains=32·327 grains HF present. In this experiment a small quantity of the acid vapour escaped during the weighing, which would probably account for the amount missing, viz. 0·853 grain; it was almost impossible to prevent a slight loss by this method on account of the great volatility of the acid, the carbonic anhydride carrying hydrofluoric acid with it.

I made several attempts to analyze the anhydrous acid by other means: thus, I gradually diluted the highly chilled acid, added to it an excess of a solution of acetate of lead, boiled the mixture, and filtered and washed the precipitate; the result was a failure owing to the fluoride of lead being sparingly soluble in water and in spirits of wine. In a second experiment a portion of the diluted acid was added to an excess and known weight of nitrate of lead in solution, and the mixture containing the precipitated fluoride was evaporated to perfect dryness and weighed: in this case, on evaporating the mixture, a reverse decomposition occurred; the whole of the fluoride of lead was reconverted into nitrate, and the hydrofluoric acid escaped; indeed I found by a subsequent experiment that fluoride of lead was wholly converted into nitrate by evaporation to dryness with nitric acid.

Another method I employed for analyzing the anhydrous acid was as follows. A six-ounce platinum bottle had a vertical open platinum tube about 4 inches (=10·2 centimetres) high cemented into its mouth, the tube being perfectly closed by a well-fitting plug of paraffin. The bottle was placed for a short time in a thin tin case immersed in a freezing-mixture, then taken out and weighed, and returned to the chilling-case; a small quantity of the acid, not exceeding 50 grains, was then poured into it through a long-necked platinum funnel, and the plug fixed securely and the whole reweighed. The bottle was now placed in the freezing-mixture itself at a temperature not above zero,

and distilled water added to it in minute drops at long intervals at first, until about a quarter or half an ounce had been added, taking great care not to cause an escape of vapour. A quantity of hydrate of potash, more than sufficient to convert all the acid into fluoride of potassium, was now dissolved in a separate vessel, and the solution chilled; some of this liquid, more highly diluted, was added in drops to the dilute acid, at intervals at first, and gradually the whole, taking care to cause no escape of vapour. The mixture (of strong alkaline reaction) was washed into a platinum beaker, and gradually evaporated until a film formed upon it. Precipitated silica, quite dry and perfectly pure, and especially free from matters soluble in hot concentrated sulphuric acid, was weighed in a stoppered bottle in more than sufficient quantity to convert the whole of the fluorine of the acid into tetrafluoride, the contents of the bottle added to the alkaline pasty liquid, and the bottle at once closed and reweighed. The mixture of silica and alkaline fluoride was incorporated by stirring, and an excess of concentrated sulphuric acid added gradually to it, and the mixture then heated until it was a thin liquid, and the sulphuric acid freely volatilized. The residuary mixture was then dissolved, transferred to a filter, and the silica washed, dried, ignited, transferred to a stoppered bottle and weighed. This method gave *nearly* accurate results.

The best method of analysis, however, consists in diluting the chilled acid with extreme care, and adding to it very gradually a known weight of pure caustic lime, and weighing the dried and ignited product; the following are the particulars of the process. A six-ounce platinum bottle provided with a neck about $2\frac{1}{2}$ inches (=6·4 centimetres) high and $\frac{1}{2}$ an inch (=1·3 centimetre) in diameter, with a flanged mouth, plate with paraffin layer (see page 175) and clamp screws, and previously weighed, was placed in a closely fitting tin case immersed in the freezing-mixture of ice and chloride of calcium (see page 173), and about 30 to 50 grains of the acid from a chilled and similar platinum bottle transferred to it. The bottle was then securely closed, removed from its case, and allowed to return to the atmospheric temperature and reweighed; it was now reimmersed in the freezing-mixture, and when quite cold the plate and screws were removed and *minute portions* of distilled water were allowed to drip down the inner side of the tube until the acid was sufficiently diluted to be able to add gradually in the whole about $\frac{1}{2}$ an ounce of water. Pure caustic lime (which had been repeatedly heated to whiteness until it ceased to lose weight), more than sufficient in quantity to neutralize all the hydrofluoric acid present, was now intensely heated in a small platinum box, and cooled with the lid (a closely fitting one) upon it, and weighed; the whole of the lime was then very gradually added to the dilute acid with shaking, and the mixture digested with shaking and heating until the liquid became alkaline to test-paper. On first heating, much heat spontaneously developed itself in the mixture, and required to be moderated by immersing the bottle in cold water, and *after* this development of heat the liquid was alkaline. The mixture was now evaporated and dried in the platinum bottle, then repeatedly ignited to redness and cooled until it ceased to lose weight, and finally weighed, every 22 grains increase of weight being estimated to represent 40

grains of HF present. By these means the loss of acid vapour did not exceed 0·05 grain; the residue was alkaline to test-paper, and the gain of weight found was 55 parts for every 100 parts of liquid acid taken. The following are the numbers experimentally found:—in the first reliable experiment 31·61 grains of the acid and 54·56 grains of lime were taken, and the weight of ignited residue found was 71·93 grains=a gain of 17·37 grains=31·58 grains of HF present=99·90 per cent. of HF in the acid. In another experiment 50·26 grains of the acid and 77·54 grains of lime were taken, and the weight of ignited residue was 105·18 grains=a gain of 27·64 grains=50·27 grains of HF=100·01 per cent. of HF.

In an analysis by the above process of a portion of non-rectified acid obtained from some double fluoride of hydrogen and potassium which had not been sufficiently fused, and which from the weight of acid obtained from a given quantity of the salt was suspected to contain traces of the double salt and a small quantity of water not exceeding at the utmost 3·47 per cent. (see page 176), 39·39 grains of the acid + 66·74 grains of lime gave 87·78 grains of ignited residue=a gain of 21·04 grains=38·25 grains of HF =97·09 per cent. of HF present. These results confirm all the former ones, and further support the conclusion that the liquid obtained by distilling properly prepared fluoride of hydrogen and potassium, and rectifying the product at a temperature not exceeding 100° Fahr., is anhydrous hydrofluoric acid. The lime for the above process was prepared as follows:—an excess of clear lime-water was added to a solution of nitrate of calcium, the filtered solution precipitated by a mixture of solution of ammonia and sesquicarbonate of ammonium; the precipitate was washed and dried, and repeatedly ignited to an incipient white heat in an open platinum crucible in a small gas-furnace. The presence of magnesia in the lime does not affect the numerical relation of the gain of weight to the amount of acid present. Various attempts were made to employ pure oxide of magnesium in place of the lime, but its power of neutralizing the acid was found to be too slow.

Molecular Volume of the Anhydrous Acid in the gaseous state.

I have made fourteen separate attempts to ascertain the volume of gaseous product formed by the chemical union of one volume of hydrogen with fluorine. The method I have adopted has been to heat pure anhydrous fluoride of silver to fusion in a platinum vessel containing an equivalent quantity of pure and dry hydrogen. The experiments were extremely difficult owing to the destructive nature of the substances, and the consequent difficulty of eliminating the numerous interferences which presented themselves. The following are the particulars of the method finally employed. Fragments of the purest magnesium were introduced into the neck of an inverted bottle full of previously boiled and partly cooled dilute sulphuric acid, consisting of one measure of the purest acid, and about forty or fifty of distilled water. The mixture was stirred with a rod of magnesium immediately before use. When the bottle was quite full of hydrogen, the gas was transferred to a dry bottle filled with mercury and inverted over mercury; sticks

of hydrate of potash were then agitated occasionally with the gas at intervals during several hours, and the gas transferred to a graduated tube over mercury, and allowed to remain in contact with sticks of dry phosphorus during twelve hours, the phosphorus being used to remove oxygen introduced by traces of air adhering to the vessel, &c.

About 15·3 grains of fluoride of silver of a very high degree of purity (made from pure materials, viz. hydrofluoric acid, and carbonate of silver) was placed in the previously heated and weighed platinum tube A (fig. 5), which was open at the top and closed at the bottom, and constructed without solder by the process of melting. The tube was $6\frac{1}{2}$ inches (=16·6 centims.) long and $\frac{1}{8}$ of an inch (=2·1 centims.) diameter, and had a capacity of 3·4 cubic inches (=55·716 cubic centimetres); in some of the experiments, where accurate weighing was not required, it was coated externally for about 2 inches (=5·0 centims.) in length at its closed end with a thick layer of copper by the electrolytic process: the object of coating it thus was to prevent traces of hydrogen passing through the platinum at a red heat; this may, however, be dispensed with by not applying more than a barely visible red heat in the subsequent reduction part of the process. The fluoride was now melted in it at a visible red heat until it formed a tranquil liquid, and the upper part of the tube also heated to redness to expel all moisture*. The tube was then either at once affixed to the platinum receiver (see fig. 6), or as soon as possible corked, cooled, and reweighed; the loss of weight upon the above quantity of salt was usually about 0·1 grain, arising from moisture expelled from the fluoride (which is a very hygroscopic and deliquescent substance), and partly from the moisture of the air decomposing a little of the fluoride during the fusion. The tube A was now fixed tightly upon the open bent platinum neck-tube B of the graduated platinum receiver C, and the junction made perfectly gas-tight by a small quantity of the melted mixture of sulphur and lampblack, bound over tightly by a stretched ribbon of the thinnest vulcanized india-rubber secured by string. The platinum receiver, C, was 6 inches (=15·3 centims.) long and $1\frac{1}{4}$ inch (=3·8 centims.) diameter, open and flanged at the bottom end, and formed in one piece with the bent neck B without solder. Glass receivers were found to be corroded even when thickly and carefully coated with paraffin. (N.B. They were used before I succeeded in obtaining the hydrofluoric acid in a perfectly anhydrous state.) They might perhaps be used provided moisture is perfectly excluded and the acid is *not allowed to assume the liquid state*. The entire apparatus, including A, B, and C, when put together had a capacity of about 15 cubic inches (=245·805 cubic centims.).

Fig. 5.

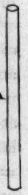
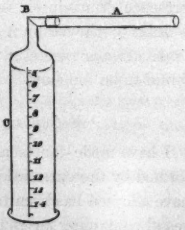


Fig. 6.



The tubes A and B were now filled as quickly as possible with dry mercury, and

* *Oxide* of silver was found to be decomposed at a temperature much below that of visible redness. H. Rose states that Ag_2O begins to evolve oxygen at 250° C. (Watts's Dictionary of Chemistry, vol. i. p. 792).

tapped persistently to remove any adhering bubbles of air; the receiver C was then also similarly filled and tapped, and inverted over mercury in a large mercurial trough. 5·5 cubic inches (=90·128 cubic centims.) of the prepared hydrogen which had lain many hours in the measuring-tube in contact with the phosphorus was now transferred to the platinum apparatus, and the tube A inclined and persistently tapped, so as to shake out of it as perfectly as possible all the mercury. A stick of dry phosphorus was again placed in the gas for one hour to remove any oxygen introduced by adhering air-bubbles, or by air lodging in the crevice at the junction of the two tubes.

Fig. 7.

The apparatus and its contents were next removed from the mercurial trough by means of the glass vessel D, which was about 8 inches high (=20·4 centims) and 2 inches (=5·1 centims.) in diameter, and were supported within this vessel by mercury at such a height that the figure "5·5 CI" of the graduation upon the platinum-receiver coincided in horizontal position with the mark — (shown at E, fig. 7) near the top of the outer vessel. Additional mercury was now poured into the outer vessel until it attained this same indicated level at the atmospheric temperature, and was therefore at the same level inside and outside the platinum-receiver. The tube A being supported horizontally, a wet rag was now laid upon it near the sealed junction and heat applied gently to the outer end by means of a spirit-lamp until a barely visible redness was attained, this being quite sufficient to melt the enclosed fluoride of silver. This degree of heat was maintained about one hour, and near the termination of the heating-process a much higher and variable temperature was applied for a short time, but not sufficiently high to melt the reduced silver. The mercury in the outer vessel was kept at the original level during the heating-process &c., by raising or lowering the platinum apparatus as occasion required. The whole apparatus was then placed in a suitable air-bath, and gradually heated to a uniform and constant temperature of about 212° Fahr. (=100° C.), a thermometer being placed in the vessel of mercury D, and another near it in the air-bath. The platinum apparatus was raised gradually as the gas expanded, to keep the mercury at the original level, and the volume of enclosed gas was noted at the various temperatures.

In each instance, during the decomposition of the silver-salt by the hydrogen, after the first expansion (produced by the heat) a very large *reduction* of volume occurred; for instance, 10·0 cubic inches (=163·87 cub. centims.) were reduced to 6·5 (=106·5 cub. centims.); and on removing the lamp and allowing the whole apparatus to cool to 50 or 60° Fahr. (=10° to 15°·5 C.), a still further and very large contraction invariably occurred; for instance, 6·5 cubic inches (=106·5 cub. centims.) became 3·0 cubic inches (=49·161 cub. centims.), even while under a rarefaction of 5 inches (=12·75 centims.) height of mercury. Similar contractions to very small volumes at the atmospheric temperature took place in all the experiments. These reductions of volume were in the earlier trials somewhat due to small quantities of air present, the oxygen of which formed water with some of the heated hydrogen and condensed a little

of the acid vapour. In all cases, however, the contraction of volume was evidently nearly wholly due to the partial condensation of the hydrofluoric-acid vapour within the vessel to a liquid state. This liquid could not of course be seen on account of the opacity of the apparatus, and being very minute in amount (probably less than half a drop for each cubic inch (=16·387 cub. centims.) of hydrogen employed) and very volatile, it was in all cases instantly converted into a dense fume on opening the vessel. The mere warmth of the hand was sufficient to make the partly liquefied acid expand very greatly. These results of contraction agree with and confirm the results already found in my experiments, viz. that anhydrous hydrofluoric acid is a very volatile liquid at 60° FAHR.

The following are the actual results obtained in one of the first reliable approximate experiments. Six cubic inches (=98·322 cub. centims.) of prepared hydrogen at 50° FAHR. and 16·4 grains of fused fluoride were employed. During the first three minutes of heating, the gas expanded to 7·8 cubic inches (=127·8 cub. centims.). Contraction then commenced and continued during about twenty-five minutes, the volume being gradually reduced to about 4½ cubic inches (=73·74 cub. centims.). The heat was continued about half an hour longer, and the apparatus then, whilst still warm, placed in the air-bath, and heated gradually to a fixed temperature of 200° FAHR.; the volume of gas was then 14·5 cubic inches (=237·61 cub. centims.) =11·2 cubic inches (=183·53 cub. centims.) at 50° FAHR. The rate of expansion was somewhat greater at the higher degrees of temperature than at the lower ones; for instance, at 139° FAHR. the volume was 12·1 cubic inches, at 160° FAHR. 12·85, at 186° FAHR. 14·0. Only a minute quantity of undecomposed silver fluoride was found after the experiment.

In another experiment, with 5·5 cubic inches (=90·128 cub. centims.) of hydrogen at 50° Fahr. and standard pressure, and 15·3 grains of fluoride, on applying heat the gas expanded to 7·3 cubic inches (=119·625 cub. centims.) at the end of three minutes; it then gradually contracted during the next twenty minutes to about 4·2 cubic inches (=68·825 cub. centims.); the heat was continued about twenty minutes longer with occasional heating to full redness (but not sufficient to fuse the enclosed reduced silver). The apparatus was then at once transferred to the hot air-bath, and heated gradually to 212° Fahr., and kept at that temperature some time. The volume of gas thus found was 12·8 cubic inches (=209·75 cub. centims.) at 176° FAHR., 13·0 (=213·03 cub. centims.) at 184° FAHR., and 14·0 (=229·418 cub. centims.) at 212° FAHR.; the latter number, after correction for the amount of displacement of the level of the mercury by the thickness of the sides of the platinum vessel between "5·5," the original depth of immersion, and "14" the final one, was 14·3 (=234·33 cub. centims); this is equal to 10·84 cubic inches (=177·635 cub. centims.) at 50° FAHR., the theoretic quantity required being 11·0 inches (=180·257 cub. centims.). On cooling the apparatus to 50° FAHR., the mercury rose and filled the platinum tube to within about 0·7 cubic inch (=11·77 cub. centims.) of its capacity, even whilst under rarefaction by a column of mercury about 3·5 inches (=8·9 centims.) in height. Minute traces only of undecomposed fluoride of silver were found in the tube after the experiment.

From the results obtained, it appears that one volume of hydrogen in uniting with fluorine, produces not simply one volume of gaseous product as it does when uniting with oxygen, but two volumes, as in the case of its union with chlorine.

According to M. Pratt* anhydrous fluoride of silver contains oxygen, its constitution being 1 equiv. of fluorine =29·6, 1 equiv. of oxygen =8, and 2 equivs. of silver =216. The results obtained in these experiments contradict this view; for according to it the volume of gases obtained should have been less, as 1 volume of hydrogen in uniting with oxygen produces only 1 volume of gaseous product.

To obtain a numerical result more nearly approaching theoretical accuracy than in the foregoing experiments, various minute corrections would have to be applied:—1st, for the increased capacity of the platinum apparatus caused by its expansion by heat; 2nd, for the rise of level of the mercury caused by its expansion by rise of temperature; 3rd, for the lowering of the said level by expansion of the glass containing-vessel; 4th, for the rise of the said level caused by the immersed bulb of the thermometer; 5th, for the variation of atmospheric pressure during the experiment; and 6th, for the meniscus, &c.

To ascertain the loss of weight by the fluoride of silver in the preceding experiments, 12·8 grains were put into a platinum boat enclosed in a roll of platinum-foil, and heated to tranquil fusion at a gentle red heat in a platinum tube-retort (similar to the tube A in fig. 5), then cooled and weighed; the loss was 0·10 grain. The boat and foil were now put into a long horizontal platinum tube and gently heated in a slow current of hydrogen (which had been generated by magnesium and pure dilute sulphuric acid, and dried by two wash-bottles of strong sulphuric acid), the current of gas and the heat being maintained until acid fumes entirely ceased; the loss of weight was 1·80 grain. This is less than the theoretic quantity by 0·10 grain, which is partly (if not wholly) accounted for by the fact that fluoride of silver, even when carefully prepared from pure materials, is liable to contain a variable amount of metallic silver, which is left as an insoluble black powder on dissolving the salt in water. This free silver is caused in the process of drying the salt, by the aqueous vapour from the imperfectly dried portions and from the surrounding air decomposing the dry and hotter portions; it is increased in quantity every time the salt is fused in contact with atmospheric air.

[To determine the amount of silver in the fluoride, 15·89 grains of perfectly anhydrous fluoride was dissolved in water, the solution filtered, the separated black precipitate washed, and the silver in the filtrate determined by addition of chloride of potassium; it gave 18·00 grains of chloride =13·55 grains of silver. The black matter weighed 0·1 grain after ignition; deducting this amount, the 15·79 grains of fluoride contained 13·55 grains of silver: theory requires 13·437 grains. On several occasions I have had reason to suspect a slight degree of solubility of the separated and finely divided metallic silver in a strong aqueous solution of silver fluoride, similar to that of lead and silver in strong aqueous solutions of their respective nitrates; and if this be correct it may account for

* Comptes Rendus, No. 9, August 1867.

the slight excess of silver found in this determination. The amount of fluorine in the filtrate was determined by means of a solution of calcium salt in the usual manner; it gave 4·40 grains of fluoride of calcium =2·1436 grains of fluorine, the quantity required to complete the weight being 2·24 grains. In my hands this process of determining the amount of fluorine has not hitherto yielded very accurate results.—Received January 9, 1869.]*

These results, combined with the foregoing experiments on molecular volume, agree with the conclusion that a volume of hydrogen weighing 1 unites with 19 parts by weight of fluorine to form 2 volumes of hydrofluoric acid, and that the specific gravity of gaseous anhydrous hydrofluoric acid is ten times the amount of that of hydrogen.

The acid vapour obtained by heating fluoride of silver in the closed vessel of hydrogen over mercury, as already described, was transferred to glass vessels over mercury; it was colourless, and quite transparent; it did not corrode the glass vessels or render them dim in the slightest degree during several weeks, provided moisture was entirely absent, but if there was the slightest trace of damp present the surface of the glass soon lost its brightness. A fragment of ice introduced into the gas instantly caused the surface of the glass near it to become corroded and opaque white. The vapour of hydrofluoric acid escaping at any minute orifice from a vessel of the dry acid, condenses moisture rapidly upon the contiguous parts of the vessel.

Fremy sought for oxygen in the anhydrous acid, but was unable to find any†. Sir H. Davy also, who prepared his acid by distilling fluor-spar with sulphuric acid containing not more than 1 equivalent of water, could find no oxygen in it, either by electrolysis, or by neutralizing it with gaseous ammonia; but when the sulphuric acid he employed contained more than 1 equivalent of water, he found water in the hydrofluoric acid. The acid obtained in my experiments also I consider to be destitute of oxygen, not only from the results obtained in the various analyses and experiments already described, but also, 1st, because the double fluoride of hydrogen and potassium from which the acid was prepared, when fused and electrolyzed with platinum electrodes, evolved abundance of inflammable gas at the cathode, but yielded no gas at the anode, although oxides are by electrolysis decomposed before fluorides; 2nd, the electrolysis of the anhydrous acid with platinum electrodes yielded no odour of ozone, whereas hydrofluoric acid containing various percentages of water evolved that odour strongly (see page 199); 3rd, the properties of hydrofluoric acid prepared by heating pure fluoride of silver in pure hydrogen (see pages 181 & 182) agreed in properties, as far as those properties were ascertained, with the acid obtained from the double salt; and 4th, because two volumes of gaseous anhydrous hydrofluoric acid were produced by the reaction of one volume of hydrogen upon its equivalent of fluoride of silver, whereas if

* Note, added May 25, 1869. I have since, by repeated and more accurate analyses, confirmed the composition of the recently fused salt to be wholly silver and fluorine, with a small quantity of enclosed metallic silver.—G. G.

† Comptes Rendus, February 27th, 1854.

oxygen had been present, less than two volumes of acid gas would have been produced. All the results also support the conclusion that the acid obtained by heat from the double fluoride, and that from pure fluor-spar and mono-hydrated sulphuric acid heated together in a platinum retort, is free from oxygen and water. The complete synthesis of the acid can only of course be performed by means of hydrogen and free fluorine.

To determine the specific gravity of the anhydrous acid, I employed a thin platinum bottle of the form and size of the annexed sketch (fig. 8); with the stopper in its place the bottle contained 1000 grains of distilled water at 60° Fahr. The stopper had a minute tube up its centre, and also two funnel-shaped cavities, one at each end, as shown in the sketch. A thin platinum wire was also soldered round the stopper to form a

Fig. 8.

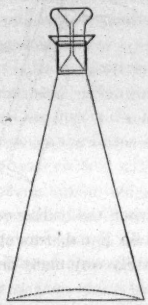


Fig. 9.

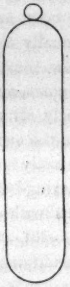
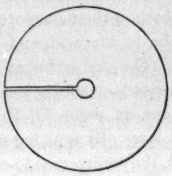


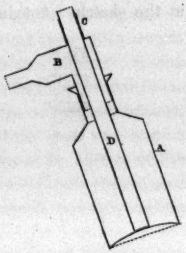
Fig. 10.



shoulder, which rested upon the neck of the bottle when the stopper was tightly down in its place. A specific-gravity bottle made of gutta percha would not resist the action of the acid. Sponge was employed to absorb the excess of acid around the stopper. The specific gravity found of the pure rectified acid in two separate experiments was 0·9885 at 56°·5 Fahr., that of water being =1000; whilst that of a less pure sample was 1·036 at 66° Fahr. (Sir H. Davy found 1·0609). These results are not perfectly accurate, owing to the unavoidable escape of acid vapour, but the amount of error is probably not more than 1 part in 1000. The specific gravity of the pure rectified acid was also found by weighing a platinum tube (which had been previously filled with paraffin by fusion) separately in the acid and in distilled water, both at 52° Fahr.; the number found was 0·9922, assuming that of water to be =1000 at 52° Fahr. By weighing a closed platinum vessel of the annexed size and form (fig. 9) separately in the acid and in distilled water, both at 55° Fahr., the specific gravity found was as 0·9879 is to 1·0000; this result is the most accurate one obtained: the pure acid is therefore a little lighter than water. The vessel containing the acid was covered by a thin plate of paraffin of the annexed form (fig. 10) during the weighing.

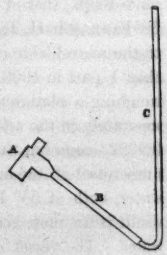
The acid obtained in my experiments was much more volatile than sulphuric ether. I ascertained its boiling-point as follows. A platinum bottle, A (fig. 11), of about 2½ ounces capacity, had a platinum angle-tube, B, which fitted tightly into its neck; the angle-tube had a short open tube, C, within which a platinum tube, D, was tightly fitted and luted with the melted sulphur mixture (see page 173); this tube was open at its upper end and closed at its lower end, and was for the purpose of receiving a thermometer. About two ounces of the perfectly pure and previously chilled acid was poured into the chilled bottle A, the angle-tube inserted and luted securely with paraffin, the smaller end of the angle-tube fixed within the platinum tube of a LIEBIG'S condenser (with freezing-mixture, see page 173) and luted with the paraffin mixture. A platinum bottle was affixed to the condenser to receive the distillate. The bottle was now immersed in a small water-bath, a thermometer placed in the water, another within the tube in the acid, and heat very gradually applied to the water. The acid boiled at 67° FAHR., the mercury of the inner thermometer remaining stationary at that point, and that of the outer one at about 77° FAHR., until nearly all the acid had passed over. By applying a stethoscope to the end of the inner tube D, the boiling could be feebly but distinctly heard; a slight escape of fume at the exit-tube of the receiving-bottle (see page 174) commenced with the boiling, continued with it, and ceased when the boiling ceased. At −30° FAHR. (=−34½° C.) the acid is still a perfectly mobile liquid, fuming but little in a narrow-necked bottle, and its solidifying point is probably very many degrees below this (see also page 198).

Fig. 11.



The vapour-tension of the acid at 60° FAHR. is very considerable, and an experiment was made in the following manner to approximately determine the degree of pressure to which the bottles containing the acid were subjected at ordinary temperatures. A six-ounce platinum bottle containing between four and five ounces of the anhydrous and pure acid in a chilled state was connected air-tight, by means of the platinum angle-tube A (fig. 12), with an inclined platinum tube B, about 16 inches (=41·7 centims.) long and $\frac{3}{8}$ of an inch (=0·95 centim.) diameter, and a vertical bent and open glass tube C, about 30 inches (=76·25 centims.) high and $\frac{1}{4}$ of an inch (=0·32 centim.) bore, the bent portion of the tubes being filled to about 6 inches (=15·3 centims.) in height with mercury. The bottle was now gradually allowed to attain a temperature of 60° FAHR. At that temperature the difference of level of the mercury in the two tubes was 15½ inches (=39·4 centims.), and therefore the pressure upon the interior of the bottle was =7·58 pounds per square inch. No leakage occurred at the cemented joints. The results of this experiment, together with those of the previous ones, show the necessity of keeping the

Fig. 12.



acid in a cool place. On loosening the lid of a bottle of the acid at 60° FAHR., the acid vapour is expelled in a jet like steam from a boiler.

The low boiling-point, the extremely dangerous and corrosive nature of the acid, and its great affinity for water, illustrate the very great difficulty of manipulating with it and retaining it in a pure state; nevertheless by the contrivance described (see page 175), and placing the bottles containing it in a cool cellar (never above a temperature of 60° FAHR.), the author has succeeded in keeping the liquid acid perfectly, without loss, and unaltered through the whole of the recent hot summer.

The following is the order of electrical relation found with different metals &c. in the pure anhydrous acid at 0° FAHR.:—zinc, tin, lead, cadmium, indium, magnesium, cobalt, aluminium, iron, nickel, bismuth, thallium, copper, iridium, silver, gas-carbon, gold, platinum, palladium. The acid in these experiments was contained in a covered deep and narrow platinum cup placed in a tin chilling case immersed deeply in the freezing-mixture.

Sir H. DAVY electrolyzed the liquid acid prepared by distilling fluor-spar with strong sulphuric acid. He cemented a platinum wire by fusion into a cup of horn silver, and inverted the cup in the acid in a tray of platinum. He used a very weak voltaic current, and kept the apparatus cool by a freezing-mixture. The platinum wire rapidly corroded and became covered with a chocolate-coloured powder. Inflammable gas like hydrogen was evolved from the cathode. He also tried a plumbago anode, but it was quickly destroyed and the liquid became turbid and black; also a charcoal anode upon the end of a platinum wire; the charcoal absorbed the acid and made the platinum wire the pole*. G. J. KNOX† also electrolyzed hydrofluoric acid with a charcoal anode attached to a platinum wire, and a platinum wire cathode, in a vessel of fluor-spar half filled with the acid, and having a perforated stopper of the same spar dipping into the acid. It is not stated in the above paper how the acid was prepared or its purity determined, nor by what means charcoal capable of resisting the acid was obtained.

I have made numerous experiments of electrolyzing the pure anhydrous acid with various electrodes. The method adopted in many of the experiments was to place a platinum cup about $3\frac{1}{2}$ inches (=9 centims.) deep and $1\frac{3}{4}$ inch (=4·5 centims.) wide in a tin case (weighted with lead) in the freezing-mixture; the cup was about half filled with the acid, and was closed by a well-fitting stopper of paraffin, through the centre of which a long rod or plate of the substance constituting the anode was closely fixed, the cup itself being the cathode; a loosely fitting cup of paraffin about half an inch deep, and provided with a handle, was immersed in the acid beneath the anode to collect particles which fell from the anode, and to prevent hydrogen ascending from the bottom of the platinum vessel and coming into contact with the positive electrode. The following are some of the results obtained. The liquid acid conducted with a platinum anode much more readily than pure water; with four SMEE's elements it began to visibly

* Works of Sir H. DAVY, vol. v. p. 416.

† Philosophical Magazine, 3rd series, vol. xvi. p. 192.

conduct, and with ten elements it conducted readily. No odour of ozone was evolved, and the platinum anode gradually acquired a thick red-brown crust. By employing forty SMEE's elements the conduction was copious; the anode rapidly corroded, and much finely divided platinum as a greyish-black powder collected in and beneath the paraffin cup; this powder appeared to be due to detached particles of the red-brown crust becoming reduced by the diffused bubbles of free hydrogen from the cathode, or was composed of particles of metal mechanically detached from the anode by the current. Portions of the soft earthy crust were scraped off at intervals into a platinum dish upon a heated slab of iron; it dried to a dark-brown earthy salt (doubtless fluoride of platinum), which rapidly deliquesced in the air to a tenacious sticky mass, similar to damp fluoride of silver: the anhydrous platinum compound was perfectly insoluble in the acid, but formed a blood-red liquid (like a solution of tetrachloride of platinum) with cold water, and the dilute aqueous solution, when heated, became full of flocks by separation of an insoluble basic salt. By far the greater portion of the platinum compound detached itself from the anode and became reduced to the metallic state, and only a small amount was left upon the anode as a soluble salt. The whole of the powder in the paraffin cup was found by appropriate tests to be metallic platinum. With an anode of very close-grained gas-carbon and six SMEE's elements, conduction occurred freely, and in less than half an hour nearly half the thickness of the carbon rod was disintegrated and fell as a loose powder to the bottom of the vessel. Numerous attempts were made to electrolyze the acid with a charcoal anode; fifteen different kinds of wood, including some of the densest kinds, were converted into charcoal by very gradual drying and equally gradual heating to full redness in a copper tube retort until all evolution of gas ceased. Some of the resulting rods were exceedingly hard and dense, and emitted a clear metallic ringing sound when struck; the denser kinds, especially those free from cracks, conducted freely even with ten SMEE's elements. The best kinds were those made from kingwood, beech, ebony, lignum-vitæ, and boxwood; that from lignum-vitæ was nearly as hard as baked earthenware*. On immersing either of the rods in the acid, even without passing an electric current, it soon began to crack and fly to pieces, and on passing the current the rods commenced breaking immediately, and in some instances with dangerous violence, projecting fragments to a considerable distance; even the hardest and densest kinds of charcoal behaved thus and became brittle throughout the immersed parts; the most resisting kind was that made from beechwood. The number of SMEE's elements employed varied from ten to fifty-six, and the kind of electrolysis-cell used was that represented by figure 13 (see page 189). The cracking of the charcoal was attended by evolution of numerous bubbles of gas (probably air previously absorbed by the carbon); on this account, and also in consequence of the fume developed by the heat of conduction-resistance in the charcoal, it was difficult to ascertain if the anode evolved gas due to electrolysis; nevertheless, with the aid of a strong light and watching

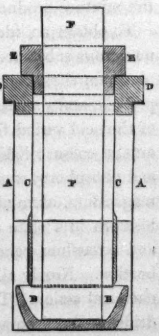
* For the preparation of articles and vessels of charcoal for these experiments, see Philosophical Magazine, September 1868.

for opportunities, it was found that the passage of the current was *not* attended by any distinct increase of gas from the anode. No special odour could be detected above the anodes, but on removing the charcoal from the liquid it evolved a feeble odour somewhat like that of chlorine, in addition to the intense one of the acid. The kinds of wood employed for making the charcoal were beech, kingwood, ebony, lignum-vitæ, box, mahogany, zebra, Memel oak, English oak, rosewood, maple, lancewood, walnut, Norwegian pine, and ironwood. The fracture of the carbons was not caused by the lowness of the temperature of the acid.

With an anode of pure sheet gold the acid scarcely conducted at all, except for a short time, even with forty SMEE's elements; by continuing the action about 1½ hour, the anode acquired a dark reddish-brown film and a few crystals upon its edges, which were at first of a green colour, but became red by exposure to the air. With a palladium anode the acid conducted somewhat less freely than with one of platinum or gas-carbon; and by employing forty SMEE's elements the palladium anode was also corroded. The salts which fall off the anodes appear to be wholly converted into metal by the disseminated minute bubbles of hydrogen, and even some of the coating of the anode itself is partly so converted if the hydrogen is allowed to come into contact with it. The action in the electrolysis-cell was sometimes watched by the aid of a magnesium light, as the fume was usually too dense to admit of inspection by ordinary light.

The most effective form of electrolytic apparatus employed is represented by the annexed figure. A A is a circular and deep platinum cup; B B is a shallow and loosely fitting cup of paraffin provided with a handle (not shown); C C is a wide tube of platinum open at both ends, and fixed at its upper end within an equally wide circular perforation in the annular paraffin stopper, D D; E E is an inner stopper of paraffin fitting within the other, and supporting a sheet of platinum or other metal, F F, which passes through it and constitutes the anode. The use of the tube C C is to prevent the hydrogen evolved at the inner surface of the cathode, A A, coming into contact with the anode, F. The tube C C is coated on its outer side at its lower end and up to a short distance above the level of the liquid with a thin layer of paraffin, to prevent the tendency of electrolytic action upon the tube C C. Electrolysis tends to first decompose any traces of water the acid may contain, and thus maintain the acid in an anhydrous state.

Fig. 13.



Two ounces of strong sulphuric acid was added to a mixture of 336 grains of pure fluoride of sodium and 472·5 grains of nitrate of potassium, both in fine powder, in a platinum retort, the mixture distilled, and the product received in a platinum bottle. The distillate weighed 678·2 grains, it should have weighed only 664 grains; the excess was probably water, from the sulphuric acid not being of maximum strength. In applying the heat, the retort was immersed in a bath of linseed-oil, the temperature of which

was not raised above 450° Fahr. No permanent gas was evolved during the process. The product was a colourless fuming liquid, much less fuming than anhydrous hydrofluoric acid, and did not corrode paraffin or platinum, but corroded vulcanized india-rubber rapidly. This liquid was electrolyzed by means of six Smee's elements and two large platinum electrodes, during five hours, the liquid being in a covered narrow and deep platinum cup immersed in a freezing-mixture; conduction occurred freely, and an odourless gas (doubtless ordinary oxygen) was freely evolved from the anode; the anode was not corroded, and both electrodes were unaltered in weight.

520 grains of anhydrous hydrofluoric acid was distilled from about 2000 grains of KF, HF in a platinum retort through a platinum tube surrounded by the freezing-mixture into a platinum bottle (surrounded by the same mixture) containing 334·17 grains of fibrous sulphuric anhydride; the first contact of the two substances was attended by a hissing sound and evolution of great heat, and the resulting compound was a colourless liquid, of a less fuming character than either of the two substances alone. In this experiment it was necessary to cement the junctions near the receiver with the melted sulphur mixture (see p. 173), as paraffin is rapidly blackened by the vapours of sulphuric anhydride*. The compound formed in this case was probably analogous in some respects to partially hydrated sulphuric acid, 2HF taking the place of H_2O, and corresponds to the third hydrate of sulphuric acid, or H_2SO_4, $2H_2O$. This liquid was also electrolyzed by means of platinum electrodes and ten Smee's elements; the conduction was moderate (less than with HF alone), and the anode was corroded. Sulphuric anhydride added to the mixture produced heat.

To obtain an idea of the *general* chemical behaviour of the pure anhydrous acid, numerous substances (generally anhydrous) were immersed in separate portions of it in platinum cups, contained in a tin chilling-case sunk deep in the freezing-mixture (see p. 173), and sometimes covered with plates of transparent fluor-spar†; the temperature of the acid varied from 20° to 0° Fahr. It slightly dissolved bromine to a liquid of an orange colour; iodine, carbon, boron, silicon, sulphur, selenium, tellurium (in powder), and phosphorus were unchanged. The noble and base metals were unaffected; and even magnesium, aluminium, zinc, cadmium, tin, lead, reduced iron, arsenic, antimony, and bismuth in a state of fine powder did not cause any evolution of hydrogen. Sodium and potassium behaved much the same as with water, the latter producing vivid combustion. Nearly all the salts of the alkali and alkaline-earth metals produced strong chemical action. The acid produced violent chemical action with the following anhydrides:—fused boracic, silicic (only when in fine precipitated powder, not with small granules of quartz or even with fine white sand), sulphuric, phosphoric, molybdic, vanadic, and niobic anhydrides; selenious and arsenious anhydrides dissolved freely; antimonious, tungstic, and titanic anhydrides were either but little or not at all affected nor dissolved;

* Gun-cotton inflamed by contact with sulphuric anhydride.

† I have obtained suitable vessels, plates, and stoppers of fluor-spar of Mr. J. Tym, Speedwell Mine, Castleton, Derbyshire.

chlorochromic acid would not mix with the acid. With concentrated nitric and sulphuric acids the acid mixed quietly, but with strong hydrochloric acid it produced active effervescence. With lime, baryta, strontia, and the hydrates of soda and potash it united powerfully, and with magnesia less strongly. No effect occurred with the peroxides of silver, iron, lead, thallium, or manganese; nor with the suboxide or protoxide of copper, the oxides of nickel, cobalt, tin (both oxides), or cerium. Teroxide of antimony, bismuth, and the oxides of silver, glucinum, and lanthanum produced strong chemical action, but did not dissolve. The oxides of mercury (both) and lead (both) were changed to fluorides, and apparently also were oxide of zinc and peroxide of barium, the latter with most violent action (probably water was present). Binoxide of platinum was slightly dissolved, and the oxides of uranium and chromium freely; chemical changes also occurred with the lower oxides of vanadium and titanium, and with alumina. The nitrates of lead, uranium, strontium, and barium exhibited no strong chemical action, and did not dissolve, but signs of free nitric acid appeared; nitrate of silver was unaffected (?), nitrate of potassium dissolved freely with a hissing sound; nitric acid was found, by suitable tests, to be set free with the nitrates of lead, barium, and potassium. The fluorides of antimony, bismuth, mercury, copper, nickel, cobalt, iron, lead, tin (both), cadmium, zinc, manganese, cerium, aluminium, magnesium, and calcium were either slightly or not at all affected, and did not dissolve; the fluorides of uranium and chromium exhibited chemical change and dissolved. Alkaline fluorides were strongly affected; the fluorides of silver, barium, and lithium, also cryolite, hissed strongly but did not dissolve; and those of thallium, strontium, sodium, potassium, rubidium, and ammonium produced different degrees of violent chemical action and solution; the acid fluoride of potassium also produced strong action and solution: the behaviour of the alkaline fluorides with the anhydrous acid may be compared to that of the alkaline oxides and hydrates with water. Solid terchloride of carbon, and the chlorides of silver, mercury (both), copper, nickel, cobalt, lead, cadmium, manganese, and the violet sesquichloride of chromium were undissolved and unaffected; the liquid tetrachloride of carbon, and the liquid chlorides of sulphur, phosphorus, arsenic, and tin would not mix with the acid, whilst the liquid perchloride of antimony and tetrachloride of titanium were decomposed with violent action and formation of deliquescent solid residues, as when water is added to them; the solid perchloride of phosphorus was also violently decomposed and a white powder formed; the chlorides of calcium, barium, lithium, sodium, potassium, and ammonium were decomposed with strong action and effervescence, the latter four with solution and especially strong action. The chlorates of sodium and potassium were rapidly decomposed with evolution of yellow fumes of peroxide of chlorine; no visible effect occurred with the chlorate of barium or perchlorate of potassium. Terbromide of antimony turned white, as on addition of water, and did not dissolve; the bromides of zinc and cadmium were but little affected; the bromides of calcium, barium, and potassium were decomposed with strong effervescence, and the latter bromide dissolved; bromate of potassium was also rapidly decomposed with copious liberation of bromine. The ter-

iodides of arsenic and antimony, the tetraiodide of platinum, iodide of silver, biniodide of mercury, subiodide of copper, and the iodides of lead, cadmium, and zinc were unaffected, whilst the iodides of calcium, barium, sodium, and potassium were strongly decomposed, and iodine freely liberated in the first three instances. Iodate of potassium also exhibited some chemical action, but no iodine was set free. The acid decomposes all carbonates; the anhydrous carbonates of silver, copper, cobalt, lead, cadmium, zinc, manganese, calcium, strontium, barium, lithium, sodium, potassium, and ammonium were all decomposed with effervescence; those of the alkali and alkaline-earth metals exhibited violent action. Fused borax and the borate of ammonium produced powerful chemical action. The boro-fluorides of sodium and potassium were unaffected. Glass was rapidly corroded. (N.B. It was not corroded in the *gaseous* acid: see page 184.) The silico-fluorides of sodium and potassium dissolved with effervescence; titano-fluoride of potassium also dissolved with strong action. Carbonic bisulphide would not mix with the acid; the acid had also no chemical or solvent effect upon the sulphides of arsenic (both), antimony (the black sulphide), palladium, mercury (vermilion), copper (subsulphide), iron (both), lead, tin (the bisulphide), cadmium, zinc, and molybdenum; the sulphides of calcium, barium, sodium, and potassium were violently decomposed with evolution of sulphuretted hydrogen, and in the latter instance with separation of liquid sulphur. Bisulphite of sodium dissolved with effervescence. Sulphate of cobalt was unaffected; the sulphates of calcium, strontium, and barium partly dissolved with a hissing sound; sulphate of sodium and acid sulphate of potassium exhibited strong action and dissolved. Chrome-alum dissolved quietly to a purple liquid, and sulphate of ammonium to a colourless liquid. Phosphide of copper was unaffected; phosphide of calcium was powerfully decomposed; phosphate of copper was also decomposed; phosphate of uranium produced action; phosphate of calcium was partly dissolved, and phosphate of ammonium completely. Arsenite of copper was decomposed; arsenite and arseniate of ammonium, and the arseniate of magnesium and ammonium, produced violent action; the first and third were dissolved. Damp crystals of permanganate of potassium hissed and dissolved to a deep-green liquid. Red chromate of lead became yellow and partly dissolved; yellow chromate of lead dissolved to a yellow liquid; the mono-chromates of sodium and potassium, and the acid chromates of sodium, potassium, and ammonium dissolved with violent action to blood-red liquids, with evolution of vapour of fluoride of chromium; vanadate and metavanadate of ammonium produced powerful action, the former partly, and the latter wholly dissolving to yellow liquids. Native molybdate of lead was unaffected; molybdate of ammonium produced strong action but did not dissolve. The tungstates of sodium and ammonium were violently decomposed, the former dissolved, and the latter remained insoluble. Cyanide of mercury was partly dissolved; cyanide of potassium was violently decomposed with liberation of hydrocyanic acid; anhydrous ferrocyanide of potassium dissolved with violent action, and the solution became milky on addition of water; ferricyanide of potassium dissolved to a yellow liquid, and nitrocyanide of titanium was unaffected.

The acid mixed with pyroxylic spirit with strong action, and with ether and alcohol.

Oxalic acid, oxalate, and benzoate of ammonium were but little affected. Paraffin was unaffected. Benzole would not mix with the acid. Spirit of turpentine exploded and produced a blood-red liquid. Gutta percha, india-rubber, ebonite, parkesine, white wax, sealing-wax, gum-arabic, amber, copal, dammar, gamboge, juniper, kino, kowrie, shellac, and ordinary resin were rapidly disintegrated, and in nearly all cases quickly dissolved to red liquids. Spermaceti, stearic acid, and myrtle wax were but little affected. Caffeine, indigo, sugar of milk, and cane-sugar dissolved freely and rapidly. Gun-cotton dissolved instantly to a colourless liquid, which left an inflammable film on evaporation. Silk also instantly dissolved to a glutinous colourless liquid. Paper, cotton-wool, calico, gelatine, and parchment were instantly converted into glutinous substances and dissolved. Horn instantly expanded to a bulky white mass. Irish moss, German-tinder, and flannel were less rapidly affected, and sponge was less affected than any other porous organic substance. Pine-wood instantly blackened.

The chemical behaviour of such of these numerous substances with the anhydrous liquid acid as have been further examined will be more fully described in their respective places.

The *gaseous* acid carbonized cork, and acted upon wood, paper, gutta percha, india-rubber, vulcanized india-rubber, and bees-wax more slowly, but in much the same manner as the liquid acid; sealing-wax absorbed the gas and became soft and disintegrated. It did not act upon carbon, sulphur, or selenium, nor, if perfectly dry, did it act upon glass, as has been already stated (see pages 184 & 192).

From the various experiments already described, we may conclude that hydrofluoric acid is by its properties placed between hydrochloric acid and water, but is much more closely allied to the former than to the latter. It is more readily liquefied than hydrochloric acid, but less readily than steam: like hydrochloric acid it decomposes all carbonates; like water it unites powerfully with sulphuric and phosphoric anhydrides, with great evolution of heat. The fluorides of the alkali metals unite violently with anhydrous hydrofluoric acid, like the oxides of those metals unite with water; the hydrated fluorides of the alkali metals also, like the hydrated fixed alkalies, have a strongly alkaline reaction, and are capable of expelling ammonia from its salts. It may be further remarked that the atomic number of fluorine lies between that of oxygen and chlorine; and the atomic number of oxygen, added to that of fluorine, very nearly equals that of chlorine.

B. *Aqueous Hydrofluoric Acid.*

Ordinary hydrofluoric acid is very impure; it usually contains large quantities of sulphuric acid (distilled over by excess of heat), much sulphurous acid (from action of the oil of vitriol upon the iron retort), some hydrofluosilicic acid (from silica in the spar), arsenic (from the sulphuric acid), and small amounts of hydrochloric acid, iron, lime, and other fixed matters. A sample manufactured in Birmingham, of specific gravity 1·399, was found by analysis to contain 29·58 per cent. of H_2SO_4, and 0·5 per cent. of ignited fixed matter which had been previously precipitated by excess of ammonia; it

also gave 4·08 per cent. of yellow sulphur (containing arsenic) by decomposing the sulphurous acid in it by an excess of sulphuretted hydrogen. A second sample, sent to me by the same makers, had a specific gravity of 1·310, and contained 4·8 per cent. of H_2SO_4. A third sample, made near Birmingham, of specific gravity 1·217, contained 3·27 per cent. of H_2SO_4 and 2·16 per cent. of ignited fixed matter precipitated by ammonia, including 0·76 per cent. of silica = 1·824 per cent. of 2HF, SiF_4. A fourth sample, obtained from Wolverhampton, of specific gravity 1·1097, contained 0·55 per cent. of H_2SO_4. A fifth, obtained from London, of specific gravity 1·1894, contained 10·54 per cent. of H_2SO_4. A sixth, from Görlitz, in Prussia, of specific gravity 1·1741, contained only traces of sulphuric acid. A seventh, from Mr. Horner, of Sheffield, of specific gravity 1·188, contained traces only of sulphuric, hydrochloric, or hydrofluosilicic acids, or of fixed matter. And an eighth sample, from Mr. William Collier, Sheffield, of specific gravity 1·248, contained no sulphuric or hydrochloric acid, and left only a trace of solid residue on evaporation to dryness: by neutralization with ammonia, 1·70 per cent. of fixed matter was obtained after washing and ignition; this fixed matter contained 1·46 part of SiO_2 = 3·5 per cent. of 2 HF, SiF_4 in the acid; the fixed matter also contained iron, &c. A second sample, weighing twelve pounds, from the same maker gave 723 grains of sulphur by treatment with excess of sulphuretted hydrogen. All the samples contained large but variable amounts of sulphurous acid, and gave copious deposits of sulphur &c. by treatment with excess of sulphuretted hydrogen; they also contained very variable percentages of HF, varying from about 10 to 70. Oil of vitriol is sometimes added to weak hydrofluoric acid to enable it to etch glass in a suitable manner; it imparts to it a fictitious strength, probably by abstraction of water. A sixty per cent. acid requires dilution before it can be used for etching glass; too strong an acid etches glass roughly. An acid of 70 per cent., and containing 5·76 per cent. of H_2SO_4, instantly converted filtering-paper into a jelly at 60° Fahr.; it was too strong for retention in gutta-percha bottles.

A small gutta-percha jug of the annexed form was sometimes employed for taking the specific gravities of samples of weak acid. The neck of the vessel was less than half an inch internal diameter, and was closed by placing a gutta-percha ball with a handle loosely in it. Any excess of liquid above the ball was absorbed by application of sponge previous to weighing.

Fig. 14.

Sulphurous acid is frequently present in commercial hydrofluoric acid to such an extent as to be readily detected by the odour; it may be easily found by adding shavings of metallic tin to the acid. Tin was found to be much better than zinc; it produced a cloud of sulphur at once, and evolved sulphuretted hydrogen readily. Bifluoride of tin also evolved sulphuretted hydrogen easily. Sulphuretted hydrogen gas in excess was passed through commercial hydrofluoric acid; some time after the commencement of its passage free sulphur appeared, and then accumulated quickly. No increase of sulphuric acid was found in the liquid after this reaction. If the reaction is correctly represented by the following equation, $2H_2S + SO_2 = 2H_2O + 3S$, then 96 parts by weight of sulphur

separated, equal 64 parts of sulphurous acid present. One gallon of a mixture of different samples of commercial hydrofluoric acid of specific gravity 1·25 yielded by the above treatment 3½ avoirdupoise ounces of dry sulphur=1·145 per cent. of SO_2. Another sample yielded sulphur equal to 2·72 per cent. of SO_2 thus. I have not determined the amount of hydrochloric acid in commercial hydrofluoric acid; it is, however, quite small; nor have I ascertained the quantity of arsenic, which is also small.

The total amount of all the free acids present in a sample was roughly determined by adding a known weight of pure precipitated dry carbonate of calcium to it, until the liquid exhibited a neutral reaction to test-paper after expulsion of the liberated carbonic acid by heat. A standard solution of carbonate of sodium is inapplicable for the purpose, because the fluoride of sodium produced has an alkaline reaction.

To find the amount of free sulphuric acid, 100 grains of the acid was carefully evaporated in a platinum dish at a gentle heat until all sulphurous, hydrofluoric, hydrochloric, and hydrofluosilicic acid was expelled; the residue was then exactly neutralized by a standard solution of carbonate of sodium in the usual manner, and the amount of sulphuric acid calculated therefrom. The residuary acid liquid was in some cases diluted with water, filtered, acidified with a little hydrochloric acid, and the total amount of sulphuric acid in it determined by the ordinary gravimetric process with solution of chloride of barium.

To determine the amount of hydrofluosilicic acid, an excess of dilute aqueous ammonia was added to 100 grains of the acid previously diluted in a large platinum dish, and the silica separated from the washed precipitate (which usually contains iron and sometimes lime) in the usual manner and weighed. 30 parts of $Si O_2$ equal 72 parts of HF, $Si F_4$.

The most feasible method of determining the amount of HF in the impure acid consists in neutralizing it with hydrate of potash, evaporating the liquid to a small bulk, adding an excess of precipitated silica, heating the mixture with sulphuric acid, and ascertaining the amount of silica dissolved, and calculating it as fluoride of silicon (see pages 176, 177, & 178). Numerous attempts were made to determine the amount of HF by diluting the acid, digesting it with pure precipitated silica, and calculating from the amount of silica dissolved the amount of HF present; but the results were not even approximately accurate, they were calculated too highly, apparently owing to the formation of a variable mixture of $Si F_2$ and HF, $Si F_2$.

The sulphate-of-calcium residue from the manufacture of hydrofluoric acid is liable to contain minute amounts of potassium, sodium, lithium, thallium, and copper; also iron freely (derived from the action of the ingredients upon the iron retort), and some free sulphuric acid. Upwards of fifty-six pounds weight of the residue was digested and washed with eight or ten gallons of distilled water, the clear liquid evaporated to small bulk in a leaden pan, and the greater portion of the free sulphuric acid expelled by heating in a platinum dish, the residue diluted, filtered, and precipitated by sulphuretted hydrogen (the yellowish-brown precipitate contained copper and thallium); most of the iron was then separated as sulphate by crystallization, and nearly all the remainder pre-

cipitated by ammonia; the filtrate was evaporated and ignited in a platinum dish to expel the sulphate of ammonium; the ignited residue was dissolved in water and precipitated by carbonate of ammonium; the precipitate contained thallium distinctly, and the filtrate contained thallium, sodium, lithium, and potassium salts.

The following is the process employed by me for purifying ordinary hydrofluoric acid. Pass sulphuretted hydrogen through it until it contains the gas in excess; this separates the sulphurous acid (and also the arsenic if the acid is not too strong). Stir thoroughly. Find the amounts of free sulphuric and hydrofluosilicic acids present by the means already described (see page 195), and add a little more than sufficient carbonate of potassium to neutralize them, *i. e.* 138 parts of anhydrous carbonate to 98 parts of H_2SO_4, or 60 parts of SiO_3*. Subside until quite clear and cold, and decant the clear liquid. The effervescence, stirring, and exposure to the air expels nearly all the excess of sulphuretted hydrogen; careful addition of carbonate, oxide (or fluoride) of silver will remove the remainder. Distil the clear liquid in a leaden retort immersed in a bath of linseed-oil at a temperature varying from about 320° Fahr. to about 460° Fahr., using a Liebig's condenser, containing a platinum tube to condense the vapour†. The acid requires to be redistilled once to render it quite pure. If the liquid product contains more than about 40 per cent. of anhydrous acid it will probably contain arsenic; this may be precipitated by diluting the liquid and passing a prolonged current of sulphuretted hydrogen gas through it. Avoid contact of the pure acid with leaden vessels. (Or the following modification of the process might probably be employed:—Find amounts of sulphuric and hydrofluosilicic acids present; add more than sufficient carbonate or sulphide of potassium to neutralize them. Subside, decant the liquid, and dilute it with water until it contains not more than about 40 per cent. of anhydrous acid. Pass an excess of sulphuretted hydrogen through it, filter, and distil.) I have made several gallons of the pure acid on various occasions by the first of these processes.

Hot junctions in the distillatory apparatus are best made gas-tight by vulcanized india-rubber washers, or by binding them round with ribbon of the thinnest and best quality of vulcanized rubber secured by tarred string. Strong hydrofluoric acid containing traces of lead yielded no precipitate on addition of sulphuretted hydrogen water; nor did I succeed in removing the lead by electrolysis; it was partly precipitated by hydrochloric acid and by sulphuric acid.

The acid prepared by the foregoing process was very pure; it yielded only a trace of fixed matter (probably derived from the gutta-percha bottle, see pages 193 & 194) on evaporation to dryness in a bright platinum dish; it evolved no sulphuretted hydrogen or sulphur by addition of zinc or tin; it gave no precipitate or coloration by addition of the following substances:—excess of aqueous ammonia or of pure anhydrous carbonate

* One gallon of mixed samples of ordinary hydrofluoric acid required 22½ ounces of carbonate of potassium of 92 per cent. to neutralize its sulphuric and hydrofluosilicic acids.

† The tube employed by me was 6 feet long and ½ inch diameter, and was kindly lent to me by Messrs. Matthey and Co.

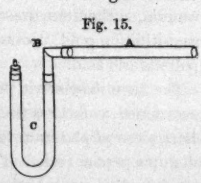
of potassium, strong solution of fluoride of potassium, excess of sulphuretted hydrogen gas (either before or after neutralization of the acid by ammonia), dilute hydrochloric acid, solution of nitrate of silver, nor by adding a single drop of solution of nitrate of barium acidulated with nitric acid; it was therefore free from all impurities at all likely to be present. Additional tests were employed which need not be enumerated. This aqueous acid was employed in making the various fluorides described in this paper. I have found a plentiful supply of the pure aqueous acid of considerable service for removing (by digestion and evaporation) the silica in minerals, especially in searching for minute ingredients in those containing much silica.

The amount of anhydrous hydrofluoric acid contained in the pure aqueous acid of various degrees of strength was ascertained both by the silica-process (see pages 176, 177, 178, and 195) and also by the lime-process (see page 178).

Aqueous hydrofluoric acid of a high percentage was also on two occasions prepared by heating dry fluoride of chromium in a suitable platinum apparatus, and condensing the vapour by immersing the condensing-tube in a mixture of snow and salt. The chromium salt was prepared by precipitating a solution of chrome-alum by aqueous ammonia, and boiling with excess of the ammonia, washing the precipitate with boiling water, dissolving it whilst still wet in pure dilute hydrofluoric acid, and evaporating the solution to perfect dryness in a platinum dish. The residue was a pale green earthy powder, non-deliquescent, and very slowly soluble in water or dilute hydrofluoric acid; it lost upwards of 40 per cent. in weight of acid vapour when heated to redness in platinum vessels, without contact of the substance with the products of combustion from the gas-burner.

The following apparatus was employed:—A (fig. 15) is a retort-tube 7 inches (=17·8 centimetres) long and $\frac{5}{8}$ of an inch (=1·6 centim.) diameter; B is an angle-tube, and C is a curved tube about 5 inches (=12·8 centims.) high and $\frac{1}{2}$ an inch (=1·2 centim.) diameter. Each tube was formed of platinum without the use of solder. The tubes fitted tightly upon each other, and the junctions were made gas-tight by means of the melted mixture of sulphur and lampblack. The outer end of the tube C was closed by a paraffin stopper containing a small platinum escape-tube.

Fig. 15.



50 grains weight of the dry chromium salt was placed in the tube A at its closed end, and the whole tube heated somewhat above the boiling-point of water; no perceptible loss of weight occurred: the tubes were then joined together, the curved tube C immersed in snow, a wet rag laid upon A near its junction with B, and heat very *gradually* applied to the outer end of A during about twenty minutes until about 3 inches in length of the tube was red-hot; the tube A was then taken off, the adhering cement removed, and the tube weighed; the loss of weight was 22·10 grains=44·20 per cent. (this number, however, is a little too high because traces of the solid substance were carried forward mechanically into the angle-tube); only a trace of vapour escaped con-

densation. Various other fluorides besides that of chromium evolve much hydrofluoric acid when heated.

A very weak acid became concentrated to a large extent by desiccation over strong sulphuric acid in a closed leaden chamber at 60° FAHR.; but as the concentration proceeded the acid itself began to volatilize along with the water, until finally the vessel became empty. The sulphuric acid acquired an acid odour as if it had absorbed some of the hydrofluoric acid.

Pure aqueous hydrofluoric acid of about 40 per cent. in strength did not show any signs of solidifying by prolonged immersion in a freezing-mixture at −24° FAHR.; nor did a mixture of two measures of that acid with one of water freeze at that temperature; but with $1\frac{1}{2}$ measure it froze imperfectly, and with 2 measures became nearly solid: no vapour of hydrofluoric acid appeared to escape during the solidification. It is evident from these experiments that a comparatively small quantity of hydrofluoric acid lowers the freezing-point of water very considerably.

The chemico-electric series of metals &c. in pure dilute hydrofluoric acid at 60° FAHR. containing about 10 per cent. of anhydrous acid was determined. The order obtained was as follows, the most positive substance being named first:—Aluminium, zinc, magnesium, thallium, cadmium, tin, lead, silicon, iron, nickel, cobalt, antimony, bismuth, copper, silver, gold, gas-carbon, and platinum. The aluminium evolved gas rapidly. The following series was obtained with pure hydrofluoric acid containing about 30 per cent. of anhydrous acid:—Zinc, magnesium, aluminium, thallium, indium, cadmium, tin, lead, silicon, iron, nickel, cobalt, antimony, bismuth, mercury, silver, copper, arsenic, osmium, ruthenium, gas-carbon, platinum, fused rhodium, palladium, tellurium, fused osmi-iridium, gold, iridium*. Magnesium was remarkably unacted upon by aqueous hydrofluoric acid.

The chemico-electric position of aqueous hydrofluoric acid to that of other acids was ascertained as follows:—A diaphragm of parchment-paper was fixed upon the end of a short piece of platinum tube, about $\frac{1}{4}$ an inch wide and 1 inch long, by means of a ring of gutta percha sealed at its edge with paraffin. The acid to be compared with it was put into this porous vessel, and the vessel immersed in aqueous hydrofluoric acid contained in an outer cup of platinum. With the outer vessel connected with one pole of a galvanometer and a sheet of platinum immersed in the inner liquid and connected with the other pole, and with acid of $28\frac{1}{2}$ per cent. in the outer vessel, the following electrical order of acids was found, the first-named being the most electro-positive:—Strong aqueous phosphoric acid, aqueous hydrofluoric acid, concentrated sulphuric acid, strong hydrochloric acid, strong nitric acid. The current in each case passed from the positive acid through the diaphragm to the other acid.

According to FARADAY (GMELIN'S Handbook, vol. i. p. 455) hydrated hydrofluoric acid is not decomposed by electrolysis, only the water in it is decomposed. I electrolyzed

* I am indebted to the kindness of Messrs. MATTHEY and Co. for the specimens of osmium, ruthenium, rhodium, and iridium.

dilute pure hydrofluoric acid of about 10 per cent. by means of six SMEE's cells, a platinum-foil anode, and a platinum crucible cathode. Gas was liberated freely from each electrode, and a very strong odour like that of ozone was evolved. No corrosion of either electrode took place during twelve hours' action. I also electrolyzed the same acid by six GROVE's cells and platinum electrodes, the anode being enclosed air-tight within a gutta-percha funnel inverted in the acid, an exit-tube with a tap being in the side of the neck of the funnel for regulating the escape of the gas. Free conduction occurred. Plenty of gas was evolved. The gas from the anode smelt powerfully of ozone, and inflamed a red-hot charcoal splint vividly. Paper wet with spirits of turpentine was not blackened in the strongly issuing stream of gas, nor was bright silver affected in it. No corrosive action occurred upon the anode. I prepared some pure hydrofluoric acid containing about 80 per cent. of anhydrous acid, by taking one part by weight of 40 per cent. pure hydrofluoric acid in a bent tube C (of the apparatus, page 197) immersed in a stream of cold water, and passing through it about two parts by weight of anhydrous hydrofluoric acid vapour, obtained by very gradually fusing and heating to redness eight parts of anhydrous double fluoride of hydrogen and potassium contained in the platinum tube-retort A, to which the bent platinum tube was fixed. The acid thus obtained was electrolyzed by ten SMEE's cells with platinum electrodes in a narrow and deep platinum cup, the cup being the negative pole. Copious conduction occurred; a powerful odour of ozone was evolved, indicating decomposition of the water and evolution of oxygen gas at the anode; much gas was liberated, and doubtless consisted chiefly of hydrogen. Heat was evolved in the liquid by the conduction-resistance. The anode dissolved, but not very rapidly; in three hours it lost 1·58 grain, and indicated the simultaneous decomposition of the acid with that of the water, the latter being the greatest in amount. The smell of ozone disappeared if the battery was weak, and reappeared on first contact. In a further eleven hours the anode lost 5·05 grains, and was covered with a blackish crust; this crust was partly soluble in water to a brownish solution. In a further twenty-two hours the loss had increased from 5·05 to 15·00 grains without any signs of an electro-deposit of platinum upon the platinum cathode tube. The total loss of the anode was 1·58+15·00 grains=16·58 grains. Found 15·5 grains of black-brown powder which showed some metal by pressure between surfaces of agate. I also electrolyzed pure 30 per cent. hydrofluoric acid by platinum electrodes and ten SMEE's cells, the liquid being mixed with other pure acids. With equal volumes of hydrofluoric acid and concentrated nitric acid; gas was evolved freely from the anode; it had no odour of ozone, and was probably ordinary oxygen. No gas was visible at the cathode; the liberated hydrogen was doubtless absorbed by the nitric acid. Only a trace of platinum was dissolved during sixteen hours' action; the liquid was then pale yellow, but became dark green, evolved abundance of red-brown fumes, and became hot on addition of water. With equal volumes of aqueous hydrofluoric and strong hydrochloric acids, much chlorine was evolved from the anode and hydrogen from the cathode; this agrees with the usual results that chlorides (as well as oxides) are decomposed before fluorides, and hydro-

chloric acid before water in electrolysis. Only a trace of chloride of platinum was formed at the anode in four hours, and the anode was not visibly corroded. With equal volumes of aqueous hydrofluoric and strong sulphuric acids much gas and a strong odour of ozone was evolved from the anode; hydrogen also from the cathode. The anode corroded very slowly, without being covered with an insoluble film, and the liquid became cloudy with suspended particles, which were light brown at first, then brown, and after many hours' action nearly black. The fumes from this mixture rapidly blackened gutta percha. With hydrofluoric acid containing much selenious acid dissolved in it, gas was evolved from both electrodes, and a liberal deposit of red selenium formed at the cathode. No odour of ozone occurred until a large quantity of red and black selenium (probably nearly all) was deposited upon the sides and bottom of the platinum cup, then ozone occurred strongly. The anode was not visibly corroded in twenty-eight hours' active electrolysis. The solution was filtered and evaporated to a small bulk; it then fumed densely: the odour was like that of hydrofluoric acid, and when cold the liquid was odourless and of a somewhat oily consistence. The final liquid evolved a little heat on adding water to it. I found traces of platinum in the deposited selenium. With aqueous hydrofluoric acid, to which some phosphoric anhydride had been freely added, ozone was evolved from the anode, and hydrogen from the cathode; the anode was slowly corroded, and a small quantity of black precipitate formed; the liquid remained nearly colourless.

VI. *A Memoir on the Theory of Reciprocal Surfaces.* By Professor CAYLEY, *F.R.S.*

Received November 12, 1868,—Read January 14, 1869.

THE present Memoir contains some extensions of Dr. SALMON's theory of Reciprocal Surfaces. I wish to put the formulæ on record, in order to be able to refer to them in a "Memoir on Cubic Surfaces," but without at present attempting to completely develope the theory.

Extension of SALMON's *Fundamental Equations.* Article Nos. 1 to 5.

1. The notation made use of is that of SALMON's 'Geometry,' pp. 450–459, with the additions presently referred to; the significations of all the symbols are explained by way of recapitulation at the end of the Memoir. I remark that my chief addition to SALMON's theory consists in a modification of his fundamental formulæ (A) and (B); these in their original form are

$$\begin{aligned}
a(n-2) &= \alpha + \varrho + 2\sigma,\\
b(n-2) &= \varrho + 2\beta + 3\gamma + 3t,\\
c(n-2) &= 2\sigma + 4\beta + \gamma,\\
a(n-2)(n-3) &= 2\delta + 3[ac] + 2[ab],\\
b(n-2)(n-3) &= 4k + [ab] + 3[bc],\\
c(n-2)(n-3) &= 6h + [ac] + 2[bc],
\end{aligned}$$

where

$$\begin{aligned}
[ab] &= ab - 2\varrho,\\
[ac] &= ac - 3\sigma,\\
[bc] &= bc - 3\beta - 2\gamma - i.
\end{aligned}$$

2. I take account of conical and biplanar nodes, or, as I call them, cnicnodes, and binodes; of pinch-points* on the nodal curve; and of close-points and off-points on the cuspidal curve: viz. I assume that there are

C, cnicnodes,
B, binodes,
j, pinch-points,
χ, close points,
θ, off-points,

* This addition to the theory is in fact indicated in SALMON, see the note, p. 445; the i there employed, which is of course different from the i of his text, is the j of the present Memoir.

deferring for the present the explanation of these singularities. The same letters, accented, refer to the reciprocal singularities. Or using "trope" as the reciprocal term to node, these will be

C′, cnictropes,
B′, bitropes,
j', pinch-planes,
χ', close-planes,
θ', off-planes;

but these present themselves, not in the equations above referred to, but in the reciprocal equations.

3. The resulting alterations are that we must in the formulæ write $\varkappa - B$, $\delta - C$ in place of $\varkappa$, δ respectively; and change the formulæ for $c(n-2)$, $[ab]$, $[bc]$, into

$$c(n-2) = 2\sigma + 4\beta + \gamma + \theta,$$
$$[ab] = ab - 2\varrho - j,$$
$$[ac] = ac - 3\sigma - \chi$$

respectively.

4. Making these changes, and substituting for $[ab]$, $[ac]$, $[bc]$ their values, the formulæ become

$$a(n-2) = \varkappa - B + \varrho + 2\sigma,$$
$$b(n-2) = \varrho + 2\beta + 3\gamma + 3t,$$
$$c(n-2) = 2\sigma + 4\beta + \gamma + \theta,$$

$$a(n-2)(n-3) = 2(\delta - C) + 3(ac - 3\sigma - \chi) + 2(ab - 2\varrho - j),$$
$$b(n-2)(n-3) = 4k + (ab - 2\varrho - j) + 3(bc - 3\beta - 2\gamma - i),$$
$$c(n-2)(n-3) = 6h + (ac - 3\sigma - \chi) + 2(bc - 3\beta - 2\gamma - i),$$

which replace the original formulæ (A) and (B).

5. For convenience I annex the remaining equations; viz. these are

$$a' = n(n-1) - 2b - 3c,$$
$$\varkappa' = 3n(n-2) - 6b - 8c,$$
$$\delta' = \tfrac{1}{2}n(n-2)(n^2-9) - (n^2-n-6)(2b+3c) + 2b(b-1) + 6bc + \tfrac{3}{2}c(c-1);$$

the equations

$$q = b^2 - b - 2k - 3\gamma - 6t,$$
$$r = c^2 - c - 2h - 3\beta,$$

(q, r in place of SALMON's R, S respectively); the equation

$$a = a';$$

and the corresponding equations, interchanging the accented and unaccented letters, in

all 23 equations between the 42 quantities

$$n, a, \delta, \varkappa; \quad b, k, t, \varrho, j; \quad c, h, r, \sigma, \theta, \chi; \quad \beta, \gamma, i'; \quad \mathrm{B}, \mathrm{C},$$
$$n', a', \delta', \varkappa'; \quad b', k', t', \varrho', j'; \quad d, h', r', \sigma', \theta', \chi'; \quad \beta', \gamma', i'; \quad \mathrm{B}', \mathrm{C}'.$$

Developments. Article Nos. 6 to 12

6. We have

$$\begin{aligned}(a-b-c\ )(n-2) &= (\varkappa-\mathrm{B}-\theta)-6\beta-4\gamma-3t,\\ (a-2b-3c)(n-2)(n-3) &= 2(\delta-\mathrm{C})\\ &\quad -8k-18h-6(bc-3\beta-2\gamma-i);\end{aligned}$$

and substituting these values of δ, $\varkappa$ in the formula

$$n'=a(a-1)-2\delta-3\varkappa,$$

and for a its value, $=n(n-1)-2b-3c$, we find

$$\begin{aligned}n'=n(n-1)^2-n(7b+12c)+4b^2+8b+9c^2+15c\\ -8k-18h+18\beta+12\gamma+12i-9t\\ -2\mathrm{C}-3\mathrm{B}-3\theta,\end{aligned}$$

where the foregoing equations for $a-b-c$ and $a-2b-3c$ show clearly the origin of the new terms $-2\mathrm{C}-3\mathrm{B}-3\theta$; these express that there is in the value of n' a reduction $=2$ for each cnicnode, $=3$ for each binode, and $=3$ for each off-point.

7. We have $(n-2)(n-3)=n^2-n+(-4n+6)=a+2b+3c+(-4n+6)$; and making this substitution in the equations which contain $(n-2)(n-3)$, these become

$$\begin{aligned}a(-4n+6)&=2(\delta-\mathrm{C})- \ a^2-4\varrho-9\sigma-2j-3\chi,\\ b(-4n+6)&= \quad 4k-2b^2-9\beta-6\gamma-3i+2\varrho-j,\\ c(-4n+6)&= \quad 6h-3c^2-6\beta-4\gamma-2i-3\sigma-\chi,\end{aligned}$$

(Salmon's equations (C)); and adding to each equation 4 times the corresponding equation with the factor $(n-2)$, these become

$$\begin{aligned}a^2-2a&=2(\delta-\mathrm{C})+ \ 4(\varkappa-\mathrm{B})-\sigma-2j-3\chi,\\ 2b^2-2b&= \quad 4k- \ \beta+6\gamma+12t-3i+2\varrho-j,\\ 3c^2-2c&= \quad 6h+10\beta+4\theta-2i+5\sigma-\chi.\end{aligned}$$

Writing in the first of these $a^2-2a=a(a-1)-a$, $=n'+2\delta+3\varkappa-a$, and reducing the other two by means of the values of q, r, the equations become

$$\begin{aligned}n'-a&=-2\mathrm{C}-4\mathrm{B}+\varkappa-\sigma-2j-3\chi,\\ 2q+\beta+3i+j&=2\varrho,\\ 3r+c+2i+\chi&=5\sigma+\beta+4\theta,\end{aligned}$$

(Salmon's equations (D)).

I attend in particular to the first of these, or rather to the reciprocal equation, which will be

$$\sigma' = a - n + \varkappa' - 2\jmath' - 3\chi' - 2C' - 4B',$$

which, writing therein $a = n(n-1) - 2b - 3c$, and $\varkappa = 3n(n-2) - 6b - 8c$, becomes

$$\sigma' = 4n(n-2) - 8b - 11c - 2\jmath' - 3\chi' - 2C' - 4B'.$$

The singularity σ' is not *explicitly* defined in SALMON; σ' is the reciprocal of σ, and (as such) it denotes the number of common tangent planes of the spinode torse and of the torse generated by the tangent planes along a plane section of the surface; or, what is the same thing, it is the number of the spinode planes which touch the plane section; that is, it is equal to the number of points of intersection of the spinode curve and the plane section; or, finally, σ' is the order of the spinode curve. The spinode curve is in fact for a surface of the order n without singularities the intersection of the surface by the Hessian surface of the order $4(n-2)$, and is thus a curve of the order $4n(n-2)$, which agrees with the formula.

8. But the formula shows that there is in the order a reduction $8b + 11c$ arising from the nodal and cuspidal curves of the surface, or, what is the same thing, that the Hessian surface meets the surface in the nodal curve taken 8 times, and in the cuspidal curve taken 11 times—a result which I had arrived at by other means, and also as appears *post*, No. 44. The formula shows further that there is a reduction $2\jmath' + 3\chi' + 2C' + 4B'$, or say there are reductions $= 2, 3, 2, 4$, for the reciprocals of a pinch-point, a close-point, a cnicnode, and a binode respectively. Geometrically this must signify that the surface and its Hessian partially intersect in certain curves which are not regarded as belonging to the spinode curve. It will at once suggest itself that for the reciprocal of a cnicnode this curve is a conic, and for the reciprocal of a binode it is a line counting 4 times; while for the reciprocal of a pinch-point it is a line counting 2 times, and for the reciprocal of a close-point, a line counting 3 times.

9. It is clear that ϱ' will in like manner denote the order of the node-couple curve.

10. I express in terms of

$$n,\ b,\ c,\ h,\ k,\ \beta,\ \gamma,\ j,\ \vartheta,\ \chi,\ \mathrm{C},\ \mathrm{B}$$

such quantities and combinations of quantities as can be so expressed. We have

$$a = a' = n(n-1) - 2b - 3c,$$

$$\varkappa' = 3n(n-2) - 6b - 8c,$$

$$\delta' = \tfrac{1}{2}n(n-2)(n^2-9) - (n^2-n-6)(2b+3c) + 2b(b-1) + 6bc + \tfrac{3}{2}c(c-1),$$

$$4i = 12h + c(5n-6) - 6c^2 - 5\gamma + 3\vartheta - 2\chi,$$

$$24t = (-8n+8)b + (15n-18)c + 8b^2 - 18c^2 - 2(8k-18h) + 20\beta - 15\gamma + 4j + 9\vartheta + 6\chi,$$

$$q = b^2 - b - 2k - 3\gamma - 6t, \ (t \text{ suprà}),$$

$$r = c^2 - c - 2h - 3\beta,$$

$$2\sigma = c(n-2) - (4\beta + \gamma) - \vartheta,$$

$$8\varrho = (16n-24)b + (-15n+18)c - 8b^2 + 18c^2 + 2(8k-18h) - 9(4\beta+\gamma) - 4j - 9\theta - 6\chi,$$

$$8\varkappa = 8n(n-1)(n-2) + b(-32n+56) + c(-17n+46) + 8b^2 - 18c^2 - 2(8k-18h) + 17(4\beta+\gamma) + 4j + 17\theta + 6\chi + 8B,$$

$$2\delta = n(n-1)(n-2)(n-3) + b(-4n^2+20n-24) + c(-6n^2+15n-18) + 12bc + 18c^2 + (8k-18h) - 9(4\beta+\gamma) - 9\theta + 2C,$$

$$8n' = 8n(n-1)^2 + (-32n+40)b + (-21n+30)c + 8b^2 - 18c^2 - 2(8k-18h) + 21(4\beta+\gamma) - 12j + 21\theta - 18\chi - 16C - 24B,$$

$$c' = 4n(n-1)(n-2) + (-16n+28)b + (-10n+26)c + 4b^2 - 9c^2 - (8k-18h) + 10(4\beta+\gamma) - 4j + 10\theta - 6\chi - 6C - 8B,$$

$$2b' = -a + n'(n'-1) - 3c', \quad (n', c' \text{ suprà}),$$

$$\sigma' + 2j' + 3\chi' + 2C' + 4B' = 4n(n-2) - 8b - 11c,$$

$$\varrho' - 4j' - 6\chi' - 4C' - 9B' = -11n(n-2) + a(n'-2) + 22b + 30c \quad (n', a \text{ suprà}),$$

$$2\sigma' + 4\beta' + \gamma' + \theta' = c'(n'-2), \quad (n', c' \text{ suprà})$$

$$4k' - 3(i' + 3\beta' + 2\gamma') - 2\varrho' - j' = (-4n'+6)b' + 2b'^2, \quad (n', b' \text{ suprà}),$$

$$6h - 2(i' + 3\beta' + 2\gamma') - 3\sigma' - \chi' = (-4n'+6)c' + 3c'^2, \quad (n', c' \text{ suprà});$$

[or in place of either of these,

$$8k' - 18h' - 4\varrho' + 9\sigma' - 2j' + 3\chi' = (2b' - 3c')\{(n'-2)(n'-3) - a\}, \quad (n', b', c', a \text{ suprà})],$$

$$\varrho' + 2\beta' + 3\gamma' + 3t' = b'(n'-2), \quad (n', b' \text{ suprà}),$$

$$2q' + \beta' + 3i' + j' - 2\varrho' = 0,$$

$$3r' + 2i' + \chi' - 5\sigma' - \beta' - 4\theta' = c', \quad (c' \text{ suprâ}),$$

(twenty-three equations, being a transformation of the original system of twenty-three equations).

11. Forming the combinations $4i+6r$, $24t-8q+18r$ (the last of which introduces on the opposite side the term $+48t$), we obtain

$$4i + 6r = c(5n-12) - 5\gamma - 18\beta + 3\theta - 2\chi,$$

$$-24t - 8q + 18r = -(8n-16)b + (15n-36)c - 34\beta + 9\gamma + 4j + 9\theta + 6\chi,$$

equations which are used *post*, No. 53.

12. I remark that if there be on a surface a right line which is such that the tangent plane is different at different points of the line, the line is said to be *scrolar*: the section of the surface by any plane through the line contains the line *once*. But if there is at each point of the line one and the same tangent plane, then the section of the surface by the tangent plane contains the line at least twice; if it contain it twice only, the line is *torsal*; if three times the line is *osculer*; and the tangent plane containing the torsal or osculer line may in like manner be termed a torsal or an osculer tangent plane. These epithets, scrolar, torsal and osculer, will be convenient in the sequel.

Explanation of the New Singularities. Article Nos. 13 to 39.

I proceed to the explanation of the new singularities.

13. The cnicnode, or singularity C=1, is an ordinary conical point; instead of the tangent plane we have a proper quadricone.

14. The cnictrope, or reciprocal singularity C'=1, is also a well known one; it is in fact the conic of plane contact, or say rather the plane of conic contact, viz. the cnictrope is a plane touching a surface, not at a single point, but along a conic.

15. Consider a surface having the cnicnode C=1, and the reciprocal surface having the cnictrope C'=1. There are on the quadricone of the cnicnode six directions of closest contact*, and reciprocal thereto we have six tangents of the cnictrope conic, touching it at six points. The plane of the cnictrope meets the surface in the conic twice, and in a residual curve which touches the conic at each of the six points. It would appear that these six contacts are part of the notion of the cnictrope.

16. We may of course have a surface with a conic of plane contact, but such that the residual curve of intersection in the plane of the conic does not touch the conic six times or at all; for instance the general equation of a surface with a conic of plane contact is PM+V²N=0, where P=0 is a plane, V=0 a quadric surface; and here the conic P=0, V=0 does not touch the residual curve P=0, N=0. The reciprocal surface will in this case have a cnicnode, but there is some special circumstance doing away with the six directions of closest contact which in general belong thereto. I do not further pursue this inquiry.

17. For a surface having the cnictrope C'=1, the Hessian surface passes through the conic, which is thus thrown off from the spinode curve; or there is a reduction =2 in the order of the curve, which agrees with a foregoing result.

18. The binode, or singularity B=1, is a biplanar node, where instead of the proper quadricone we have two planes; these may be called the biplanes, and their line of intersection, the edge of the binode. The biplanes form a plane-pair.

19. The bitrope, or reciprocal singularity B'=1, is the plane of point-pair contact; but this needs explanation.

20. Consider a surface having a binode, and the reciprocal surface having a bitrope. We have the bitrope, a plane the reciprocal of the binode; in this plane a line, the reciprocal of the edge; in the line two points, or say a point-pair, the reciprocal of the biplanes: these points may be called the bipoints. There are in each biplane three directions of closest contact; the reciprocals of these are in the bitrope three directions through each of the two points. The section of the reciprocal surface by the bitrope is made up of the line counting three times (or the line is osculer), and of a curve passing in the three directions (having therefore a triple point) through each of the two bipoints. The bitrope contains thus an osculer line; but it is part of the notion that

* Taking for greater simplicity coordinates $x, y, z, 1$, then for a surface having a cnicnode at the origin, the equation is U_2+U_3+ &c. $=0$, the suffixes showing the degree in the coordinates; the equation of the quadricone is $U_2=0$, and the six directions are given as the lines of intersection of the two cones $U_2=0$, $U_3=0$.

there are on this line two points each a triple point on the residual curve of intersection.

21. We may however have on a surface an oscular line without upon it two or any triple points of the residual curve of intersection. Such a surface is $Mx+Ny^2=0$; the intersections of the line $x=0$, $y=0$ with the curve $x=0$, $N=0$ will be all of them ordinary points. The reciprocal surface will have a binode, but there will be some special circumstance doing away with the existence of the directions of closest contact in the two biplanes respectively. I do not at present pursue the question.

22. For a surface having a bitrope $B'=1$, it appears from what precedes, that the oscular line must count 4 times in the intersection of the surface with the Hessian; for only in this way can the reduction 4 in the order of the spinode curve arise.

23. The pinch-point, or singularity $j=1$, is in fact mentioned in SALMON; it is a point on the nodal curve such that the two tangent planes coincide, or say it is a cuspidal point on the nodal curve. If, to fix the ideas, we take the nodal curve to be a complete intersection $P=0$, $Q=0$, then the equation of the surface is $(A, B, C)(P, Q)^2=0$ (A, B, C functions of the coordinates); we have a surface $AC-B^2=0$, which may be called the critic surface, intersecting the nodal curve in the points $P=0$, $Q=0$, $AC-B^2=0$, which are the pinch-points thereof; or if there be a cuspidal curve, then such of these points as are not situate on the cuspidal curve are the pinch-points: see my paper "On a Singularity of Surfaces," Quart. Math. Journ. vol. ix. (1868) pp. 332–338. The single tangent plane at the pinch-point meets the surface (see p. 338) in a curve having at the pinch-point a triple point, =cusp+2 nodes, viz. there is a cuspidal branch the tangent to which coincides with that of the nodal curve; and there is a simple branch the tangent to which may be called the cotangent at the pinch-point. In the particular case where the nodal curve is a right line the section is the line twice (representing the cuspidal branch), and a residual curve of the order $\overline{n-2}$, the tangent to which is the cotangent.

24. The pinch-plane, or reciprocal singularity $j'=1$, is in fact a torsal plane touching the surface along a line, or meeting it in the line twice and in a residual curve. Let the line and curve meet in a point P; for the reason that the section by the plane is the line twice and the residual curve, the section has at P two coincident nodes; that is, the plane is a node-couple plane with two coincident nodes. The plane meets the consecutive node-couple plane in a line μ passing through P and touching at this point the residual curve. Considering now the reciprocal figure, the reciprocal of the pinch-plane is thus a point of the nodal curve, and is a pinch-point; the tangent plane at the pinch-point is the reciprocal of the point P; the tangent to the nodal curve is the reciprocal of the line μ, that is, of the tangent at P to the residual curve; and the cotangent at the pinch-point is the reciprocal of the torsal line.

25. There is in this theory the difficulty that for a surface of the order n, the torsal plane meets the residual curve of intersection in $(n-2)$ points P, and if each of these be a point on the node-couple curve, then in the reciprocal figure the pinch-point would be

a multiple point on the nodal curve. I apprehend that starting with a pinch-point, a simple point on the nodal curve, we have in the reciprocal figure a pinch-plane or torsal plane as above, but with some specialty in virtue of which only one of the $(n-2)$ points of intersection of the torsal line with the residual plane curve is a point of the node-couple curve of the reciprocal surface. In the case of a pinch-plane or torsal plane of a cubic surface, $n-2$ is $=1$, and the question of multiplicity does not arise.

26. For a surface with a pinch-plane or torsal plane as above ($j'=1$), the Hessian surface not only passes through the torsal line, but it touches the surface along this line, causing, as already mentioned, a reduction $=2$ in the order of the spinode curve. That the surfaces *touch* along the line is an important theorem*, and I annex a proof.

27. Let $x=0$, $y=0$ be the torsal line, $x=0$ being the torsal plane; the equation of the surface therefore is $x\varphi+y^2\psi=0$; and if A, B, C, D be the first derived functions of φ, $(a, b, c, d, f, g, h, l, m, n)$ the second derived functions, and if (A′, B′, C′, D′), $(a', b', c', d', f', g', h', l', m', n')$ refer in like manner to ψ, then the equation of the Hessian is

$$0=\begin{vmatrix} 2A+xa+y^2a' & , B+xh+2yA'+y^2h', & C+xg+y^2g', & D+xl+y^2l' \\ B+xh+2yA'+y^2h', & xb+2\psi+4yB'+y^2b', & xf+2yC'+y^2f', & xm+2yD'+y^2m' \\ C+xg+y^2g' & , xf+2yC'+y^2f' & , xc+y^2c' & , xn+y^2n' \\ D+xl+y^2l' & , xm+y^2n'+2yD' & , xn+y^2n' & , xd+y^2d' ; \end{vmatrix}$$

and representing this for a moment by

$$\begin{vmatrix} A, & H, & G, & L \\ H, & B, & F, & M \\ G, & F, & C, & N \\ L, & M, & N, & D \end{vmatrix}=0,$$

then in the developed equation

$$D(ABC-AF^2-BG^2-CH^2+2FGH)$$
$$-(BC-F^2, CA-G^2, AB-H^2, GH-AF, HF-BG, FG-CH)(L, M, N)^2=0,$$

observing that C, F, M, N, D are of the first order in x, y, the only terms of the first order are contained in $B(-DG^2-CL^2+2NGL)$; and since C, D, N are of the first order, we obtain all the terms of the first order by reducing B, G, L to the values 2ψ, C, D; viz. the terms of the first order are

$$2\psi(-C^2dx-D^2cx+2CDnx), \quad =-2\psi(C^2d+D^2c-2CDn)x.$$

Hence the complete equation is of the form

$$-2\psi(C^2d+D^2c-2CDn)x+(x, y)^2=0,$$

* See SALMON, p. 218, where it is only stated that the Hessian passes through the line.

or, what is the same thing, $x\Phi + y^2\Psi = 0$; the Hessian has therefore along the line $x=0$, $y=0$ the same tangent plane $x=0$ as the surface; or it *touches* the surface along this line; that is, the line counts twice in the intersection of the two surfaces.

28. If instead of the right line we have a plane curve, say if the equation be $x\varphi + P^2\psi = 0$, then the value of the Hessian is $x\Phi + P\Psi = 0$ (viz. the second term divides by P only, not by P^2), so that, as before mentioned in regard to a conic of contact, the surface and the Hessian merely cut but do not touch along the curve $x=0$, $P=0$. To show this in the most simple manner take the equation to be $x\varphi + \frac{1}{2}P^3 = 0$; let A′, B′, C′, D′ be the first derived functions of φ, and (A, B, C, D), $(a, b, c, d, f, g, h, l, m, n)$ the first and second derived functions of P; then if in the equation of the Hessian we write for greater simplicity $x=0$, the equation is

$$\begin{vmatrix} 2A' + Pa + A^2, & B' + Ph + AB, & C' + Pg + AC, & D' + Pl + AD \\ B' + Ph + AB, & Pb + B^2, & Pf + BC, & Pm + BD \\ C' + Pg + AC, & Pf + BC, & Pc + C^2, & Pn + CD \\ D' + Pl + AD, & Pm + BD, & Pn + CD, & Pd + D^2 \end{vmatrix} = 0.$$

The equation contains for example the term

$$-(D' + Pl + AD)^2 \{P^2(bc - f^2) + P(bC^2 + cB^2 - 2fBC)\},$$

dividing as it should do by P, but not dividing by P^2; and considering the portion hereof $-D'^2P(bC^2 + cB^2 - 2fBC)$, there are no other terms in D'^2P which can destroy this, and to make the whole equation divide by P^2; which proves the required negative.

29. For the off-point or singularity $\theta=1$; this is a point on the cuspidal curve at which the second derived functions all of them vanish. In further explanation hereof consider a surface $U=0$, and the second polar of an arbitrary point $(\alpha, \beta, \gamma, \delta)$; viz. this is $(\alpha\partial_x + \beta\partial_y + \gamma\partial_z + \delta\partial_w)^2 U = 0$, or say for shortness $\Delta^2 U = 0$, where the coefficients of the powers and products of $(\alpha, \beta, \gamma, \delta)$ are of course the second derived functions of U; this equation, when reduced by means of the equations of the cuspidal curve, may acquire a factor Λ, thus assuming the form $\Lambda(\alpha P + \beta Q + \gamma R + \delta S)^2 = 0$, and if so the intersections of the cuspidal curve with the second polar $(=2\sigma + \theta$, if, as for simplicity is supposed, there is no nodal curve) will be made up of the intersections of the cuspidal curve with the surface $\Lambda=0$, and of those with the surface $\alpha P + \beta Q + \gamma R + \delta S = 0$ each twice; the latter of these, depending on the coordinates $(\alpha, \beta, \gamma, \delta)$ of the arbitrary points, are the points σ each twice; the former of them, or intersections of the cuspidal curve with the surface $\Lambda=0$, are the points θ, or off-points of the cuspidal curve. If there is a nodal curve, the only difference is that the off-points are such of the above points as do not lie on the nodal curve.

30. As the most simple instance of the manner in which this singularity may present itself, consider a surface $FP^2 + GQ^3 = 0$, where the degrees of the functions are f, p, g, q, and therefore $n = f + 2p = g + 3q$, if n be the order of the surface. This has a cuspidal curve $P=0, Q=0$ of the order pq; the equation $\Delta^2(FP^2 + GQ^3) = 0$ of the second polar,

when reduced by the equations $P=0$, $Q=0$ of the cuspidal curve, becomes simply $F(\Delta P)^3=0$; and we have thus the off-points $F=0$, $P=0$, $Q=0$, consequently $\theta=fpq$.

31. But suppose, as before, the case of a surface $(A, B, C\text{\textbf{)(}}P, Q)^3=0$ having a cuspidal curve $P=0$, $Q=0$, and therefore $AC-B^2$ being $=0$ for $P=0$, $Q=0$. The equation of the second polar, writing therein $P=0$, $Q=0$, becomes $(A, B, C\text{\textbf{)(}}\Delta P, \Delta Q)^2=0$, and if for any given surface this assumes the form $\Lambda(M\Delta P+N\Delta Q)^3=0$ (observe that M, N may be fractional provided only the $M\Delta P+N\Delta Q$ is integral), then there will be on the cuspidal curve the off-points $\Lambda=0$, $P=0$, $Q=0$.

32. An interesting example is afforded by a surface which presents itself in the Memoir on Cubic Surfaces: the surface

$$\begin{aligned} &4y^6\\ &-4y^3x(x^2+3zw)\\ &+zw(3x^2+zw)^2=0 \end{aligned}$$

has the cuspidal conic $y=0$, $3x^2+zw=0$, and (as coming under the form $FP^2+GQ^3=0$) has the off-points $zw=0$, $y=0$, $3x^2+zw=0$; that is, the points $(x=0, y=0, z=0)$, $(x=0, y=0, w=0)$ each twice; $\theta=4$.

But writing the same equation in the form

$$(4,\ 6x,\ 8x^2+zw\text{\textbf{)(}}y^2-2x^2,\ x^2-zw)^2=0,$$

where

$$4.(8x^2+zw)-(6x)^2=-4(x^2-zw),$$

it appears that there are also the three cuspidal conics $y^2-2x^2=0$, $x^2-zw=0$. Reducing by means of these two equations, the equation of the second polar is at first obtained in the form

$$(4,\ 6x,\ 8x^2+zw\text{\textbf{)(}}3y^2\Delta y-6x^2\Delta x,\ 2x\Delta x-z\Delta\omega-\omega\Delta z)^2=0;$$

but further reducing by the same equations and writing for this purpose $y=\omega x(\omega^2=2)$, the equation becomes

$$(4,\ 6x,\ 9x^2\text{\textbf{)(}}x^2(3\omega^2\Delta y-6\Delta x),\ 2x\Delta x-z\Delta w-w\Delta z^2)^2=0,$$

that is

$$x^2[2x(3\omega^2\Delta y-6\Delta x)+3(2x\Delta x-z\Delta w-w\Delta z)]^2=0,$$

and we have thus the off-points $x^2=0$, $y^2-2x^2=0$, $x^2-zw=0$, in fact the before-mentioned two points each 6 times; and the complete value of θ is $\theta=(4+12=)16$; viz. the off-points are the points $(x=0, y=0, z=0)$, $(x=0, y=0, w=0)$ each 8 times. On account of this union of points the singularity is really one of a higher order, but equivalent to $\theta=16$.

I am not at present able to explain the off-plane or reciprocal singularity $\theta'=1$.

33. As to the close-point or singularity $\chi=1$. I remark that at an ordinary point of the cuspidal curve the section by the tangent plane *touches*, at the point of contact, the cuspidal curve: the point of contact is on the curve of section a singular point [in the nature of a triple point, viz. taking the point of contact as origin, the form of the branch in the vicinity thereof is $y^3-x^4=0$, where $y=0$ is the equation of the tangent to the

cuspidal curve], such that the point of contact counts 4 times in the intersection of the cuspidal curve with the curve of section. At a close-point the form of the curve of section is altered; viz. the point of contact is here in the nature of a quadruple point with two distinct branches, one of them a triple branch of the form $y^3=x^4$, but such that the tangent thereof, $y=0$, is not the tangent of the cuspidal curve; the other of them a simple branch, the tangent of which is also distinct from the tangent of the cuspidal branch: the point of contact counts $3+1$ times, that is 4 times, as before, in the intersection of the cuspidal curve and the curve of section. The tangent to the simple branch may conveniently be termed the cotangent at the close-point; that of the other branch the cotriple tangent.

34. We may look at the question differently thus: to fix the ideas, let the cuspidal curve be a complete intersection $P=0$, $Q=0$; the equation of the surface is $(A, B, C\text{XP}, Q)^3=0$, where $AC-B^2=0$, in virtue of the equations $P=0$, $Q=0$ of the cuspidal curve, that is, $AC-B^2$ is $=MP+NQ$ suppose. We have (as in the investigation regarding the pinch-point) a critic surface $AC-B^2=0$, this meets the surface in the cuspidal curve and in a residual curve of intersection; the residual curve by its intersection with the cuspidal curve determines the close-points; the tangent at the close-point is I believe the tangent of the residual curve. Analytically the close-points are given by the equations $P=0$, $Q=0$, $(A, B, C\text{XN}, -M)^2=0$. It is proper to remark that if besides the cuspidal curve there be a nodal curve, only such of the points so determined as do not lie on the nodal curve are the close-points.

35. I take as an example a surface which is substantially the same as one which presents itself in the Memoir on Cubic Curves, viz. the surface $(1, w, xy\text{X}w^2-xy, z)^3=0$, having the cuspidal conic $w^2-xy=0$, $z=0$. Since in the present case $AC-B^2=P$, we have $M=1$, $N=0$, and the close-points are given by $P=0$, $Q=0$, $C=0$; that is, they are the points $(z=0, w=0, x=0)$ and $(z=0, w=0, y=0)$.

36. I first however consider an ordinary point on the cuspidal curve, or conic $w^2-xy=0$, $z=0$; the coordinates of any point on the conic are given by $x:y:z:w=1:\theta^2:0:\theta$, where θ is an arbitrary parameter; we at once find $\theta^2x+y-\theta(z+2w)=0$ for the equation of the tangent plane of the surface or cuspidal tangent plane at the point $(1, \theta^2, 0, \theta)$. Proceeding to find the intersection of this plane with the surface, the elimination of z gives

$$(\theta^2, \theta w, xy\text{X}w^2-xy, \theta^2x+y, -2\theta w)^2=0,$$

which is of course the cone, vertex $(x=0, y=0, w=0)$, which passes through the required curve of intersection. In place of the coordinates x, y take the new coordinates

$$\theta^2x-y=2p, \text{ and } \theta^2x+y-2\theta w=2q; \text{ we have}$$

$$\theta^2x= \quad \theta w+p-q,$$

$$-y=-\theta w+p-q,$$

and thence

$$-\theta^2xy=p^2-(q+\theta w)^2=p^2-q^2-2\theta qw-\theta^2w^2,$$

$$\theta^2(w^2-xy) \qquad =p^2-q^2-2\theta qw,$$

and the equation thus is

$$(\theta^4, \theta^2 w, -p^2+q^2+2\theta qw+\theta^2w^2)(p^2-q^2-2\theta qw, 2\theta^2 q)=0,$$

or, what is the same thing,

$$(1, \theta w, -p^2+q^2+2\theta qw+\theta^2w^2)(p^2-q^2-2\theta qw, 2q)^2=0;$$

viz. this is

$$(p^2-q^2-2\theta qw)^2+4\theta qw(p^2-q^2-2\theta qw)+4q^2(-p^2+q^2+2\theta qw+\theta^2w^2)=0;$$

or reducing, it is

$$(p^2-q^2)(p^2-5q^2)+8\theta q^2w=0,$$

the equation of the section in terms of the coordinates p, q, w. The equation is satisfied by the values $p=0$, $q=0$ which belong to the assumed point $(1, \theta^2, 0, \theta)$ of the conic, and in the vicinity of this point we have $p^4+8\theta q^2w=0$, which is a triple branch of the form $y^3=x^4$, the tangent $q=0$ being, it will be observed, the tangent of the conic. But at the close-points, or when $\theta=0$ or $\theta=\infty$, the transformation fails; and these points must be considered separately.

37. At the first of these, viz. the point $z=0$, $w=0$, $x=0$, the tangent plane of the surface or cuspidal tangent plane is $x=0$, and this meets the surface in the curve $x=0$, $w^3(w+2z)=0$, that is in the line $x=0$, $w=0$ three times, and in the line $x=0$, $w+2z=0$ (that the section consists of right lines is of course a speciality, and it is clear that considering in a more general surface the section as defined by an equation in (w, z, y), the line $w=0$ represents the tangent to a triple branch $w^3=z^4+$ &c., and the line $w+2z=0$ the tangent to a simple branch); these lines are each of them, it will be observed, distinct from the tangent to the cuspidal conic, which is $x=0$, $z=0$. And similarly the tangent plane at the other of the two points is $y=0$, meeting the surface in the curve $y=0$, $w^3(w+2z)=0$, that is in the line $y=0$, $w=0$ three times, and in the line $y=0$, $w+2z=0$.

38. The close-plane or reciprocal singularity $\chi'=1$ is (like the pinch-plane) a torsal plane, meeting the surface in a line twice and in a residual curve; the distinction is that the line and curve have an intersection P lying on the spinode curve; the close-plane is thus a spinode plane; it meets the consecutive spinode plane in a line μ passing through P, and which is *not* the tangent of the residual curve. In the reciprocal figure, the reciprocal of the close-plane is on the cuspidal curve, and is a close-point; the reciprocal of the point P is the cuspidal tangent plane; that of the line μ the tangent of the cuspidal curve; that of the tangent of the residual curve the cotriple tangent; that of the torsal line the cotangent.

39. The torsal line of a close-plane is not a mere torsal line; in fact by what precedes it appears that the surface and the Hessian intersect in this line, counting not twice but three times, and it is thus that the reduction in the order of the spinode curve caused by the close-plane is $=3$.

Application to a Class of Surfaces. Article Nos. 40 & 41.

40. Consider the surface $FP^n+GR^nQ^n=0$, where f, p, g, r, q being the degrees of the several functions, and n the order of the surface, we have of course $n=f+2p=g+2r+3q$.

There is here a nodal curve, the complete intersection of the two surfaces $P=0, R=0$; hence $b=pr$, $k=\frac{1}{2}pr(p-1)(r-1)$, $=\frac{1}{2}b(b-p-r+1)$; $t=0$; whence $(q)^*=pr(p+r-2)$. There is also a cuspidal curve the complete intersection of the two surfaces $P=0, Q=0$; hence $c=pq$, $h=\frac{1}{2}pq(p-1)(q-1)=\frac{1}{2}c(c-p-q+1)$; whence $(r)^*=pq(p+q-2)$. The two curves intersect in the pqr points $P=0, Q=0, R=0$, which are not stationary points on either curve; that is, $\beta=0$, $\gamma=0$, $i=pqr$.

There are on the nodal curve the $j=(f+g)pr$ pinch-points $F=0, P=0, R=0$, and $G=0, P=0, R=0$. There are on the cuspidal curve $\theta=fpq$ off-points $F=0, P=0, Q=0$; and there the gpq singular points $G=0, P=0, Q=0$. I find that these last, and also the θ points each three times, must be considered as close-points, that is, that we have $\chi=(g+3f)pq$.

41. We ought then to have

$$\begin{aligned} b(n-2) &= \varrho, \\ c(n-2) &= 2\sigma+\theta; \\ 2(q)+3i+j &= 2\varrho, \\ 3(r)+c+2i+\chi &= 5\sigma+4\theta; \end{aligned}$$

the first two of which give ϱ, σ, and then, substituting their values, the other two equations should become identities. In fact, attending to the values $pr=b$, $pq=c$, the equations become

$$\begin{aligned} 2b(p+r-2)+3bq+b(f+g) &= 2b(n-2), \\ 3c(p+q-2)+c+2cr+b(g+3f) &= \tfrac{5}{2}\{c(n-2)-cf\}+4cf. \end{aligned}$$

The first of these is

$$2n=2p+2r+3q+f+g, \quad =(2p+f)+(2r+3q+g),$$

and the second is

$$\tfrac{5}{2}n=3p+3q+2r+g+\tfrac{3}{2}f, \quad =\tfrac{3}{2}(2p+f)+(2r+3q+g),$$

so that the equations are satisfied.

The Flecnodal Curve. Article No. 42.

42. A point on a surface may be flecnodal, viz. the tangent plane may meet the surface in a curve having at the point a flecnode, that is, a node with an inflexion on one of the branches. SALMON has shown that, for a surface of the order n without singularities, the locus of the flecnodal points, or flecnodal curve, is the complete intersection of the surface by a surface of the order $11n-24$, which may be called the flecnodal surface, the order of the curve being thus $=n(11n-24)$. I have succeeded in showing, in a somewhat peculiar way by consideration of a surface of revolution, that if the surface of the order n has a nodal curve of the order b, and a cuspidal curve of the order c, then

* I have written for distinction (q), (r), to denote the q, r of the fundamental equations.

that the order of the flecnodal curve is $=n(11n-24)-22b-27c$; before giving this investigation, I will by the like principles demonstrate the above-mentioned theorem that the order of the spinode curve is $=4n(n-2)-8b-11c$.

Surfaces of Revolution, in connexion with the Spinode Curve and the Flecnodal Curve.
Article Nos. 43 to 47.

43. Consider a plane curve of the order m with δ nodes and $\varkappa$ cusps, and let this be made to revolve about an axis in its own plane, so as to generate a surface of revolution. The complete meridian section is made up of the given curve and of an equal curve situate symmetrically therewith on the other side of the axis; the order of the surface is thus $=2m$. The two curves intersect in m points on the axis and in m^2-m points, forming $\frac{1}{2}(m^2-m)$ pairs of points, situate symmetrically on opposite sides of the axes; these last generate $\frac{1}{2}(m^2-m)$ circles, nodal curves on the surface; the nodes generate δ circles, which are nodal curves on the surface, and the cusps generate $\varkappa$ circles, cuspidal curves on the surface. There are $m^2-m-2\delta-3\varkappa$ circles of plane contact corresponding in the plane curve to the tangents perpendicular to the axis. Each of the m points on the axis gives in the surface a pair of (imaginary) lines; and we have thus two sets each of m lines, such that along the lines of each set the surface is touched by an (imaginary) meridian plane; viz. these are the circular planes $x+iy=0, x-iy=0$ passing through the axis. I assume without stopping to show it that these $2m$ lines are lines not j' but χ', that is, that they each reduce the order of the spinode curve by 3*. The inflexions generate $3m^2-6m-6\delta-8\varkappa$ circles which constitute the spinode curve on the surface.

44. And we can thus verify that the complete intersection of the surface with the Hessian is made up in accordance with the foregoing theory; viz.

Order of surface $=2m$,		
Order of Hessian $=4(2m-2)$,		
whence order of intersection		$=16m^2-16m$
Nodal curve, $\frac{1}{2}(m^2-m)+\delta$ circles,	8 times	$8m^2-8m+16\delta$
Cuspidal curve, $\varkappa$ circles,	11 times	$+22\varkappa$
Circles of contact $m^2-m-2\delta-3\varkappa$,		$2m^2-2m-4\delta-6\varkappa$
Lines $2m$,	3 times	$+6m$
Spinode curve, $3m^2-6m-6\delta-8\varkappa$ circles,		$6m^2-12m-12\delta-16\varkappa$
		$16m^2-16m$

45. We may by a similar reasoning show that the surface and the flecnode surface intersect in the nodal curve taken 22 times, and in the cuspidal curve taken 27 times;

* Observe that the terms in m cannot be got rid of in a different manner, by any alteration of the numbers 8 & 11 to which the present investigation relates.

and consequently that the order of the residual intersection or flecnodal curve is

$$=n(11n-24)-22b-27c.$$

To effect this, observe that at any point whatever of a quadric surface the tangent plane meets the surface in a pair of lines, that is, in a curve having at the point of contact a node with an inflexion on each branch, or say, a fleflecnode. Imagine in the plane figure a conic having its centre on the axis of rotation and its axis coincident therewith, and the conic having with the curve of the order m a 4-pointic intersection at any point P; the point P generates a circle, such that along this circle the surface is osculated by a quadric surface of revolution in such wise that the meridian sections have a four-pointic contact; the circle in question is thus on the surface a fleflecnode circle; and I assume that it counts twice as a flecnode circle. Hence if the number of the points P be $=\theta$, we have on the surface θ fleflecnode circles, $=2\theta$ flecnode circles, that is, a flecnode curve of the order 4θ. I wish to show that we have $\theta=5m^2-9m-10\delta-12\varkappa$.

46. The problem is as follows: given a curve of the order m with δ nodes and $\varkappa$ cusps; it is required to find the number of the conics, centre on a given line, and an axis coincident in direction with this line, which have with the given curve a 4-pointic intersection, or contact of the third order. This may be solved by means of formulæ contained in my "Memoir on the Curves which satisfy given Conditions"*.

Taking $x=0$ for the given line, the conic $(a, b, c, f, g, h)(x, y, 1)^2=0$ will have its centre on the given line and an axis coincident therewith, if only $h=0, g=0$; and denoting these two conditions by 2X, it is easy to see that we have

$$(2X\,\therefore)=1,\quad (2X:/)=2,\quad (2X\cdot//)=2,\quad (2X///)=1.$$

But in general if the conic satisfy any other three conditions 3Z, then the number of the conics (2X, 3Z) is

$$\begin{aligned} = \quad & \alpha'(\qquad\qquad -\tfrac{1}{4}\gamma+\tfrac{1}{2}\delta) \\ & +\beta'(-\tfrac{3}{8}\alpha+\tfrac{5}{16}\beta+\tfrac{5}{16}\gamma-\tfrac{3}{8}\delta) \\ & +\gamma'(\ \tfrac{1}{2}\alpha\ -\tfrac{1}{4}\beta\qquad\quad), \end{aligned}$$

where $\alpha, \beta, \gamma, \delta$ denote $(2X\,\therefore)$, $(2X:/)$, $(2X\cdot//)$, $(2X///)$, viz. in the present case the values are 1, 2, 2, 1 respectively, and where α', β', γ' denote $(3Z:)$, $(3Z\cdot/)$, $(3Z//)$ respectively.

47. Substituting for $\alpha, \beta, \gamma, \delta$ their values, the number of the conics in question is $=\frac{1}{2}\beta'$, that is $=\frac{1}{2}(3Z\cdot/)$. Suppose that 3Z, or say 3, denotes the condition of a contact of the third order with a given curve $(m, \delta, \varkappa)$, or say with a given curve (m, n, α) (m the order, n the class $=m^2-m-2\delta-3\varkappa$, $\alpha=3n+\varkappa$), then we have

$$\begin{aligned} (3:\ ) &= -4m-3n+3\alpha, \\ (3\cdot/) &= -8m-8n+6\alpha, \\ (3//\ ) &= -3m-4n+3\alpha; \end{aligned}$$

and from the second of these the number of the conics in question is $=-4m-4n+3\alpha$, that is, it is $=-4m+5n+3\varkappa$, or finally it is $=5m^2-9m-10\delta-12\varkappa$.

* Philosophical Transactions, t. 158 (1868), pp. 75–144: see p. 88.

Hence, assuming that the $2m$ lines each counts 6 times*,

Order of surface $=2m$

Order of flecnode surface $=11(2m-24)$ or $22m-24$

Order of intersection	$=44m^2-48m$
Nodal curve, $\frac{1}{2}(m^2-m)+\delta$ circles, 11 times	$22m^2-22m+44\delta$
Cuspidal curve $\varkappa$ circles, 27 times	$+54\varkappa$
Circles of contact $m^2-m-2\delta-3\varkappa$,	$2m^2-2m-4\delta-6\varkappa$
Lines of contact $2m$, 6 times	$+12m$
Flecnodal curve, $5m^2-9m-10\delta-12\varkappa$ circles each twice	$20m^2-36m-40\delta-48\varkappa$
	$44m^2-48m$

The Flecnodal Torse. Article Nos. 48 & 49.

48. Starting from

$$\begin{aligned} 22b'+27c' &= 6(6b'+8c')-7(2b'+3c') \\ &= 6(3n'^2-6n'-\varkappa)-7(n'^2-n'-\delta) \\ &= 11n'^2-29n'+7-6\varkappa, \end{aligned}$$

that is

$$11n'^2-24n'-22b'-27c' = 5n'-7\delta+6\varkappa,$$

I find

$$\begin{aligned} &n'(11n'-24)-22b'-27c' \\ &\quad = n(n-1)(11n-24)+b(-59n+96)+c(-94n+156)+26b^2+87c^2 \\ &\qquad -52k-114h+141\beta+94\gamma+77i+3j+4\chi \\ &\qquad -15\vartheta-45t-10\mathrm{C}-9\mathrm{B}. \end{aligned}$$

49. For a surface of the order n without singularities this equation is

$$n'(11n'-24)-22b'-27c' = n(n-1)(11n-24);$$

to explain the meaning of it, I say that the reciprocal of a flecnode is a flecnodal plane, and *vice versâ*: the reciprocal of the flecnodal torse of the surface n (viz. the torse generated by the flecnodal planes of the surface) is thus the flecnodal curve of the reciprocal surface n'; and the class of the torse must therefore be equal to the order of the curve. The flecnodal torse is generated by the tangents of the surface n along the curve of intersection with a surface of the order $11n-24$; the number of tangent planes which pass through an arbitrary point, or class of the torse, is at once found to be $n(n-1)(11n-24)$; for the reciprocal surface the order of the flecnodal curve is by what precedes $n'(11n'-24)-22b'-27c'$; and the equation thus expresses that the order of the curve is equal to the class of the torse.

* See last foot note: the same remark applies to the present terms in m, which cannot be got rid of by an alteration of the numbers 22 and 27 to which the investigation relates.

The general surface of the Order n *without Singularities.* Article No. 50.

50. In the general surface of the order n without singularities, we have

$n = n$,
$a = n^2 - n$,
$\delta = \frac{1}{2}n(n-1)(n-2)(n-3)$,
$\varkappa = n(n-1)(n-2)$,
$b = 0$,
$k = 0$,
$t = 0$,
$q = 0$,
$\varrho = 0$,
$j = 0$,
$c = 0$,
$h = 0$,
$r = 0$,
$\sigma = 0$,
$\theta = 0$,
$\chi = 0$,
$C = 0$,
$B = 0$,
$\beta = 0$,
$\gamma = 0$,
$i = 0$,
$n' = n(n-1)^2$,
$a' = n(n-1)$,
$\delta' = \frac{1}{2}n(n-2)(n^2-9)$,
$\varkappa' = 3n(n-2)$,
$b' = \frac{1}{2}n(n-1)(n-2)(n^2-n^2+n-12)$,
$k' = \frac{1}{8}n(n-2)(n^{10}-6n^9+16n^8-54n^7+164n^6-288n^5+547n^4-1058n^3+1068n^2-1214n+1464)$,
$t' = \frac{1}{6}n(n-2)(n^7-4n^6+7n^5-45n^4+114n^3-111n^2+548n-960)$,
$q' = n(n-2)(n-3)(n^2+2n-4)$,
$\varrho' = n(n-2)(n^3-n^2+n-12)$,
$j' = 0$,
$c' = 4n(n-1)(n-2)$,

$N' = \frac{1}{2}n(n-2)(16n^4-64n^3+80n^2-108n+156)$,
$r' = 2n(n-2)(3n-4)$,
$\sigma' = 4n(n-2)$,
$\theta' = 0$,
$\chi' = 0$,
$C' = 0$,
$B' = 0$,
$\beta' = 2n(n-2)(11n-24)$,
$\gamma' = 4n(n-2)(n-3)(n^2-3n+16)$,
$t' = 0$.

Investigation of Formula for β'. Article Nos. 51 to 64.

51. The value $\beta' = 2n(n-2)(11n-24)$ for a surface without singularities was obtained by SALMON by independent geometrical considerations, viz. he obtains

$$2\beta' = 4n(n-2)(11n-24)$$

as the number of intersections of the spinode curve $\left(\text{order } = 4n(n-2)\right)$ by the flecnode surface of the order $11n-24$.

52. The value of β' must be obtainable in the case of a surface with singularities, and I have been led to conclude that we have

$$\begin{aligned}\beta' = {} & 2n(n-2)(11n-24) \\ & -(110n-272)b+44q \\ & -(116n-303)c+\tfrac{6\,3}{2}r \\ & +\tfrac{6\,9\,1}{2}\beta+248\gamma+198t \\ & + \text{linear function } (i, j, \theta, \chi, C, B, i', j', \chi', C', B'),\end{aligned}$$

but I have not yet completely determined the coefficients of the linear function. The reciprocal formula in the case of a surface of the order n without singularities, $i, j, \theta, \chi, C, B, i', j', \theta', \chi', C', B'$ then all vanishing, is the identity

$$\begin{aligned}0 = {} & 2n'(n'-2)(11n'-24) \\ & -(110n'-272)b'+44q' \\ & -(116n'-303)c'+\tfrac{6\,3}{2}r' \\ & +\tfrac{6\,9\,1}{2}\beta'+248\gamma'+198t'\end{aligned}$$

($n', b', q', c', r', \beta', \gamma', t'$ having the values in the foregoing Table). It was by assuming for β an expression of the above form but with indeterminate coefficients, and then determining these in such wise that the reciprocal equation should be an identity, that the foregoing formula for β' was arrived at.

53. I assume

$$\begin{aligned}\beta' = \;& 2n(n-2)(11n-24)\\ & -b(An-B)+Cq\\ & -c(Dn-E)+Fr\\ & -G\beta-H\gamma-It\\ & + \text{linear function } (i, j, \theta, \chi, C, B, i', j', \theta', \chi', C', B'),\end{aligned}$$

where it is to be remarked that, in virtue of the equations obtained No. 11, two of the coefficients of this form are really arbitrary: I cannot recall the considerations which led me to write D=116, E=303.

54. Forming the reciprocal equation

$$\begin{aligned}\beta = \;& 2n'(n'-2)(11n'-24)\\ & -b'(An'-B)+Cq\\ & -c'(Dn'-E)+Fr\\ & -G\beta'-H\gamma'-It'\\ & + \text{linear function } (i', j', \theta', \chi', C', B', i, j, \theta, \chi, C, B),\end{aligned}$$

and substituting herein the values which belong to the surface of the order n without singularities, we should have identically

$$\begin{aligned}0 = \;& 2n(n-1)^3(n-2)(n^2+1)(11n^3-22n^2+11n-24)\\ & -\tfrac{1}{2}n(n-1)(n-2)(n^3-n^2+n-12)[An(n-1)^3-B]\\ & + n(n-2)(n-3)(n^2+2n-4)C\\ & -4n(n-1)(n-2)[Dn(n-1)^3-E]\\ & +2n(n-2)(3n-4)F\\ & -2n(n-2)(11n-24)G\\ & -4n(n-2)(n-3)(n^3+3n-16)H\\ & -\tfrac{1}{6}n(n-2)(n^7-4n^6+7n^5-45n^4+114n^3-111n^2+548n-960)I;\end{aligned}$$

or dividing the whole by $n(n-2)$, this is

$$\begin{aligned}0 = \;& 2(n-1)^3(n^2+1)(11n^3-22n^2+11n-24)\\ & -\tfrac{1}{2}(n-1)(n^3-n^2+n-12)[An(n-1)^3-B]\\ & + (n-3)(n^2+2n-4)C\\ & -4(n-1)[Dn(n-1)^3-E]\\ & +2(3n-4)F\\ & -2(11n-24)G\\ & -4(n-3)(n^3+3n-16)H\\ & -\tfrac{1}{6}(n^7-4n^6+7n^5-45n^4+114n^3-111n^2+548n-960)L\end{aligned}$$

55. And then, expanding in powers of n and equating to zero the coefficients of the several powers $n^7, \ldots n^0$, we obtain

$$\begin{array}{cccccccc}
22 & -88 & +154 & -224 & +250 & -184 & +118 & -48 \\
-\frac{1}{2}A & +2A & -\frac{7}{2}A & +\frac{19}{2}A & -20A & +\frac{37}{2}A & -6A & \\
 & & & +\frac{1}{2}B & -B & +B & -\frac{13}{2}B & +6B \\
 & & & & +C & -C & -10C & +12C \\
 & & & -4D & +12D & -12D & +4D & \\
 & & & & & & +4E & -4E \\
 & & & & & & +6F & -8F \\
 & & & & & & -22G & +48G \\
 & & & -4H & +12H & -12H & +100H & -192H \\
-\frac{1}{6}I & +\frac{3}{3}I & -\frac{7}{6}I & +\frac{15}{2}I & -19I & +\frac{37}{2}I & -\frac{214}{3}I & +160I \\
\| & \| & \| & \| & \| & \| & \| & \| \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}$$

viz. the equations are read vertically downwards. The first, second, and third equations, and the sum of the fourth and fifth, all give the same relation, $132-3A-I=0$; there are consequently, inclusive of this, five independent relations. By combining the equations so as to simplify the numbers, I find these to be

$$\begin{aligned}
&3A+I-132=0,\\
&4A-B-2C-80=0,\\
&7A-B+2E+2F+2G-476=0,\\
&26A-B+8D+8H-1532=0,\\
&6A-E-2F+12G-48H+40I-132=0.
\end{aligned}$$

56. I found, as presently mentioned, $A=110$, $B=272$, $C=44$; values which satisfy (as they should do) the second equation; and then assuming $D=116$ and $E=303$, we have $F=\frac{63}{2}$, $G=-\frac{691}{2}$, $H=-248$, $I=-198$; and the formula is

$$\begin{aligned}
\beta' = {} & 2n(n-2)(11n-24)\\
& -(110n-272)b+44q\\
& -(116n-303)c+\tfrac{63}{2}r\\
& +\tfrac{691}{2}\beta+248\gamma+198t\\
& + \text{linear function } (i, j, \theta, \chi, C, B, i', j', \theta', \chi', C', B'),
\end{aligned}$$

the process not enabling the determination of the coefficients of the linear function.

57. The values of A, B, C were found from the general theorem that if three surfaces of the orders μ, ν, ϱ respectively intersect in a curve of the order m and class r which is α-tuple on u, β-tuple on v, and γ-tuple on ϱ, then the number of the points of inter-

section of the three surfaces is

$$=\mu\nu\varrho-m(\beta\gamma\mu+\gamma\alpha\nu+\alpha\beta\varrho-2\alpha\beta\gamma)+\alpha\beta\gamma r.$$

Apply this to the case of a surface of the order n with a nodal curve of the order b and class q, intersecting the Hessian and flecnodal surfaces, we have

	Order.	Passing through (b, q), times
Surface	n	2
Hessian	$4n-8$	4
Flecnodal	$11n-24$	11

whence number of intersections is

$$=4n(n-2)(11n-24)-b\{n\,.\,4\,.\,11+(4n-8)11\,.\,2+(11n-24)2\,.\,4-2\,.\,2\,.\,4\,.\,11\}$$
$$+2\,.\,4\,.\,11q,$$

that is

$$=4n(n-2)(11n-24)-(220n-544)b-88q;$$

and the value of β' is one half of this,

$$=2n(n-2)(11n-24)-(110n-272)b+44q.$$

I have not succeeded in applying the like considerations to the cuspidal curve.

58. As regards the general theorem, we know (SALMON, p. 274) that if two surfaces of the orders μ, ν partially intersect in a curve of the order m and class r, and besides in a curve of the order m', then the curves m, m' meet in $m(\mu+\nu-2)-r$ points.

Suppose that the curve m is α-tuple on the surface μ; then to find the number I of the intersections of the curves m and m', we may imagine through m a surface of the order ϱ; the surfaces μ, ν intersect in the curve m α times, and in a residual curve of the order $\mu\nu-m\alpha$, this last meets the surface ϱ in $\varrho(\mu\nu-m\alpha)$ points, and thence the three surfaces meet in $\mu\nu\varrho-m\alpha\varrho-\mathrm{I}$ points. But since m is a simple curve on each of the surfaces ν, ϱ, the three surfaces meet in $\mu(\nu\varrho-m)-\alpha[m(\nu+\varrho-2)-r]$ points, whence equating the two values

$$\mathrm{I}=m(\mu+\alpha\nu-2\alpha)-\alpha r.$$

Next, let the curve m be α-tuple on the surface μ, β-tuple on the surface μ. Considering the new surface ϱ through m, then μ, ν intersect in the curve m $\alpha\beta$ times, and in a residual curve of the order $\mu\nu-m\alpha\beta$; this last meets the surface ϱ in $\varrho(\mu\nu-m\alpha\beta)$ points; whence the three surfaces meet in $\varrho(\mu\nu-m\alpha\beta)-\mathrm{I}$ points. But the curve m being a β-tuple curve on ν, and a simple curve on ϱ, these meet in the curve m β times and in a residual curve of the order $\nu\varrho-\beta m$, whence the three surfaces meet in

$$\mu(\nu\varrho-\beta m)-\alpha[m(\nu+\beta\varrho-2\beta)-\beta r]$$

points; and equating the two values, we have

$$\mathrm{I}=m(\beta\mu+\alpha\nu-2\alpha\beta)-2\alpha\beta r.$$

Lastly, if the curve m be γ-tuple on ϱ, then the surfaces μ, ϱ meet in m $\alpha\gamma$ times and in a residual curve of the order $\mu\varrho-m\alpha\gamma$; this last meets ν in

$$\nu(\mu\varrho-\alpha\gamma m)-\beta[m(\gamma\mu+\alpha\varrho-2\alpha\gamma)-\alpha\gamma r]$$

points, that is, the number of points of intersection of the three surfaces is

$$=\mu\nu\varrho - m(\beta\gamma\mu+\gamma\alpha\nu+\alpha\beta\varrho-2\alpha\beta\gamma)+\alpha\beta\gamma\tau.$$

59. I represent the complete value of β' by

$$\begin{aligned}\beta' = \ & 2n(11n-24)\\ & -(110n-272)b+44q\\ & -(116n-303)c+\tfrac{63}{2}r\\ & +\tfrac{69}{2}\beta+248\gamma+198t\\ & -h\mathrm{C}-g\mathrm{B}-x\mathrm{i}-\lambda j-\mu\chi-\nu\theta\\ & -h'\mathrm{C}'-g'\mathrm{B}'-x'\mathrm{i}'-\lambda' j'-\mu'\chi'-\nu'\theta',\end{aligned}$$

and (observing that the Table of Singularities in my Memoir on Cubic Surfaces was obtained without the aid of the formula now in question) I endeavour by means of the results therein contained to find the values of the unknown coefficients h, g, x, λ, μ, ν, h', g', x', λ', μ' ν'.

60. For a cubic surface $n=3$, and for a cubic surface without singular lines (in fact for all the cases except the cubic scrolls XXII and XXIII), the formula is

$$\beta'=54-h\mathrm{C}-g\mathrm{B}-\lambda' j'-\mu'\chi'-\nu'\theta-h'\mathrm{C}'-g'\mathrm{B}';$$

and applying this to the several cases of cubic surfaces as grouped together in the Table, and referred to by the affixed roman numbers, the resulting equations are

$$\begin{aligned}54&=54, && \text{(I)}\\ 30&=54-h, && \text{(II)}\\ 18&=54-g-16\nu', && \text{(III)}\\ 13&=54-2h-\lambda', && \text{(IV)}\\ 6&=54-h-g-\mu'-8\nu', && \text{(VI)}\\ 3&=54-3h-3\lambda', && \text{(VIII)}\\ 0&=54-2g-16\nu'-g', && \text{(IX)}\\ 1&=54-2h-g-\lambda'-2\mu', && \text{(XIII)}\\ 0&=54-4h-6\lambda', && \text{(XVI)}\\ 0&=54-h-2g-2\mu'-g', && \text{(XVII)}\\ 0&=54-3g-3g', && \text{(XXI)}\end{aligned}$$

which are all satisfied if only

$$\begin{aligned}h &= 24,\\ g+16\nu' &= 36,\\ g+2\mu' &= 12,\\ g+g' &= 18,\\ \lambda' &= -7.\end{aligned}$$

61. If we apply to the same surfaces the reciprocal equation for β, or, what is the same

thing, apply the original equation to the reciprocal surfaces, as given by interchanging the upper and lower halves of the Table of Singularities, we have another series of equations, viz. this is

$$\begin{aligned}
&0=54432-54432, && && \text{(I)}\\
&0=27851-27846-h', && h'=5, && \text{(II)}\\
&0=18180-18318-g'-16\nu, && g'+16\nu=-138, && \text{(III)}\\
&0=11765-11756-2h'-\lambda, && 2h'+\lambda=9, && \text{(IV)}\\
&0=6917-6584-h'-g'-\mu-8\nu, && h'+g'+\mu+8\nu=45, && \text{(VI)}\\
&0=3534-3522-3h'-3\lambda, && 3h'+3\lambda=12, && \text{(VIII)}\\
&0=3024-3144-2g'-16\nu-g, && 2g'+16\nu+g=-120, && \text{(IX)}\\
&0=1433-1386-2h'-g'-\lambda-2\mu, && 2h'+g'+\lambda+2\mu=47, && \text{(XIII)}\\
&0=518-504-4h'-6\lambda, && 4h'+6\lambda=14, && \text{(XVI)}\\
&0=383-322-h'-2g'-g-2\mu, && h'+2g'+g+2\mu=61, && \text{(XVII)}\\
&0=54-3g-3g', && 3g+3g'=54, && \text{(XXI)}
\end{aligned}$$

all satisfied if only

$$\begin{aligned}
h' &= 5,\\
g'+16\nu &= -138,\\
g'+2\mu &= 38,\\
g+g' &= 18,\\
\lambda &= -1.
\end{aligned}$$

62. I remark however that the cubic scroll XXII or XXIII gives

$$0=54-(330-272)-2(\lambda+\lambda'),$$

that is, $\lambda+\lambda'=-2$, instead of $\lambda+\lambda'=-8$. The investigation is in fact really inapplicable to a scroll, for every point of a scroll has the property of a flecnode; whence if $U=0$ be the equation of the scroll, that of the flecnodal surface is $M.U=0$, containing U as a factor, and there is not any definite curve of intersection constituting the flecnodal curve; but I am nevertheless surprised at the numerical contradiction.

63. Combining the two sets of results, we find

$$\begin{aligned}
h &= 24,\\
g &= g,\\
x &= x,\\
\lambda &= -1,\\
\mu &= 10+\tfrac{1}{32}g,\\
\nu &= -\tfrac{3}{4}\tfrac{9}{2}+\tfrac{1}{16}g,\\
h' &= 5,
\end{aligned}$$

$$g' = 18 - g,$$
$$\varkappa' = \varkappa,$$
$$\lambda' = -7,$$
$$\mu' = 6 - \tfrac{1}{2}g,$$
$$\nu' = \tfrac{9}{4} - \tfrac{1}{16}g;$$

and the formula thus is

$$\begin{aligned}\beta' = {} & 2n(n-2)(11n-24) \\ & -(110n-272)b+44g \\ & -(116n-303)c+\tfrac{63}{2}r \\ & +\tfrac{691}{4}\beta+248\gamma+198t \\ & -24\mathrm{C}+j-10\chi+\tfrac{329}{4}\theta-5\mathrm{C}'-18\mathrm{B}'-6\chi'-\tfrac{9}{4}\theta' \\ & -xi-x'i'+\tfrac{1}{16}g(-16\mathrm{B}-8\chi-\theta+16\mathrm{B}'-8\chi'-\theta'),\end{aligned}$$

where x, x', g are constants which remain to be determined. The cubic surfaces fail to determine them, for the reason that in all of them we have $i=0$, $i'=0$; and $16\mathrm{B}+8\chi+\theta=16\mathrm{B}'+8\chi'+\theta'$: this last is a very remarkable relation, for the existence of which I do not perceive any *à priori* reason.

Substituting herein for q, r their values from No. 11, this may be written in the form

$$\begin{aligned}\beta' = {} & 2n(n-2)(11n-24)+b(-66n+184)+c(-\tfrac{369}{4}n+240) \\ & +141\beta+\tfrac{359}{4}\gamma+66t \\ & -x' \qquad i'+\ 7j'-\ 6\chi'-\ \tfrac{9}{4}\theta'-\ 5\mathrm{C}'-18\mathrm{B}' \\ & -(x+87)i-21j-\tfrac{41}{2}\chi+\tfrac{51}{2}\theta-24\mathrm{C} \\ & +\tfrac{1}{16}g(-16\mathrm{B}-8\chi-\theta+16\mathrm{B}'+8\chi'+\theta').\end{aligned}$$

64. We have of course by interchanging the unaccented and accented letters, the reciprocal equation giving the value of β.

Recapitulation. Article Nos. 65 to 68.

65. In recapitulation, I say that we have between the 42 quantities

$$n, a, \mathfrak{d}, \varkappa;\ b, k, t, q, \varrho, j;\ c, h, r, \sigma, \theta, \chi;\ \beta, \gamma, i;\ \mathrm{B}, \mathrm{C},$$
$$n', a', \mathfrak{d}', \varkappa';\ b', k', t', q', \varrho', j';\ c', h', r', \sigma', \theta', \chi';\ \beta', \gamma', i';\ \mathrm{B}', \mathrm{C}',$$

in all 25 equations, viz. these are

$$a = a',$$
$$a' = n(n-1)-2b-3c,$$
$$\varkappa' = 3n(n-2)-6b-8c,$$
$$\mathfrak{d}' = \tfrac{1}{2}n(n-2)(n^2-9)-(n^2-6)(2b+3c)+2b(b-1)+6bc+\tfrac{9}{2}c(c-1),$$

$$a(n-2) = \varkappa - B + \varrho + 2\sigma,$$
$$b(n-2) = \varrho + 2\beta + 3\gamma + 3t,$$
$$c(n-2) = 2\sigma + 4\beta + \gamma + \vartheta,$$

$$a(n-2)(n-3) = 2(\delta - C) + 3(ac - 3\sigma - \chi) + 2(ab - 2\varrho - j),$$
$$b(n-2)(n-3) = 4k + (ab - 2\varrho - j) + 3(bc - 3\beta - 2\gamma - i),$$
$$c(n-2)(n-3) = 6h + (ac - 3\sigma - \chi) + 2(bc - 3\beta - 2\gamma - i),$$

$$q = b^2 - b - 2k - 3\gamma - 6t,$$
$$r = c^2 - c - 2h - 3\beta,$$

$$a = n'(n'-1) - 2b' - 3c',$$
$$\varkappa = 3n'(n'-2) - 6b' - 8c',$$
$$\delta = \tfrac{1}{2}n'(n'-2)(n'^2-9) - (n'^2 - n' - 6)(2b' + 3c') + 2b'(b'-1) + 6b'c' + \tfrac{9}{2}c'(c'-1),$$

$$a'(n'-2) = \varkappa' - B' + \varrho' + 2\sigma',$$
$$b'(n'-2) = \varrho' + 2\beta' + 3\gamma' + 3t',$$
$$c'(n'-2) = 2\sigma' + 4\beta' + \gamma' + \vartheta',$$

$$a'(n'-2)(n'-3) = 2(\delta' - C') + 3(a'c' - 3\sigma' - \chi') + 2(a'b' - 2\varrho' - j'),$$
$$b'(n'-2)(n'-3) = 4k' + (a'b' - 2\varrho' - j') + 3(b'c' - 3\beta' - 2\gamma' - i'),$$
$$c'(n'-2)(n'-3) = 6h' + (a'c' - 3\sigma' - \chi') + 2(b'c' - 3\beta' - 2\gamma' - i'),$$

$$q' = b'^2 - b' - 2k' - 3\gamma' - 6t',$$
$$r' = c'^2 - c' - 2h' - 3\beta',$$

together with the equations for β and β'.

66. The symbols signify as follows; viz.

n, order of the surface.
a, order of the tangent cone drawn from any point to the surface.
δ, number of nodal edges of the cone.
$\varkappa$, number of its cuspidal edges.
b, order of nodal curve.
k, number of its apparent double points.
t, number of its triple points.
q, its class.
ϱ, number of points where nodal curve is met by curve of contact of tangent cone.
j, number of pinch-points.
c, order of cuspidal curve.
h, number of its apparent double points.
r, its class.
σ, number of points where cuspidal curve is met by curve of contact of tangent cone.

θ, number of off-points.
χ, number of close-points.
β, number of intersections of nodal and cuspidal curves, stationary points on cuspidal curve.
γ, number of intersections, stationary points on nodal curve.
i, number of intersections, not stationary on either curve.
B, number of binodes of surface.
C, number of cnicnodes.

67. And the accented letters have the like significations in regard to the reciprocal surface; or, referring them to the original surface, we have

n', class of the surface.
α', class of curve of intersection by any plane.
δ', number of double tangents of curve of intersection.
$\varkappa'$, number of its inflexions.
b', class of node-couple torse.
k', number of its apparent double planes.
t', number of its triple planes.
q', its order.
ϱ', order of node-couple curve.
j', number of pinch-planes.
c', class of spinode torse.
h', number of its apparent double planes.
r', its order.
σ', order of spinode curve.
θ', number of off-planes.
χ', number of close-planes.
β', number of common planes of node-couple and spinode torses, stationary planes of the spinode torse.
γ', number of common planes, stationary planes of node-couple torse.
i', number of common planes, not stationary planes of either torse.
B', number of bitropes of surface.
C', number of cnictropes.

68. It is hardly necessary to recall that a spinode plane is a tangent plane meeting the surface in a curve having at the point of contact a spinode or cusp; the envelope of the spinode planes is the spinode torse, and the locus of their points of contact the spinode curve. And similarly a node-couple plane is a double tangent plane, or plane meeting the surface in a curve having two nodes; the envelope of the planes is the node-couple torse, and the locus of the points of contact the node-couple curve; the other terms made use of are all explained in the present Memoir.

Addition, August 3, 1869.

As in the theory of Curves, so in that of Surfaces, there are certain functions of the order, class, &c. and singularities which have the same values in the original and the reciprocal figures respectively; for convenience I represent any such identity by means of the symbol Σ, viz. $\phi(n, a, b, \ldots)=\Sigma$ denotes that the function $\phi(n, a, b, \ldots)$ is equal to the same function $\phi(n', a', b', \ldots)$ of the accented letters. By what precedes we have $a=\Sigma$; and it is moreover clear that any function of the unaccented letters which is $=0$, or which is equal to a symmetrical function of any of the accented and unaccented letters, or to a function of a, is $=\Sigma$; for instance, from the equations of No. 5 we have $3a'-\varkappa'=3n-c$, and thence $3n-c-\varkappa=3a'-\varkappa-\varkappa'$, that is, $3n-c-\varkappa=\Sigma$; and from one of the equations of No. 11 we have $n-2\mathrm{C}-4\mathrm{B}+\varkappa-\sigma-2j-3\chi=n+n'-a$, $=\Sigma$; we have thus the system of eight equations,

$$\begin{aligned}
&a && =\Sigma,\\
&3n-c-\varkappa && =\Sigma,\\
&a(n-2)-\varkappa+\mathrm{B}-\varrho-2\sigma && =\Sigma,\\
&b(n-2)-\varrho-2\beta-3\gamma-3t && =\Sigma,\\
&c(n-2)-2\sigma-4\beta-\gamma-\theta && =\Sigma,\\
&n+\varkappa-\sigma-2\mathrm{C}-4\mathrm{B}-2j-3\chi && =\Sigma,\\
&2q-2\varrho+\beta+3i+j && =\Sigma,\\
&3r+c-5\sigma-\beta-4\theta+2i+\chi && =\Sigma.
\end{aligned}$$

Or if from these we eliminate $\varkappa$, ϱ, σ, then the system of five equations,

$$\begin{aligned}
&a && =\Sigma,\\
&n(c-8)-4\beta-\gamma-\theta+4\mathrm{C}+8\mathrm{B}+6\chi+4j && =\Sigma,\\
&(a-b)(n-2)-11n+3c+4\mathrm{C}+9\mathrm{B}+2\beta+3\gamma+3t+6\chi+4j && =\Sigma,\\
&3r-20n+6c-\beta+2i+10\mathrm{C}+20\mathrm{B}+16\chi+10j-4\theta && =\Sigma,\\
&2q-2b(n-2)+5\beta+6\gamma+6t+3i+j && =\Sigma.
\end{aligned}$$

By means of a theorem of Dr. Cleebsch's I was led to the following expression for the "deficiency" of a surface of the order n having the singularities considered in the foregoing Memoir:

$$\text{Deficiency}=\tfrac{1}{6}(n-1)(n-2)(n-3)-(n-3)(b+c)+\tfrac{1}{2}(q+r)+2t+\tfrac{7}{2}\beta+\tfrac{5}{2}\gamma+i-\tfrac{1}{6}\theta;$$

This should be equal to the deficiency of the reciprocal surface, viz. we must have

$$2(n-1)(n-2)(n-3)-12(n-3)(b+c)+6q+6r+24t+42\beta+30\gamma+12i-\tfrac{3}{5}\theta=\Sigma;$$

but from a combination of the last mentioned five equations we have

$$\begin{aligned}
&-2n^3+6n^2+4n+(12n-36)b+(12n-48)c-6q-6r-24t\\
&\qquad -41\beta-30\gamma-13i-7j-8\chi+2\theta-4\mathrm{C}-10\mathrm{B}=\Sigma;
\end{aligned}$$

and adding to the last preceding equation we have

$$26n-12c+\beta-i-7j-8\chi+\tfrac{1}{2}\vartheta-4C-10B=\Sigma.$$

Substituting for Σ its value in terms of the accented letters, we obtain for β' the value

$$\begin{aligned}\beta'=\beta+26n-12c+i'+7j'+8\chi'-\tfrac{1}{2}\vartheta'+4C'+10B'\\ -26n'+12c'-i-7j-8\chi+\tfrac{1}{2}\vartheta-4C-10B.\end{aligned}$$

We have

$$c'=-3a+\varkappa+3n',$$

and thence

$$12c'-26n'=-36a+12\varkappa+10n';$$

writing herein

$$n'=a+\varkappa-\sigma-2C-4B-2j-3\chi-\sigma,$$

the value is

$$=-26a+22\varkappa-20C-40B-20j-30\chi-10\sigma.$$

Or substituting for $\varkappa$ its value $=a(n-2)+B-\varrho-2\sigma$, we have

$$12c'-26n'=a(22n-70)-20C-18B-20j-30\chi-22\varrho-44\sigma;$$

or substituting for a, ϱ, σ their values, this is

$$\begin{aligned}=\{n(n-1)-2b-3c\}(22n-70)-20C-18B-20j-30\chi\\ -22b(n-2)+44\beta+66\gamma+66t\\ -27c(n-2)+108\beta+27\gamma\qquad+27\vartheta,\end{aligned}$$

and adding hereto the remaining terms,

$$\begin{aligned}\beta+26n-12c+i'+7j'+8\chi'-\tfrac{1}{2}\vartheta'+4C'+10B'\\ -i-7j-8\chi+\tfrac{1}{2}\vartheta-4C-10B,\end{aligned}$$

we have

$$\begin{aligned}\beta'=2n(n-2)(11n-24)+b(-66n+184)+c(-93n+252)+153\beta+93\gamma+66t\\ +i'+7j'+8\chi'-\tfrac{1}{2}\vartheta'+4C'+10B'\\ -i-27j-38\chi+\tfrac{55}{2}\vartheta-24C-28B.\end{aligned}$$

Comparing this with the value of β', No. 63 of the foregoing Memoir, we should have

$$\begin{aligned}0=\tfrac{13}{4}cn-12c-12\beta-\tfrac{13}{4}\gamma\\ -(x'+1)i'\qquad-14\chi-\tfrac{7}{4}\vartheta'-9C'-28B'\\ -(x+86)i+6j+\tfrac{35}{4}\chi-2\vartheta\qquad+28B\\ +\tfrac{1}{16}g(-16B-8\chi-\vartheta+16B'+8\chi'+\vartheta'),\end{aligned}$$

or, what is the same thing,

$$0=13cn-48c-48\beta-13\gamma+\Phi,$$

if for shortness

$$\begin{aligned}\Phi=-(4x'+4)i'\qquad-56\chi'-7\vartheta'-36C'-112B'\\ -(4x+344)i+24j+70\chi-8\vartheta\qquad+112B\\ +\tfrac{1}{4}g(-16B-8\chi-\vartheta+16B'+8\chi'+\vartheta').\end{aligned}$$

I do not attempt to verify this equation, but I will partially verify a result deducible from it; viz. if Φ' is the like function of the accented letters, then we have

$$\Phi - \Phi' = \Pi - \Pi',$$

where

$$\Pi = (4x^2 - 4x - 340)i + 24j + 126\chi + 224\mathrm{B} + 36\mathrm{C} - \theta - \tfrac{1}{2}g(16\mathrm{B} + 8\chi + \theta);$$

and Π' is the like function of the accented letters. And this being so, we should have

$$13cn - 48c - 48\mathrm{B} - 13\gamma + \Pi = 13c'n' - 48c' - 48\beta' - 13\gamma' + \Pi',$$

or, as this may be written,

$$13cn - 48c - 48\beta - 13\gamma + \Pi = \Sigma.$$

We have

$$26n - 12c + \beta - i - 7j - 8\chi + \tfrac{1}{2}\theta - 4\mathrm{C} - 10\mathrm{B} = \Sigma;$$

and multiplying by -4 and adding, the equation to be verified is

$$13n(c-8) - 13(4\beta + \gamma) + \Pi + 4i + 28j + 32\chi - 2\theta + 16\mathrm{C} + 40\mathrm{B} = \Sigma.$$

But we have from the Memoir

$$-13n(c-8) + 13(4\beta + \gamma) \qquad -52j - 78\chi + 13\theta - 52\mathrm{C} - 104\mathrm{B} = \Sigma,$$

which reduces the equation to

$$\Pi + 4i - 24j - 46\chi + 11\theta - 36\mathrm{C} - 64\mathrm{B} = \Sigma;$$

or substituting for Π its value, this is

$$(4x' - 4x - 336)i + 80\chi + 10\theta + 160\mathrm{B} - 2\lambda(16\mathrm{B} + 8\chi + \theta) = \Sigma,$$

that is

$$4(x' - x - 84)i - (\tfrac{1}{2}g - 10)(16\mathrm{B} + 8\chi + \theta) = \Sigma,$$

an equation which is satisfied if

$$i' = i, \quad x' = x,$$

and

$$g = 20, \text{ or else } 16\mathrm{B} + 8\chi + \theta = 16\mathrm{B}' + 8\chi' + \theta'.$$

VII. *A Memoir on Cubic Surfaces.* By Professor CAYLEY, *F.R.S.*

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THE present Memoir is based upon, and is in a measure supplementary to that by Professor SCHLÄFLI, "On the Distribution of Surfaces of the Third Order into Species, in reference to the presence or absence of Singular Points, and the reality of their Lines," Phil. Trans. vol. cliii. (1863) pp. 193–241. But the object of the Memoir is different. I disregard altogether the ultimate division depending on the reality of the lines, attending only to the division into (twenty-two, or as I prefer to reckon it) twenty-three cases depending on the nature of the singularities. And I attend to the question very much on account of the light to be obtained in reference to the theory of Reciprocal Surfaces. The memoir referred to furnishes in fact a store of materials for this purpose, inasmuch as it gives (partially or completely developed) the equations in plane-coordinates of the several cases of cubic surfaces, or, what is the same thing, the equations in point-coordinates of the several surfaces (orders 12 to 3) reciprocal to these repectively. I found by examination of the several cases, that an extension was required of Dr. SALMON'S theory of Reciprocal Surfaces in order to make it applicable to the present subject; and the preceding "Memoir on the Theory of Reciprocal Surfaces" was written in connexion with these investigations on Cubic Surfaces. The latter part of the Memoir is divided into sections headed thus:—"Section I=12, equation $(X, Y, Z, W)^3=0$" &c. referring to the several cases of the cubic surface; but the paragraphs are numbered continuously through the Memoir.

The twenty-three Cases of Cubic Surfaces—Explanations and Table of Singularities. Article Nos. 1 to 13.

1. I designate as follows the twenty-three cases of cubic surfaces, adding to each of them its equation:

I $=12$,	$(X, Y, Z, W)^3=0$,
II $=12-C_2$,	$W(a, b, c, f, g, h)(X, Y, Z)^2+2kXYZ=0$,
III $=12-B_2$,	$2W(X+Y+Z)(lX+mY+nZ)+2kXYZ=0$,
IV $=12-2C_2$,	$WXZ+Y^2(\gamma Z+\delta W)+(a, b, c, d)(X, Y)^3=0$,
V $=12-B_4$,	$WXZ+(X+Z)(Y^2-aX^2-bZ^2)=0$,
VI $=12-B_2-C_2$,	$WXZ+Y^2Z+(a, b, c, d)(X, Y)^3=0$,
VII $=12-B_3$,	$WXZ+Y^2Z+YX^2-Z^3=0$,

VIII	$=12-3C_2$,	$Y^3+Y^2(X+Z+W)+4aXZW=0$,
IX	$=12-2B_3$,	$WXZ+(a, b, c, d)(X, Y)^3=0$,
X	$=12-B_4-C_2$,	$WXZ+(X+Z)(Y^2-X^2)=0$,
XI	$=12-B_6$,	$WXZ+Y^2Z+X^3-Z^3=0$,
XII	$=12-U_6$,	$W(X+Y+Z)^2+XYZ=0$,
XIII	$=12-B_3-2C_2$,	$WXZ+Y^2(X+Y+Z)=0$,
XIV	$=12-B_5-C_2$,	$WXZ+Y^2Z+YX^2=0$,
XV	$=12-U_7$,	$WX^2+XZ^2+Y^2Z=0$,
XVI	$=12-4C_2$,	$W(XY+XZ+YZ)+XYZ=0$,
XVII	$=12-2B_3-C_2$,	$WXZ+XY^2+Y^3=0$,
XVIII	$=12-B_4-2C_2$,	$WXZ+(X+Z)Y^2=0$,
XIX	$=12-B_6-C_2$,	$WXZ+Y^2Z+X^3=0$,
XX	$=12-U_8$,	$WX^2+XZ^2+Y^3=0$,
XXI	$=12-3B_3$,	$WXZ+Y^3=0$,
XXII	$=3, S(1, 1)$,	$WX^2+ZY^2=0$,
XXIII	$=3, S(\overline{1, 1})$,	$X(WX+YZ)+Y^2=0$;

2. Where C_2 denotes a conic-node diminishing the class by 2; B_3, B_4, B_5, B_6 a biplanar node diminishing (as the case may be) the class by 3, 4, 5, or 6; and U_6, U_7, U_8 a uniplanar node diminishing (as the case may be) the class by 6, 7, or 8. The affixed explanation, which I shall usually retain in connexion with the Roman number, shows therefore in each case what the class is, and also the singularities which cause the reduction: thus XIII$=12-B_3-2C_2$ indicates that there is a biplanar node, B_3, diminishing the class by 3, and two conic-nodes, C_2, each diminishing the class by 2; and thus that the class is $12-3-2.2, =5$. As regards the cases XXII and XXIII, these are surfaces having a nodal right line, and are consequently scrolls, each of the class 3, viz. XXII is the scroll $S(1, 1)$ having a simple directrix right line distinct from the nodal line, and XXIII is the scroll $S(\overline{1, 1})$ having a simple directrix right line coincident with the nodal line: see as to this my "Second Memoir on Skew Surfaces, otherwise Scrolls," Phil. Trans. vol. cliv. (1864) pp. 559–577.

3. The nature of the points C_2, B_3, B_4, B_5, B_6, U_6, U_7, U_8 requires to be explained.

$C(=C_2)$ is a conic-node, where, instead of the tangent plane, we have a proper quadric cone.

$B(=B_3, B_4, B_5$ or $B_6)$ is a biplanar-node, where the quadric cone becomes a plane-pair (two distinct planes): the two planes are called the biplanes, and their line of intersection is the edge:

In B_3, the edge is not a line on the surface—in the other cases it is; this implies that the surface is touched along the edge by a plane, viz. in B_4, B_5 the edge is torsal, in B_6 it is oscular:

In B_4, the tangent plane is distinct from each of the biplanes:

In B_5, the tangent plane coincides with one of the biplanes; we have thus an ordinary biplane, and a torsal biplane:

In B_6, the tangent plane coinciding with one of the biplanes becomes oscular; we have thus an ordinary biplane, and an oscular biplane.

$U(=U_6, U_7$ or $U_8)$ is a uniplanar-node, where the quadric cone becomes a coincident plane-pair; say, the plane is the uniplane. It is to be observed that there is not in this case any edge. The uniplane meets the cubic surface in three lines, or say "rays," passing through the uniplanar-node, viz.

In U_6, the rays are three distinct lines:

In U_7, two of them coincide:

In U_8, they all three coincide.

4. To connect these singular points with the theory of the preceding Memoir, it is to be observed that they are respectively equivalent to a certain number of the cnicnodes $C(=C_3)$ and binodes $B(=B_3)$, viz. we have

$$\begin{aligned} C_3 &= C, \\ B_3 &= B, \\ B_4 &= 2C, \\ B_5 &= C + B, \\ \left\{\begin{aligned} B_6 &= 3C, \\ U_6 &= 3C, \end{aligned}\right. & \\ U_7 &= 2C + B, \\ U_8 &= C + 2B. \end{aligned}$$

5. I take the opportunity of remarking that although the expressions cnicnode and binode properly refer to the simple singularities C and B, yet as $C_3=C$, C_3 is properly spoken of as a cnicnode, and we may (using the term binode as an abbreviation for biplanar-node) speak of any of the singularities B_3, B_4, B_5, B_6 as a binode. Thus the surface $X=12-B_4-C_3$ has a binode B_4 and a cnicnode C_3; although theoretically the binode B_4 is equivalent to two cnicnodes, and the surface belongs to those with three cnicnodes, or for which $C=3$. I use also the expression unode for shortness, instead of uniplanar-node, to denote any of the singularities U_6, U_7, U_8.

6. The foregoing equations (substantially the same as SCHLÄFLI's) are *Canonical forms*; the reduction of the equation of any case of surface to the above form is not always obvious. It would appear that each equation is from its simplicity in the form best adapted to the separate discussion of the surface to which it belongs; there is the disadvantage that the equations do not always (when from the geometrical connexion of the surfaces they ought to do so) lead the one to the other; for instance, $V=12-B_3$ includes $VII=12-B_3$, but we cannot from the equation $WXZ+(X+Z)(Y^2-aX^2-bZ^2)=0$ of the former pass to the equation $WXZ+Y^2Z+YX^2-X^3=0$ of the latter. This would be a serious imperfection if the object were to form a theory of the quaternary function

$(X, Y, Z, W)^3$; but the equations are in the present Memoir used only as means to an end, the establishment of the geometrical theory of the surfaces to which they respectively belong, and the imperfection is not material.

7. I have used the capital letters (X, Y, Z, W) in place of SCHLÄFLI'S (x, y, z, w), reserving these in place of his (p, q, r, s) for plane-coordinates of the cubic surfaces, or (what is the same thing) point-coordinates of the reciprocal surfaces; but I have in several cases interchanged the coordinates (X, Y, Z, W) so that they do not *in this order* correspond to SCHLÄFLI'S (x, y, z, w): this has been done so as to obtain a greater uniformity in the representation of the surfaces. To explain this, let A, B, C, D be the vertices of the tetrahedron formed by the coordinate planes A=YZW, B=ZWX, C=WXY, D=XYZ; the coordinate planes have been chosen so that determinate vertices of the tetrahedron shall correspond to determinate singularities of the surface.

8. Consider first the surfaces which have no nodes B or U. It is clear that the nodes C_2 might have been taken at any vertices whatever of the tetrahedron; they are taken thus: there is always a node C_2 at D; when there is a second node C_2, this is at C, the third one is at A, and the fourth at B.

9. Consider next the surfaces which have a binode B_2, B_4, B_5, or B_6; this is taken to be at D, and the biplanes to be X=0, Z=0* (the edge being therefore DB), viz. in B_5 or B_6, where the distinction arises, X=0 is the ordinary biplane, Z=0 the torsal or (as the case may be) oscular biplane. If there is a second node, this of necessity lies in an ordinary biplane; it may be and is taken to be in the biplane X=0, at C. I suppose for a moment that this is a node C_2. It is only when the binode is B_5 or B_4 that there can be a third node, for it is only in these cases that there is a second ordinary biplane Z=0; but in these cases respectively the third node, a C_2, may be and is taken to be in the biplane Z=0, at A.

10. The only case of two binodes is when each is a B_2. Here the first is as above at D, its biplanes being X=0, Z=0; and the second is as above in the biplane X=0, at C; the biplanes thereof are then X=0 (which is thus a biplane common to the two binodes, or say a common biplane), and a remaining biplane which may be and is taken to be W=0. If there is a third node, this may be either C_2 or B_2, but it will in either case lie in the biplane Z=0 of the first binode, and also in the biplane W=0 of the second binode, that is, in the line BA; and it may be and is taken to be at A; if a binode, then its biplanes are of necessity Z=0, W=0; and the plane Y=0 will be the plane through the three binodes D, C, A.

11. If there is a unode, then this may be and is taken to be at D, and its uniplane may be taken to be X=0; in the surface $XII=12-U_6$ the uniplane is, however, taken to be X+Y+Z=0. There is never, besides the unode, any other node.

12. The result is that the nodes, in the order of their speciality, are in the equations taken to be at D, C, A, B respectively; and that (except in the case $III=12-B_3$) the biplanes of the first binode are X=0, Z=0 (for a binode B_5 or B_6, X=0 being the ordinary biplane, Z=0 the special biplane), those of the second binode X=0, W=0,

* In the case, however, of a single B_3, $III=12-B_3$, the biplanes are taken to be X+Y+Z=0, $lX+mY+nZ=0$.

those of the third binode $Z=0$, $W=0$, and that (except in the case $XII=12-U_4$) the uniplane is $X=0$. For example, in the surface $XVII=12-2B_2-C_2$, as represented by its equation $WXZ+Y^2Z+X^2=0$, we have a B_2 at D, the biplanes being $X=0$, $Z=0$, a B_2 at C, the biplanes being $X=0$, $W=0$ (therefore $X=0$ the common biplane), and a C_2 at A.

13. It will be convenient (anticipating the results of the investigations contained in the present Memoir) to give at once the following Table of Singularities; the several symbols have of course the significations explained in the former Memoir.

	I $=12$	II $=12-C_2$	III $=12-B_2$	IV $=12-2C_2$ / V $=12-B_2-C_2$	VI $=12-B_2-C_2$ / VII $=12-3B_2$	VIII $=12-3C_2$ / X $=12-B_2-C_2$ / XI $=12-B_2$ / XII $=12-U_2$	IX $=12-2B_2$	XIII $=12-B_2-2C_2$ / XIV $=12-B_2-C_2$ / XV $=12-U_2$	XVI $=12-4C_2$ / XVIII $=12-B_2-2C_2$ / XIX $=12-B_2-C_2$	XVII $=12-2B_2-C_2$ / XX $=12-U_2$	XXI $=12-3B_2$	XXII $=8(1, 1)$.	XXIII $=8(1, 1)$.	
κ	3	3	3	3	3	3	3	3	3	3	3	3	3	κ
$\kappa\sigma$	6	6	6	6	6	6	6	6	6	6	6	6	4	$\kappa\sigma$
$\kappa\tau$	0	1	0	2	1	3	0	2	4	1	0	0	0	$\kappa\tau$
κ	6	6	7	6	7	6	8	7	6	8	9	3	3	κ
δ	0	0	0	0	0	0	0	0	0	0	0	0	1	δ
$\delta\kappa$	0	0	0	0	0	0	0	0	0	0	0	0	0	$\delta\kappa$
$\delta\sigma$	0	0	0	0	0	0	0	0	0	0	0	0	0	$\delta\sigma$
$\delta\tau$	0	0	0	0	0	0	0	0	0	0	0	0	0	$\delta\tau$
δ	0	0	0	0	0	0	0	0	0	0	0	0	1	δ
γ	0	0	0	0	0	0	0	0	0	0	0	0	2	γ
θ	0	0	0	0	0	0	0	0	0	0	0	0	0	θ
$\theta\kappa$	0	0	0	0	0	0	0	0	0	0	0	0	0	$\theta\kappa$
$\theta\sigma$	0	0	0	0	0	0	0	0	0	0	0	0	0	$\theta\sigma$
$\theta\tau$	0	0	0	0	0	0	0	0	0	0	0	0	0	$\theta\tau$
θ	0	0	0	0	0	0	0	0	0	0	0	0	0	θ
χ	0	0	0	0	0	0	0	0	0	0	0	0	0	χ
β	0	0	0	0	0	0	0	0	0	0	0	0	0	β
$\gamma\iota$	0	0	0	0	0	0	0	0	0	0	0	0	0	$\gamma\iota$
ι	0	0	0	0	0	0	0	0	0	0	0	0	0	ι
C	0	1	0	2	1	3	0	2	4	1	0	0	0	C
B	0	0	1	0	1	0	2	1	0	2	3	0	0	B
	I	II	III	IV V	VI VII	VIII X XI XII	IX	XIII XIV XV	XVI XVIII XIX	XVII XX	XXI	XXII	XXIII	
κ'	12	10	9	8	7	6	6	5	4	4	3	3		κ'
$\kappa\sigma'$	6	6	6	6	6	6	6	6	6	6	6	4		$\kappa\sigma'$
$\kappa\tau'$	0	0	0	0	0	0	0	0	0	0	0	0		$\kappa\tau'$
κ'	9	9	9	9	9	9	9	9	9	9	9	3		κ'
δ'	27	15	9	7	3	3	0	1	3	0	0	1		δ'
$\delta\kappa'$	216	60	18	12	3	0	0	0	0	0	0	0		$\delta\kappa'$
$\delta\sigma'$	45	15	6	3	0	1	0	0	1	0	0	0		$\delta\sigma'$
$\delta\tau'$	0	0	0	0	0	0	0	0	0	0	0	0		$\delta\tau'$
γ'	27	15	9	7	3	3	0	1	3	0	0	1		γ'
ι'	0	0	0	1	0	3	0	1	6	0	0	2		ι'
θ'	24	18	16	12	10	6	8	4	0	2	0	0		θ'
$\theta\kappa'$	180	96	72	38	24	6	12	2	0	0	0	0		$\theta\kappa'$
$\theta\sigma'$	30	24	42	17	24	9	32	5	0	2	0	0		$\theta\sigma'$
$\theta\tau'$	12	12	12	10	9	6	8	4	0	2	0	0		$\theta\tau'$
θ'	0	0	16	0	8	0	16	0	0	0	0	0		θ'
χ'	0	0	0	0	1	0	0	2	0	2	0	0		χ'
β'	54	30	18	13	6	3	0	1	0	0	0	0		β'
$\gamma\iota'$	0	0	0	0	0	0	0	0	0	0	0	0		$\gamma\iota'$
ι'	0	0	0	0	0	0	0	0	0	0	0	0		ι'
C′	0	0	0	0	0	0	0	0	0	0	0	0		C′
B′	0	0	0	0	0	0	1	0	0	1	3	0		B′

Explanation in regard to the determination of the number of certain singularities.
Article Nos. 14 to 19.

14. In the several cases I to XXI, we have a cubic surface ($n=3$), with singular points C and B but without singular lines. The section by an arbitrary plane is thus a curve, order $n=3$, that is, a cubic curve, without nodes or cusps, and therefore of the class $a'=6$, having $\gamma=0$ double tangents and $\varkappa'=9$ inflexions. The tangent cone with an arbitrary point as vertex is a cone of the order $a=6$, having in the case I=12, $\delta=0$ nodal lines and $\varkappa=6$ cuspidal lines, but with (in the several other cases) C nodal lines and B cuspidal lines (or rather singular lines tantamount to C double lines and B cuspidal lines): the class of the cone, or order of the reciprocal surface, is thus $n'=6.5-2(0+C)-3(6+B)=12-2B-3C$.

15. In the general case I=12, there are on the cubic surface 27 lines, lying by 3's in 45 planes; these 27 lines constitute the node-couple curve of the order $\varrho'=27$, and the node-couple torse consists of the pencils of planes through these lines respectively, being thus of the class $\varrho'=b'=27$; the 45 planes are triple tangent planes of the node-couple torse, which has thus $t'=45$ triple tangent planes. But in the other cases it is only certain of the 27 lines, say the "facultative lines" (as will be explained), which constitute the node-couple curve of the order ϱ': the pencils of planes through these lines constitute the node-couple torse of the class $b'=\varrho'$; the t' planes, each containing three facultative lines, are the triple tangent planes of the node-couple torse. Or if (as is somewhat more convenient) we refer the numbers b', t' to the reciprocal surface, then the lines, reciprocals of the facultative lines, constitute the nodal curve of the order b'; and the points t', each containing three of these lines, are the triple points of the nodal curve. Inasmuch as the nodal curve consists of right lines, the number k' of its apparent double points is given by the formula $2k'=b'^2-b'-6t'$; and comparing with the formula $q'=b'^2-b'-2k'-3\gamma'-6t'$, we have $q'+3\gamma'=0$, that is, $q'=0$ (q' the class of the nodal curve), and also $\gamma'=0$.

16. In the general case I=12, the spinode curve is the complete intersection of the cubic surface by the Hessian surface of the order 4, and it is thus of the order $\sigma'=12$; but in the other cases the complete intersection consists of the spinode curve together with certain right lines not belonging to the curve, and the spinode curve is of an order σ' less than 12: this will be further explained, and the reduction accounted for (see *post*, Nos. 24 *et seq.*).

17. Again, in the general case I=12, each of the 27 lines is a double tangent of the spinode curve, and the tangent planes of the surface at the points of contact are common tangent planes of the spinode torse and the node-couple torse, stationary planes of the spinode torse; or we have $\beta'=2\varrho'=54$. In the other cases, however, instead of the 27 lines we must take only the facultative lines, each of which is or is not a double or a single tangent of the spinode curve; and the tangent planes of the surface at the points of contact are the common tangent planes as above—that is, the number of contacts gives β', not in general $=2\varrho'$.

18. There are not, except as above, any common tangent planes of the two torses, that is, not only $\gamma'=0$ as already mentioned, but also $i'=0$. I do not at present account *à priori* for the values $\delta'=16$, 8, and 16, which present themselves in the Table. The cubic surface cannot have a plane of conic contact, and we have thus in every case $C'=0$; but the value of B′ is not in every case $=0$.

19. In what precedes we see how a discussion of the equation of the cubic surface should in the several cases respectively lead to the values b', t', ϱ', σ', β', j', χ', B′, and how in the reciprocal surface the nodal curve of the order b' is known by means of the facultative lines of the original cubic surface. The cuspidal curve c' might also be obtained as the reciprocal of the spinode-torse; but this would in general be a laborious process, and it is the less necessary, inasmuch as the equation of the reciprocal surface is in each case obtained in a form putting in evidence the cuspidal curve.

The Lines and Planes of a Cubic Surface; Facultative Lines; Explanation of Diagrams. Article Nos. 20 to 23.

20. In the general surface I=12, we have 27 lines and 45 triple-tangent planes, or say simply, planes: through each line pass 5 planes, in each plane lie 3 lines. For the surfaces II to XXI (the present considerations do not of course apply to the Scrolls) several of the lines come to coincide with each other, and several of the planes also come to coincide with each other; but the number of the lines is always reckoned as 27, and that of the planes as 45. If we attend to the distinct lines and the distinct planes, each line has a multiplicity, and the sum of these is=27; and so each plane has a multiplicity, and the sum of these is=45. Again, attending to a particular line in a particular plane, the line has a frequency 1, 2, or 3, that is, it represents 1, 2, or 3 of the 3 lines in the plane (this is in fact the distinction of a scrolar, torsal, or osculer line); and similarly, the plane has a frequency 1, 2, 3, 4, or 5, according to the number which it represents of the 5 planes through the line. It requires only a little consideration to perceive that the multiplicity of the plane into its frequency in regard to the line is equal to the multiplicity of the line into its frequency in regard to the plane. Observe, further, that if M be the multiplicity of the plane, then, considering it in regard to the lines contained therein, we get the products (M, M, M), (2M, M), or 3M, according as the three lines are or are not distinct, but that the sum of the products is always=3M, and that in regard to all the planes the total sum is 3×45, =135. And so if M′ be the multiplicity of the line, then, considering it in regard to the planes which pass through it, we get the products (M′, M′, M′, M′, M′), (2M′, M′, M′, M′) . . . (5M′), as the case may be, but that the sum of the products is =5M′, and that in regard to all the lines the sum is 5×27, =135, as before.

21. The mode of coincidence of the lines and planes, and the several distinct lines and planes which are situate in or pass through the several distinct planes and lines respectively, are shown in the annexed diagrams I to XXI*: the multiplicity of each line

* See the commencements of the several sections.

appears by the upper marginal line, and that of each plane by the left-hand marginal column (thus in diagram I, $27\times1=27$ and $45\times1=45$, 1 is the multiplicity of each line, and it is also the multiplicity of each plane); the frequencies of a line and plane in regard to each other appear by the dots in the square opposite to the line and plane in question, these being read, for the frequency of the line vertically, and for the frequency of the plane horizontally; thus $\begin{smallmatrix}\cdot&\cdot\\ &\cdot\end{smallmatrix}$ indicates that the frequency of the line is $=3$, and the frequency of the plane is $=2$. There should be and are in every line of the diagram 3 dots, and in every column of the diagram 5 dots (a symbol $\begin{smallmatrix}\cdot&\cdot\\ \cdot&\\ \cdot&\end{smallmatrix}$ being read as just explained, 2 dots in the line, 3 dots in the column).

22. For the surface I=12, there is of course no distinction between the lines, but these form only a single class, and the like for the planes; but for the other surfaces the lines and planes form separate classes, as shown in the diagrams by the lower marginal explanation of the lines, and the right-hand marginal explanation of the planes. I use here and elsewhere "ray" to denote a line passing through a single node; "axis" to denote a line joining two nodes; "edge" (as above) to denote the edge of a binode; any other line is a "mere line." An axis is always torsal or osculär; when it is torsal, the plane touching along the axis contains a third line which is the "transversal" of such axis; but a transversal may be a mere line, a ray, or an axis; in the case $XVI=12-4C_2$, each transversal is a transversal in regard to two axes.

23. In the general case I=12, each of the 27 lines is, as already mentioned, part of the node-couple curve; and the node-couple curve is made up of the 27 lines, and is thus a curve of the order 27. In fact each plane through a line meets the cubic surface in this line, and in a conic; the line and conic meet in two points, and the plane (that is in any plane) through the line is thus a double tangent plane touching the surface at the two points in question; the locus of the points of contact, that is the line itself, is thus part of the node-couple curve. But in the other cases, II to XXI, certain of the lines do not belong to the node-couple curve (this will be examined in detail in the several cases respectively); but I wish to show here how in a general way a line passing through a node, say a nodal ray, is not part of the node-couple curve. To fix the ideas, consider the surface $II=12-C_2$; there are here through C_2 six lines, or say rays: attending to any one of these, a plane through the ray meets the surface in the ray itself and in a conic; the ray and the conic meet as before in two points, one of them being the point C_2: the plane touches the surface at the other point, *but it does not touch the surface at* C_2. (I am not sure, and I leave it an open question, whether we ought to say that at a node C_2 there is *no* tangent plane, or to say that only the tangent planes of the nodal cone are tangent planes of the surface; but, at any rate, an arbitrary plane through C_2 is *not* a tangent plane.) The plane through the ray is only a single tangent plane, not a double tangent plane; and the ray is not part of the node-couple curve. We say that a line of the surface is or is not "facultative" according as it does or does not form part of the node-couple curve.

Axis; the different kinds thereof. Article Nos. 24 to 26.

24. A line joining two nodes is an axis; such a line is always a line, and it is a torsal or oscular line, of the surface. But some further distinctions are requisite; using the expressions in their strict sense, cnicnode =C, binode =B, an axis is a CC-axis joining two cnicnodes, or it is a CB-axis joining a cnicnode and a binode, or it is a BB-axis joining two binodes. A CC-axis is torsal, the transversal being a mere line, not a ray through either of the cnicnodes; a CB-axis is torsal, the transversal being a ray of the binode; a BB-axis is oscular. The distinction is of course carried through as regards the higher biplanar nodes B_4, B_5, B_6, and the uniplanar nodes U_6, U_7, U_8: thus (B_3=B) the edge of a binode B_3 is not an axis at all, but (B_4=2C) the edge of a binode B_4 is a CC-axis; (B_5=B+C) the edge of a binode B_5 is a CB-axis; (B_6=3C) the edge of a binode B_6 is a thrice-taken CC-axis; (U_6=3C) each of the rays is regarded as a CC-axis; (U_7=B+2C) the double ray is regarded as a twice-taken CB-axis, and the single ray as a CC-axis; (U_8=2B+C) the ray is regarded as a BB-axis + a twice-taken CB-axis.

25. It has been mentioned that the intersection of the surface with the Hessian consists of the spinode curve, together with certain right lines; these lines are in fact the axes—viz. the examination of the several cases shows that in the complete intersection each CC-axis presents itself twice, each CB-axis 3 times, and each BB-axis 4 times. We thus see that a CC-axis, or rather the torsal plane along such axis, is the pinch-plane or singularity $j'=1$; the CB-axis, or rather the torsal plane along such axis, the close-plane or singularity $\chi'=1$; and the BB-axis, or oscular plane along such axis, the bitrope or singularity $B'=1$; for a cubic surface with singular lines the expression of σ' being in fact $\sigma'=12-2j'-3\chi'-4B'$. There are, however, some cases requiring explanation; thus for the case VIII=12−B_3, where the edge is by what precedes a CB-axis, the complete intersection is made up of the edge 4 times and of an octic curve; the consideration of the reciprocal surface shows, however, that the edge taken once is really part of the spinode curve (viz. that this curve is made up of the edge taken once and of the octic curve, its order being thus $\sigma'=9$); and the interpretation then of course is that the intersection is made up of the edge taken 3 times (as for a CB-axis it should be) and of the spinode curve.

26. I remark in further explanation, that in the several sections, in showing how the complete intersection of the cubic surface with the Hessian is made up, I have not referred to the axes in the above precise significations; thus XIV=12−B_3−C_2, the binode B_3 is C+B, and the edge is thus a CB-axis, while the axis B_3C_2 is a CB-axis + a CC-axis ($\chi'=1+1$, $=2$, $j'=1$). The complete intersection should therefore consist of the spinode curve, + edge (as a CB-axis) 3 times + axis (as a CB-axis + a CC-axis) 2+3, =5 times: it is in the section stated (in perfect consistency herewith, but without the full explanation) that the intersection is made up of the axis 5 times, the edge 4 times, and a cubic curve—which cubic curve together with the edge once constitutes the spinode curve; and so in other cases: this explanation will, I think, remove all difficulty.

On the Determination of the Reciprocal Equation. Article Nos. 27 to 32.

27. Consider in general the cubic surface $(*)(X, Y, Z, W)^3=0$, and in connexion therewith the equation $Xx+Yy+Zz+Ww=0$, which regarding therein X, Y, Z, W as current coordinates, and x, y, z, w as constants, is the equation of a plane. If from the two equations we eliminate one of the coordinates, for instance W, we obtain

$$(*)(Xw, Yw, Zw, -(Xx+Yy+Zz))^3=0,$$

which, (X, Y, Z) being current coordinates, is obviously the equation of the cone, vertex (X=0, Y=0, Z=0), which stands on the section of the cubic surface by the plane. Equating to zero the discriminant of this function in regard to (X, Y, Z), we express that the cone has a nodal line; that is, that the section has a node, or, what is the same thing, that the plane $xX+yY+zZ+wW=0$ is a tangent plane of the cubic surface; and we thus by the process in fact obtain the equation of the cubic surface in the reciprocal or plane coordinates (x, y, z, w). Consider in the same equation x, y, z, w as current coordinates, (X, Y, Z) as given parameters, the equation represents a system of three planes, viz. these are the planes $xX+yY+zZ+wW'=0$, where W' has the three values given by the equation $(*)(X, Y, Z, W')^3=0$, or, what is the same thing, X, Y, Z, W' are the coordinates of any one of the three points of intersection of the cubic surface by the line $\frac{x}{X}=\frac{y}{Y}=\frac{z}{Z}$; (X, Y, Z, W') belongs to a point on the surface, and

$$xX+yY+zZ+wW'=0$$

is the polar plane of this point in regard to a quadric surface $X^2+Y^2+Z^2+W^2=0$; the equation

$$(*)(Xw, Yw, Zw, -(Xx+Yy+Zz))^3=0$$

is thus the equation of a system of 3 planes, the polar planes of three points of the cubic surface (which three points lie on an arbitrary line through the point $x=0, y=0, z=0$). In equating to zero the discriminant in regard to (X, Y, Z), we find the envelope of the system of three planes, or say of a plane, the polar plane of an arbitrary point on the cubic surface,—or we have the equation of the reciprocal surface, being, as is known, the same thing as the equation of the cubic surface in the reciprocal or plane coordinates (x, y, z, w). In what precedes we have the explanation of an ordinary process of finding the equation of the reciprocal surface, this equation being thereby given by equating to zero the discriminant of a function $(*)(X, Y, Z)^3$, that is, of a ternary cubic function.

28. The process, as last explained, is a special one, viz. the position of a point on the surface is determined by means of certain two parameters, the ratios X : Y : Z which fix the position of the line joining this point with the point $(x=0, y=0, z=0)$. More generally we may consider the position of the point as determined by means of any two parameters; the equation of the polar plane then contains the two parameters, and by taking the envelope in regard to the two parameters considered as variable, we have the equation of the reciprocal surface.

29. But let the parameters, say θ, φ, be regarded as varying successively; if φ alone

vary, we have on the surface a curve Θ, the equation whereof contains the parameter θ, and when θ varies this curve sweeps over the surface. The envelope in regard to φ of the polar plane of a point of the surface is a torse, the reciprocal of the curve Θ, and the envelope of the torse is the reciprocal surface. In particular the curve Θ may be the plane section by any plane through a fixed line, say, by the plane $P-\theta Q=0$; the section is a cubic curve, the reciprocal is a sextic cone having its vertex in a fixed line (the reciprocal of the line $P=0$, $Q=0$), and the reciprocal surface is thus obtained as the envelope of this cone; assuming that the equation of the sextic cone has been obtained, this is an equation of a certain order in the parameter θ; or writing $\theta=P:Q$, we obtain the equation of the reciprocal surface by equating to zero the discriminant of a *binary* function of (P, Q).

30. With a variation, this process is a convenient one for obtaining the reciprocal of a cubic surface: we take the fixed line to be one of the lines on the cubic surface; the curve Θ is then a conic, its reciprocal is a quadricone, and the envelope of this quadricone is the required reciprocal surface. This is really what SCHLÄFLI does (but the process is not explained) in the several instances in which he obtains the equation of the reciprocal surface by means of a binary function. I remark that it would be very instructive, for each case of surface, to take the variable plane successively through the several kinds of lines on the particular surface; the equation of the reciprocal surface would thus be obtained under different forms, putting in evidence the relation to the reciprocal surface of the fixed line made use of. But this is an investigation which I do not enter upon: I adopt in each case SCHLÄFLI'S process, without explanation, and merely write down the ternary or (as the case may be) binary function by means of which the equation of the reciprocal surface is obtained.

31. It is to be mentioned that there is a reciprocal process of obtaining the equation of the reciprocal surface; we may imagine, touching the cubic surface along any curve, a series of planes; that is, a torse circumscribed about the surface, and the equation whereof contains a variable parameter θ; the reciprocal figure is a curve, the equations whereof contain the parameter θ; the locus of this curve is the reciprocal surface; that is, the equation of the reciprocal surface is obtained by eliminating θ from the equations of the curve. In particular let the torse be the circumscribed cone having its vertex at any point of a fixed line; the reciprocal figure is then a plane curve, the plane of which passes through the line which is the reciprocal of the fixed line; it is moreover clear that if the position of the vertex on the fixed line be determined by the parameter θ *linearly* (for instance if the vertex be given as the intersection of the fixed line by a plane $P-\theta Q=0$), then the equation of the plane of the curve will be of the form $P'=\theta Q'$, containing the parameter θ linearly; the other equation of the plane curve will contain θ rationally, and the elimination will be at once effected by substituting in this other equation for θ its value, $=P'\div Q'$. And observe moreover that if the fixed line be a line on the cubic surface, then the cone is a quadricone having for its reciprocal a conic; the reciprocal surface is thus given as the locus of a variable conic, the plane of

which always passes through a fixed line; there are thus on the reciprocal surface series of such conics. It would be very instructive and interesting to carry out the investigation in detail.

32. The equation of the reciprocal surface is found by equating to zero the discriminant of a ternary or a binary function*, viz. this is a ternary cubic, or a binary quartic, cubic, or quadric. The equation as given in the form disct. $=0$, contains a factor which for the adopted forms of equations is always a power or product of powers of w, z, x† known *à priori*, and which is thrown out without difficulty, the equation being thereby reduced to the proper order. There is the singular advantage that the process puts in evidence the cuspidal curve of the resulting reciprocal surface, viz. for a ternary cubic, the form obtained is $S^3-T^2=0$, and for a binary quartic it is the equivalent form $I^3-27J^2=0$; but for the factor thrown out as just mentioned, we should have simply $(S=0, T=0)$, or, as the case may be, $(I=0, J=0)$ for equations of the cuspidal curve; the existence of the factor occasions however a modification, viz. the intersection of the two surfaces is not an indecomposable curve, and the cuspidal curve is in most cases, not the complete intersection, but a partial intersection of the two surfaces. In several cases it thus happens that the cuspidal curve is obtained as a curve $\left\| \begin{matrix} P, & Q, & R \\ P', & Q', & R' \end{matrix} \right\| = 0$, without or with further speciality. Similarly when the equation of the reciprocal surface is obtained by means of a binary cubic; if the coefficients hereof (functions of course of the coordinates x, y, z, w) be A, B, C, D, then the surface is

$$(AD-BC)^2-4(AC-B^2)(BD-C^2)=0,$$

having the cuspidal curve $\left\| \begin{matrix} A, & B, & C \\ B, & C, & D \end{matrix} \right\| = 0$, subject however to modification in the case of a thrown out factor.

Explanation as to the Sections of the Memoir. Article Nos. 33 & 34.

33. As regards the following Sections I to XXIII, it is to be observed that for the general surface $I=12$, I do not attempt to form the equation of the reciprocal surface, and in some of the other cases, $II=12-C_2$, &c., the equation of the reciprocal surface is either not obtained in a completely developed form, or it is too complicated to allow of its being dealt with, for instance so as to put in evidence the nodal curve of the surface. Portions of the theory given in the latter sections are consequently omitted in the earlier ones, and in particular in the Section I there is given only the diagram of the 27 lines and the 45 planes (with however developments as to notation and otherwise which have no place in the subsequent sections), and with the analytical expressions for the several lines

* In some easy cases, for instance $XVI=12-4C_2$, the equation of the reciprocal surface is obtained otherwise by a direct elimination.

† The factor is in general a power or product of powers of the linear functions which, equated to zero, give the equations of the planes reciprocal to the several nodes of the surface.

and planes, although from the want of the equation of the reciprocal surface these analytical expressions have no present application. And so in some of the next following sections, no application is made of the analytical expressions of the lines and planes.

34. I call to mind that if a line be given as the intersection of the two planes

$$AX+BY+CZ+DW=0,\ A'X+B'Y+C'Z+D'W=0,$$

then the six coordinates of the line are

$$\begin{matrix} a, & b, & c, & f, & g, & h \\ =AD'-A'D, & BD'-B'D, & CD'-C'D, & BC'-B'C, & CA'-C'A, & AB'-A'B, \end{matrix}$$

and that in terms of its six coordinates the line is given as the common intersection of the four planes

$$\left|\begin{matrix} . & h, & -g, & a \\ -h, & . & f, & b \\ g, & -f, & . & c \\ -a, & -b, & -c, & . \end{matrix}\right| (X, Y, Z, W)=0,$$

and that (reciprocating as usual in regard to $X^2+Y^2+Z^2+W^2=0$) the coordinates of the reciprocal line are (f, g, h, a, b, c); that is, this is the common intersection of the four planes

$$\left|\begin{matrix} . & c, & -b, & f \\ -c & . & a, & g \\ b, & -a, & . & h \\ -f, & -g, & -h, & . \end{matrix}\right| (x, y, z, w)=0.$$

It is in some cases more convenient to consider a line as determined as the intersection of two planes rather than by means of its six coordinates; thus, for instance, to speak of the line $X=0$, $Y=0$ rather than of the line $(0, 0, 0, 1, 0, 0)$; and in some of the sections I have preferred not to give the expressions of the six coordinates of the several lines.

§ I=12, Equation $(X, Y, Z, W)^3=0$. Article Nos. 35 to 46.

35. There is in the system of the 27 lines and the 45 planes a complicated and many-sided symmetry which precludes the existence of any unique notation: the notation can only be obtained by starting from some arrangement which is not unique, but one of a system of several like arrangements. The notation employed in my original paper "On the Simple Tangent Planes of Surfaces of the Third Order," Camb. and Dub. Math. Journ. vol. iv. 1849, pp. 118–132, and which is shown in the right hand and lower margins of the diagram, starts from such an arrangement; but it is so complicated that it can hardly be considered as at all putting in evidence the relations of the lines and planes; that of Dr. HART (SALMON "On the Triple Tangent Planes of a Surface of the Third Order," same volume, pp. 252–260), depending on an arrangement of the 27 lines according to a cube of 3 each way, is a singularly elegant one, and will be presently reproduced.

36. But the most convenient one is SCHLÄFLI'S, starting from a double-sixer; viz. we can (and that in 36 different ways) select out of the 27 lines two systems each of six lines, such that no two lines of the same system intersect, but that each line of the one system intersects all but the corresponding line of the other system; or, say, if the lines are

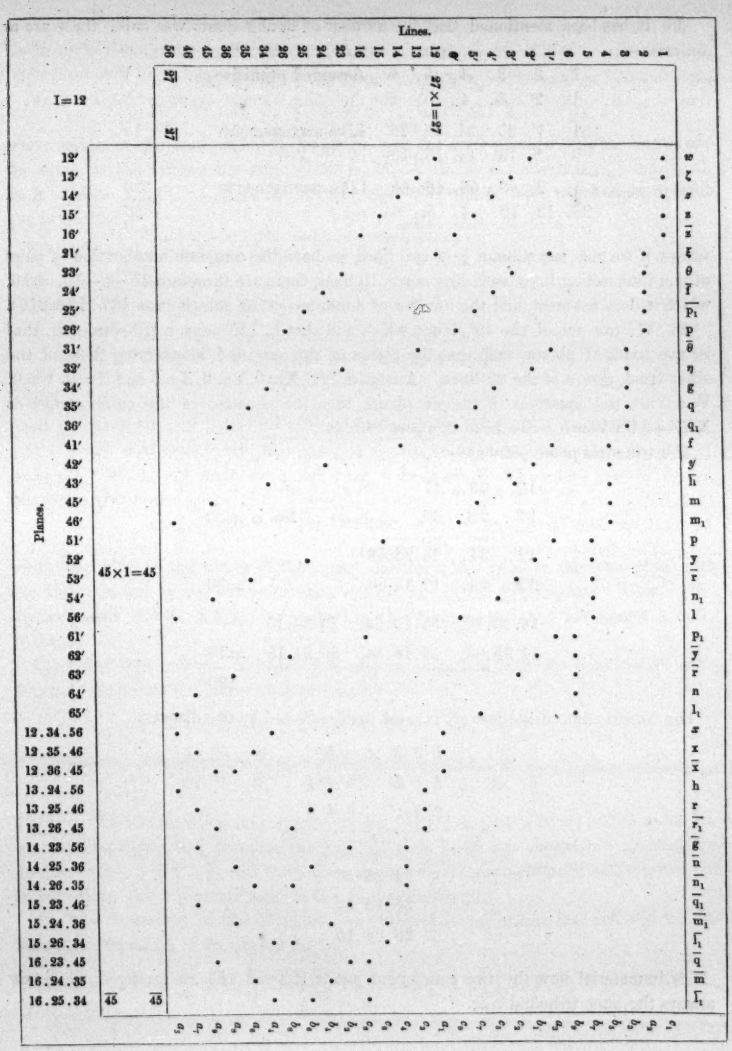
$$1, 2, 3, 4, 5, 6$$
$$1', 2', 3', 4', 5', 6',$$

then these have the thirty intersections

	$1'$	$2'$	$3'$	$4'$	$5'$	$6'$
1		•	•	•	•	•
2	•		•	•	•	•
3	•	•		•	•	•
4	•	•	•		•	•
5	•	•	•	•		•
6	•	•	•	•	•	

Any two lines such as $1, 2'$ lie in a plane which may be called $12'$; similarly the lines $1', 2$ lie in a plane which may be called $1'2$; these two planes meet in a line 12; and any three lines such as $12, 34, 56$ meet in pairs, lying in a plane $12 . 34 . 56$. We have thus the entire system of the 27 lines and 45 planes, as in effect completely explained by what has been stated, but which is exhibited in full in the diagram.

37. The diagram of the lines and planes is


Lines.
Planes.
I=12
27
27
27×1=27
45×1=45
45
45
12′
13′
14′
15′
16′
21′
23′
24′
25′
26′
31′
32′
34′
35′
36′
41′
42′
43′
45′
46′
51′
52′
53′
54′
56′
61′
62′
63′
64′
65′
12.34.56
12.35.46
12.36.45
13.24.56
13.25.46
13.26.45
14.23.56
14.25.36
14.26.35
15.23.46
15.24.36
15.26.34
16.23.45
16.24.35
16.25.34

38. It has been mentioned that the number of double-sixers was =36, these are as follows:—

1, 2, 3, 4, 5, 6 1′, 2′, 3′, 4′, 5′, 6′	Assumed primitive . .	1
1, 1′, 23, 24, 25, 26 2, 2′, 13, 14, 15, 16	Like arrangements . .	15
1, 2, 3, 56, 46, 45 23, 13, 12, 4, 5, 6	Like arrangements . .	20 36

where, if we take any column $\frac{1}{1}$ of two lines, we have the complete number 216 of pairs of non-intersecting lines (each line meets 10 lines, there are therefore 27−1−10, =16, which it does not meet, and the number of non-intersecting pairs is thus $\frac{1}{2}$27 . 16=216).

39. We can out of the 45 planes select, and that in 120 ways, a trihedral-pair, that is, two triads of planes, such that the planes of the one triad, intersecting those of the other triad, give 9 of the 27 lines. Analytically if X=0, Y=0, Z=0 and U=0, V=0, W=0 are the equations of the six planes, then the equation of the cubic surface is XYZ+kUVW=0. See as to this *post*, No. 44.

The trihedral plane pairs are—

12′, 23′, 31′ 1′2, 2′3, 3′1	No. is =20
12′, 34′, 14.23.56 2′3, 4′1, 12.34.56	=90
14.25.36, 35.16.24, 26.34.15 14.35.26, 25.16.34, 36.24.15	=10 120

The construction of the last set is most easily effected by the diagram

1 2 3 × 4 5 6
3 1 2 5 6 4
2 3 1 6 4 5

||

14 25 36
35 16 24
26 34 15

It is immaterial how the two component triads 123 and 456 are arranged, we obtain always the same trihedral pair.

40. Dr. HART arranges the 27 lines, cubically, thus:

A_1 B_1 C_1	a_1 b_1 c_1	α_1 β_1 γ_1
A_2 B_2 C_2	a_2 b_2 c_2	α_2 β_2 γ_2
A_3 B_3 C_3	a_3 b_3 c_3	α_3 β_3 γ_3

where letters of the same alphabet denote lines in the same plane, if only the letters are the same or the suffixes the same; thus A_1, A_2, A_3 lie in a plane $A_1A_2A_3$; A_1, B_1, C_1 lie in a plane $A_1B_1C_1$. Letters of different alphabets denote lines which meet according to the Table

a_1 b_2 c_3 A_1 α_1 β_2 γ_3	b_1 c_2 a_3 B_1 β_1 γ_2 α_3	c_1 a_2 b_3 C_1 γ_1 α_2 β_3
c_2 a_3 b_1 A_2 β_3 γ_1 α_2	a_2 b_3 c_1 B_2 γ_3 α_1 β_2	b_2 c_3 a_1 C_2 α_3 β_1 γ_2
b_3 c_1 a_2 A_3 γ_2 α_3 β_1	c_3 a_1 b_2 B_3 α_2 β_3 γ_1	a_3 b_1 c_2 C_3 β_2 γ_3 α_1

where the letter in the centre of the square denotes a line lying in the same plane with the lines denoted by the letters of each vertical pair in the same square. Thus A_1 lies in the planes $A_1a_1\alpha_1$, $A_1b_2\beta_2$, $A_3c_3\gamma_3$ (and in the before-mentioned two planes $A_1A_2A_3$, $A_1B_1C_1$).

41. I find that one way in which this may be identified with the double-sixer notation is to represent the above arrangement by

1, 2′, 12	3′, 4, 34	13, 24, 56
14, 25, 36	2, 6′, 26	1′, 16, 6
4′, 5, 45	23, 46, 15	3, 35, 5′

and then the identification may apparently be effected in $(720 \times 36 =)$ 25920 ways, viz. we may first in any way permute the $\frac{1}{1}$, $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, $\frac{5}{5}$, $\frac{6}{6}$, by this means not altering the double-sixer $\frac{1}{1}$, $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, $\frac{5}{5}$, $\frac{6}{6}$, and then upon the arrangements so obtained make any of the substitutions which permute *inter se* the 36 double-sixers.

42. The equations of the 45 planes are obtained in my paper last referred to, viz. taking the equation of the surface to be

$$W(1, 1, 1, 1, mn+\tfrac{1}{mn}, nl+\tfrac{1}{nl}, lm+\tfrac{1}{lm}, l+\tfrac{1}{l}, m+\tfrac{1}{m}, n+\tfrac{1}{n})(X, Y, Z, W)^3+kXZY=0,$$

where

$$k=\frac{p^2-\beta^2}{2(p-\alpha)},\ \alpha=lmn+\frac{1}{lmn},\ \beta=lmn-\frac{1}{lmn},$$

then the equations of the planes are:—

$$W=0, \qquad [12'=w]$$

$$lX+mY+nZ+W\left[1+\frac{1}{k}\left(l-\frac{1}{l}\right)\left(m-\frac{1}{m}\right)\left(n-\frac{1}{n}\right)\right]=0, \qquad [23'=\theta]$$

$$\frac{X}{l}+\frac{Y}{m}+\frac{Z}{n}+W\left[1-\frac{1}{k}\left(l-\frac{1}{l}\right)\left(m-\frac{1}{m}\right)\left(n-\frac{1}{n}\right)\right]=0, \qquad [31'=\bar{\theta}]$$

$$X=0, \qquad [12.34.56=x]$$

$$Y=0, \qquad [42' \quad =y]$$

$$Z=0, \qquad [14' \quad =z]$$

$$X+\frac{1}{k}\left(m-\frac{1}{m}\right)\left(n-\frac{1}{n}\right)W=0, \qquad [21'=\xi]$$

$$Y+\frac{1}{k}\left(n-\frac{1}{n}\right)\left(l-\frac{1}{l}\right)W=0, \qquad [32'=\eta]$$

$$Z+\frac{1}{k}\left(l-\frac{1}{l}\right)\left(m-\frac{1}{m}\right)W=0, \qquad [13'=\zeta]$$

$$lX+\frac{Y}{m}+\frac{Z}{n}+W=0, \qquad [41'=f]$$

$$\frac{X}{l}+mY+\frac{Z}{n}+W=0, \qquad [34'=g]$$

$$\frac{X}{l}+\frac{Y}{m}+Z+W=0, \qquad [13.24.56=h]$$

$$\frac{X}{l}+mY+nZ+W=0, \qquad [24'=\bar{f}]$$

$$lX+\frac{Y}{m}+nZ+W=0, \qquad [14.25.36=\bar{g}]$$

$$lX+mY+\frac{Z}{n}+W=0, \qquad [43'=\bar{h}]$$

$$X+\frac{l(p-\alpha)+2mn}{p+\beta}W=0, \qquad [12.35.46=x]$$

$$Y+\frac{m(p-\alpha)+2nl}{p+\beta}W=0, \qquad [52'=y]$$

$$Z+\frac{n(p-\alpha)+2lm}{p+\beta}W=0, \qquad [15'=z]$$

$$X+\frac{\frac{1}{l}(p-\alpha)+\frac{2}{mn}}{p+\beta}W=0, \qquad [12.36.45=\bar{x}]$$

$$Y+\frac{\frac{1}{m}(p-\alpha)+\frac{2}{nl}}{p+\beta}W=0, \qquad [62'=\bar{y}]$$

$$Z+\frac{\frac{1}{n}(p-\alpha)+\frac{2}{lm}}{p+\beta}W=0, \qquad [16'=\bar{z}]$$

$$-\frac{2n}{m(p-\alpha)}X+\frac{1}{m}Y+nZ+W=0, \qquad [56'=l]$$

$$lX-\frac{2l}{n(p-\alpha)}Y+\frac{1}{n}Z+W=0, \qquad [45'=m]$$

$$\frac{1}{l}X+mY-\frac{2m}{l(p-\alpha)}Z+W=0, \qquad [64'=n]$$

$$-\frac{2m}{n(p-\alpha)}X+mY+\frac{1}{n}Z+W=0, \qquad [15.26.34=\bar{l}]$$

$$\frac{1}{l}X-\frac{2n}{l(p-\alpha)}Y+nZ+W=0, \qquad [16.24.35=\bar{m}]$$

$$lX+\frac{1}{m}Y-\frac{2l}{m(p-\alpha)}Z+W=0, \qquad [14.25.36=\bar{n}]$$

$$-\frac{n(p-\alpha)}{2m}X+\frac{Y}{m}+nZ+W=0, \qquad [65'=l_1]$$

$$lX-\frac{l(p-\alpha)}{2n}Y+\frac{1}{n}Z+W=0, \qquad [46'=m_1]$$

$$\frac{1}{l}X+mY-\frac{m(p-\alpha)}{2l}Z+W=0, \qquad [54'=n_1]$$

$$\frac{m(p-\alpha)}{2n}X+mY+\frac{1}{n}Z+W=0, \qquad [16.25.34=\bar{l}_1]$$

$$\frac{1}{l}X-\frac{n(p-\alpha)}{2l}Y+nZ+W=0, \qquad [15.24.36=\bar{m}_1]$$

$$lX+\frac{1}{m}Y-\frac{l(p-\alpha)}{2m}Z+W=0, \qquad [14.26.35=\bar{n}_1]$$

$$-\frac{2X}{p-\alpha}+nY+mZ+(mn(p-\alpha)-2l(1-m^2-n^2))\frac{W}{p+\beta}=0, \qquad [51'=p]$$

$$nX-\frac{2Y}{p-\alpha}+lZ+(nl(p-\alpha)-2m(1-n^2-l^2))\frac{W}{p+\beta}=0, \qquad [35'=q]$$

$$mX+lY-\frac{2Z}{p-\alpha}+(lm(p-\alpha)-2n(1-l^2-m^2))\frac{W}{p+\beta}=0, \qquad [13.25.46=r]$$

$$-\frac{2X}{p-\alpha}+\frac{1}{n}Y+\frac{1}{m}Z+\left(\frac{1}{mn}(p-\alpha)-\frac{2}{l}\left(1-\frac{1}{m^2}-\frac{1}{n^2}\right)\right)\frac{W}{p-\beta}=0, \qquad [26'=\bar{p}]$$

$$\frac{1}{n}X+\frac{2Y}{p-\alpha}+\frac{1}{l}Z+\left(\frac{1}{nl}(p-\alpha)-\frac{2}{m}\left(1-\frac{1}{n^2}-\frac{1}{l^2}\right)\right)\frac{W}{p-\beta}=0, \qquad [16.23.45=\bar{q}]$$

$$\frac{1}{m}X+\frac{1}{l}Y-\frac{2Z}{p-\alpha}+\left(\frac{1}{lm}(p-\alpha)-\frac{2}{n}\left(1-\frac{1}{l^2}-\frac{1}{m^2}\right)\right)\frac{W}{p-\beta}=0, \qquad [36'=\bar{r}]$$

$$-\frac{p-\alpha}{2}X+\frac{Y}{n}+\frac{Z}{m}-lmn\left(\frac{1}{l}\left(1-\frac{1}{m^2}-\frac{1}{n^2}\right)(p-\alpha)-\frac{2}{mn}\right)\frac{W}{p+\beta}=0, \qquad [25'=p_1]$$

$$\frac{X}{n}-\frac{p-\alpha}{2}Y+\frac{Z}{l}-lmn\left(\frac{1}{m}\left(1-\frac{1}{n^2}-\frac{1}{l^2}\right)(p-\alpha)-\frac{2}{nl}\right)\frac{W}{p+\beta}=0, \qquad [15.23.46=q_1]$$

$$\frac{X}{m}+\frac{Y}{l}-\frac{p-\alpha}{2}Z-lmn\left(\frac{1}{n}\left(1-\frac{1}{l^2}-\frac{1}{m^2}\right)(p-\alpha)-\frac{2}{lm}\right)\frac{W}{p+\beta}=0, \qquad [53'=r_1]$$

$$-\frac{p-\alpha}{2}X+nY+mZ-\frac{1}{lmn}(l(1-m^2-n^2)(p-\alpha)-2mn)\frac{W}{p-\beta}=0, \qquad [61'=\bar{p}_1]$$

$$nX-\frac{p-\alpha}{2}Y+lZ-\frac{1}{lmn}(m(1-n^2-l^2)(p-\alpha)-2nl)\frac{W}{p-\beta}=0, \qquad [36'=\bar{q}_1]$$

$$mX-lY-\frac{p-\alpha}{2}Z-\frac{1}{lmn}(n(1-l^2-m^2)(p-\alpha)-2lm)\frac{W}{p-\beta}=0, \qquad [13.26.45=\bar{r}_1]$$

43. The coordinates of the 27 lines are then found to be as follows:—

(a)	(b)	(c)
1	0	0
0	1	0
0	0	1
$1-\frac{1}{kl}\left(m-\frac{1}{m}\right)\left(n-\frac{1}{n}\right)$	$-\frac{m}{k}\left(m-\frac{1}{m}\right)\left(n-\frac{1}{n}\right)$	$-\frac{n}{k}\left(m-\frac{1}{m}\right)\left(n-\frac{1}{n}\right)$
$-\frac{l}{k}\left(n-\frac{1}{n}\right)\left(l-\frac{1}{l}\right)$	$1-\frac{1}{km}\left(n-\frac{1}{n}\right)\left(l-\frac{1}{l}\right)$	$-\frac{n}{k}\left(n-\frac{1}{n}\right)\left(l-\frac{1}{l}\right)$
$-\frac{l}{k}\left(l-\frac{1}{l}\right)\left(m-\frac{1}{m}\right)$	$-\frac{m}{k}\left(l-\frac{1}{l}\right)\left(m-\frac{1}{m}\right)$	$1-\frac{1}{km}\left(l-\frac{1}{l}\right)\left(m-\frac{1}{m}\right)$
$1-\frac{l}{k}\left(m-\frac{1}{m}\right)\left(n-\frac{1}{n}\right)$	$-\frac{1}{mk}\left(m-\frac{1}{m}\right)\left(n-\frac{1}{n}\right)$	$-\frac{1}{nk}\left(m-\frac{1}{m}\right)\left(n-\frac{1}{n}\right)$
$-\frac{1}{lk}\left(n-\frac{1}{n}\right)\left(l-\frac{1}{l}\right)$	$1-\frac{m}{k}\left(n-\frac{1}{n}\right)\left(l-\frac{1}{l}\right)$	$-\frac{1}{nk}\left(n-\frac{1}{n}\right)\left(l-\frac{1}{l}\right)$
$-\frac{1}{lk}\left(l-\frac{1}{l}\right)\left(m-\frac{1}{m}\right)$	$-\frac{1}{mk}\left(l-\frac{1}{l}\right)\left(m-\frac{1}{m}\right)$	$1-\frac{n}{k}\left(l-\frac{1}{l}\right)\left(m-\frac{1}{m}\right)$
1	0	0
0	1	0
0	0	1

(f)	(g)	(h)	
0	0	0	$(12=a_1)$
0	0	0	$(2'=b_1)$
0	0	0	$(1=c_1)$
0	$-n$	m	$(2=a_2)$
n	0	$-l$	$(23=b_2)$
$-m$	l	0	$(3'=c_2)$
0	$-\frac{1}{n}$	$\frac{1}{m}$	$(1'=a_3)$
$\frac{1}{n}$	0	$-\frac{1}{l}$	$(3=b_3)$
$-\frac{1}{m}$	$\frac{1}{l}$	0	$(13=c_3)$
0	$-\frac{1}{n}$	m	$(34=a_4)$
n	0	$-\frac{1}{l}$	$(24=b_4)$
$-\frac{1}{m}$	l	0	$(14=c_4)$

(a)	(b)	(c)
1	0	0
0	1	0
0	0	1
$2m\left(l-\frac{1}{l}\right)$ $-m(p-\alpha)\left(1+\frac{2nl}{m(p-\alpha)}\right)$ $2\left(1+\frac{2lm}{n(p-\alpha)}\right)$	$2\left(1+\frac{2mn}{l(p-\alpha)}\right)$ $2n\left(m-\frac{1}{m}\right)$ $-n(p-\alpha)\left(1+\frac{2lm}{n(p-\alpha)}\right)$	$-l(p-\alpha)\left(1+\frac{2mn}{l(p-\alpha)}\right)$ $2\left(1+\frac{2nl}{m(p-\alpha)}\right)$ $2l\left(n-\frac{1}{n}\right)$
$2n\left(l-\frac{1}{l}\right)$ $2\left(1+\frac{2nl}{m(p-\alpha)}\right)$ $-n(p-\alpha)\left(1+\frac{2lm}{n(p-\alpha)}\right)$	$-l(p-\alpha)\left(1+\frac{2mn}{l(p-\alpha)}\right)$ $2l\left(m-\frac{1}{m}\right)$ $2\left(1+\frac{2lm}{n(p-\alpha)}\right)$	$2\left(1+\frac{2mn}{l(p-\alpha)}\right)$ $-m(p-\alpha)\left(1+\frac{2nl}{m(p-\alpha)}\right)$ $2m\left(n-\frac{1}{n}\right)$
$\frac{2}{n}\left(l-\frac{1}{l}\right)$ $-2\left(1+\frac{2m}{nl(p-\alpha)}\right)$ $\frac{p-\alpha}{n}\left(1+\frac{2n}{lm(p-\alpha)}\right)$	$\frac{p-\alpha}{l}\left(1+\frac{2l}{mn(p-\alpha)}\right)$ $\frac{2}{l}\left(m-\frac{1}{m}\right)$ $-2\left(1+\frac{2n}{lm(p-\alpha)}\right)$	$-2\left(1+\frac{2l}{mn(p-\alpha)}\right)$ $\frac{p-\alpha}{m}\left(1+\frac{2m}{nl(p-\alpha)}\right)$ $\frac{2}{m}\left(n-\frac{1}{n}\right)$
$\frac{2}{m}\left(l-\frac{1}{l}\right)$ $\frac{p-\alpha}{m}\left(1+\frac{2m}{nl(p-\alpha)}\right)$ $-2\left(1+\frac{2n}{lm(p-\alpha)}\right)$	$-2\left(1+\frac{2l}{mn(p-\alpha)}\right)$ $\frac{2}{n}\left(m-\frac{1}{m}\right)$ $\frac{p-\alpha}{n}\left(1+\frac{2n}{lm(p-\alpha)}\right)$	$\frac{p-\alpha}{l}\left(1+\frac{2l}{mn(p-\alpha)}\right)$ $-2\left(1+\frac{2m}{nl(p-\alpha)}\right)$ $\frac{2}{l}\left(n-\frac{1}{n}\right)$

(f)	(g)	(h)	
0	$-n$	$\frac{1}{m}$	$(56 = a_8)$
$\frac{1}{n}$	0	$-l$	$(\ 4 = b_8)$
$-m$	$\frac{1}{l}$	0	$(\ 4' = c_8)$
0	$-(p+\beta)$	$-\frac{2(p+\beta)}{l(p-\alpha)}$	$(35 = a_6)$
$-\frac{2(p+\beta)}{m(p-\alpha)}$	0	$-(p+\beta)$	$(25 = b_6)$
$-(p+\beta)$	$-\frac{2(p+\beta)}{n(p-\alpha)}$	0	$(15 = c_6)$
0	$\frac{2(p+\beta)}{l(p-\alpha)}$	$p+\beta$	$(46 = a_7)$
$p+\beta$	0	$\frac{2(p+\beta)}{m(p-\alpha)}$	$(\ 5 = b_7)$
$\frac{2(p+\beta)}{n(p-\alpha)}$	$p+\beta$	0	$(\ 5' = c_7)$
0	$-\frac{2l(p-\beta)}{p-\alpha}$	$-(p-\beta)$	$(36 = a_9)$
$-(p-\beta)$	0	$-\frac{2m(p-\beta)}{p-\alpha}$	$(26 = b_9)$
$-\frac{2n(p-\beta)}{p-\alpha}$	$-(p-\beta)$	0	$(16 = c_9)$
0	$p-\beta$	$\frac{2l(p-)\beta}{p-\alpha}$	$(45 = a_9)$
$\frac{2m(p-\beta)}{p-\alpha}$	0	$p-\beta$	$(\ 6 = b_9)$
$p-\beta$	$\frac{2n(p-\beta)}{p-\alpha}$	0	$(\ 6' = c_9)$

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44. We have $X=0$, $Y=0$, $Z=0$, $W=0$ for the equations of the planes

$$(12.34.56=x),\ (42'=y),\ (14'=z),\ (12'=w);$$

and representing by $f=lX+\frac{1}{m}Y+\frac{1}{n}Z+W=0$ the equation of any other plane $(41'=f)$, the equation of the cubic surface may be presented in the several forms:

$$\begin{aligned}
0=U&=Wf\bar{f} &&+k\xi YZ\\
&=Wg\bar{g} &&+k\eta ZX\\
&=Wh\bar{h} &&+k\zeta XY\\
&=W\theta\bar{\theta} &&+k\xi\eta\zeta\\
&=Wl\bar{l}_1 &&+ky\bar{z}x\\
&=Wm\bar{m}_1 &&+kz\bar{x}y\\
&=Wn\bar{n}_1 &&+kx\bar{y}z\\
&=Wl_1\bar{l} &&+k\bar{y}zx\\
&=Wm_1\bar{m} &&+k\bar{z}xy\\
&=Wn_1\bar{n} &&+k\bar{x}yz\\
&=Wpp_1 &&+k\xi yz\\
&=Wqq_1 &&+k\eta zx\\
&=Wrr_1 &&+k\zeta xy\\
&=W\bar{p}\bar{p}_1 &&+k\xi\bar{y}\bar{z}\\
&=W\bar{q}\bar{q}_1 &&+k\eta\bar{z}\bar{x}\\
&=W\bar{r}\bar{r}_1 &&+k\zeta\bar{x}\bar{y},
\end{aligned}$$

which are the 16 forms containing W, out of the complete system of 120 trihedral-pair forms.

45. The 27 lines are each of them facultative; we have therefore $b'=\varrho'=27$; $t'=45$; moreover each of the lines is a double tangent of the spinode curve, and therefore $\beta'(=2\varrho')=54$.

46. The equation of the reciprocal surface is not here investigated; its form is

$$S^3-T^2=0,$$

where $S=(*)(x, y, z, w)^4$, $T=(*)(x, y, z, w)^6$; wherefore $n'=12$.

The nodal curve is composed of the lines which are the reciprocals of the original 27 lines ($b'=27$, $t'=45$ *ut suprà*). It may be remarked that the reciprocal of a double-sixer is a double-sixer. Hence the 27 lines of the reciprocal surface may be (and that in 36 different ways) represented by

1, 2, 3, 4, 5, 6

1′, 2′, 3′, 4′, 5′, 6′

12, 13, ... 56,

where 12 is now the line joining the points 12′ and 1′2; and so for the other lines. The lines 12, 34, 56 meet in a point 12 . 34 . 56; the 30 points 12′, 1′2... 56′, 5′6, and the fifteen points 12.34.56 make up the 45 points t'.

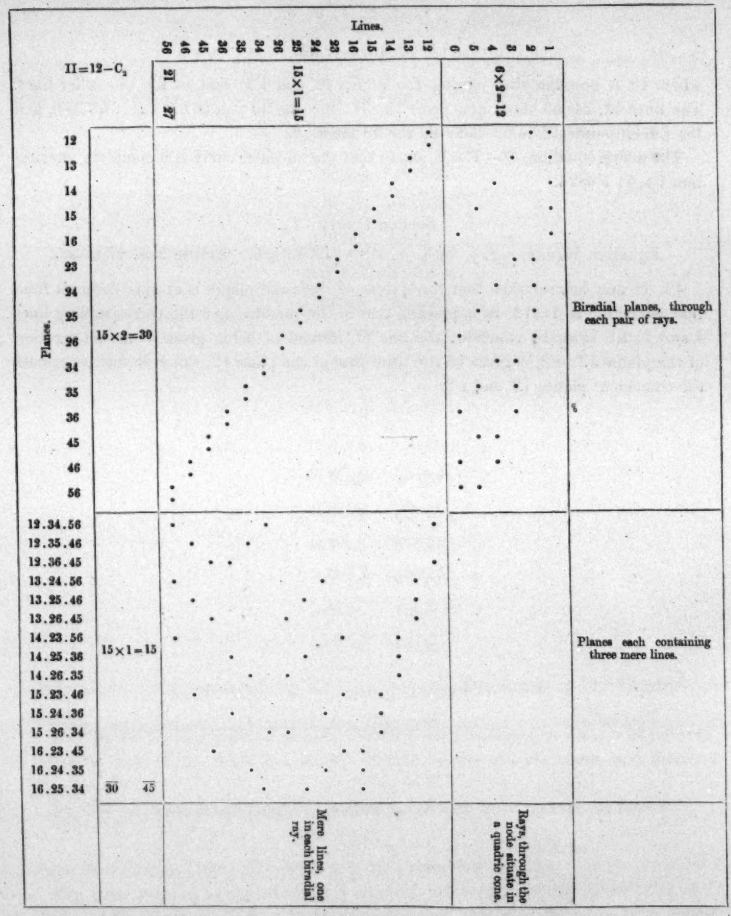
The above equation, $S^3-T^2=0$, shows that the cuspidal curve is a complete intersection 6×4; $c'=24$.

Section II$=12-C_2$.

Equation $W(a, b, c, f, g, h)(X, Y, Z)^2+2kXYZ=0$. Article Nos. 47 to 59.

47. It may be remarked that the system of lines and planes is at once deduced from that belonging to I$=12$, by supposing that in the double-sixer the corresponding lines 1 and 1′, &c. severally coincide; the line 12, instead of being given as the intersection of the planes 12′, 1′2, is given as the third line in the plane 12, which in fact represents the coincident planes 12′ and 1′2.

48. The diagram is



49. Putting the equation of the surface in the form

$$W\left(1, 1, 1, l+\frac{1}{l}, m+\frac{1}{m}, n+\frac{1}{n}\right)(X, Y, Y)^2+\frac{\alpha\beta\gamma\delta}{p}XYZ=0,$$

where for shortness

$$\begin{aligned} \alpha &= mn - l, \\ \beta &= nl - m, \\ \gamma &= lm - n, \\ \delta &= lmn - 1, \\ p &= lmn, \end{aligned}$$

then taking $X=0$ as the equation of the plane [12], $Y=0$ as that of the plane [34], $Z=0$ as that of the plane [56], the equations of the 30 distinct planes are found to be

$$\begin{aligned}
X &= 0, && [12] \\
Y &= 0, && [34] \\
Z &= 0, && [56] \\
m\,X + l\,Y + Z &= 0, && [23] \\
m^{-1}X + l\,Y + Z &= 0, && [24] \\
m\,X + l^{-1}Y + Z &= 0, && [13] \\
m^{-1}X + l^{-1}Y + Z &= 0, && [14] \\
X + n\,Y + m\,Z &= 0, && [45] \\
X + n^{-1}Y + m\,Z &= 0, && [46] \\
X + n\,Y + m^{-1}Z &= 0, && [35] \\
X + n^{-1}Y + m^{-1}Z &= 0, && [36] \\
n\,X + Y + l\,Z &= 0, && [16] \\
n^{-1}X + Y + l\,Z &= 0, && [15] \\
n\,X + Y + l^{-1}Z &= 0, && [26] \\
n^{-1}X + Y + l^{-1}Z &= 0, && [25] \\
W &= 0, && [12.34.56] \\
X + \beta\gamma W &= 0, && [12.36.45] \\
X - \alpha\delta W &= 0, && [12.35.46] \\
Y + \alpha\gamma W &= 0, && [16.25.34] \\
Y - \beta\delta W &= 0, && [15.26.34] \\
Z + \alpha\beta W &= 0, && [14.23.56] \\
Z - \gamma\delta W &= 0, && [13.24.56] \\
mn\,X + nl\,Y + lm\,Z + \alpha\beta\gamma\delta W &= 0, && [16.23.45] \\
p\,X + n\,Y + m\,Z + \beta\gamma\delta W &= 0, && [13.26.45] \\
n\,X + p\,Y + l\,Z + \gamma\alpha\delta W &= 0, && [16.24.35] \\
m\,X + l\,Y + p\,Z + \alpha\beta\delta W &= 0, && [15.23.46] \\
X + lm\,Y + ln\,Z - \beta\gamma\delta W &= 0, && [15.24.36] \\
lm\,X + Y + mn\,Z - \gamma\alpha\delta W &= 0, && [13.25.46] \\
nl\,X + mn\,Y + Z - \alpha\beta\delta W &= 0, && [14.26.35] \\
l\,X + m\,Y + n\,Z - \alpha\beta\gamma W &= 0, && [14.25.36]
\end{aligned}$$

50. And the coordinates of the 21 distinct lines are

(a)	(b)		(f)	(g)	(h)	whence equations may be taken to be
l	0	0	0	-1	1	(1) $X=0$, $Y+lZ=0$
0	m	0	1	0	-1	(3) $Y=0$, $Z+mX=0$
0	0	n	-1	1	0	(5) $Z=0$, $X+nY=0$
l^{-1}	0	0	0	-1	1	(2) $X=0$, $Y+l^{-1}Z=0$
0	m^{-1}	0	1	0	-1	(4) $Y=0$, $Z+m^{-1}X=0$
0	0	n^{-1}	-1	1	0	(6) $Z=0$, $X+n^{-1}Y=0$
1	n	m	0	$\frac{m}{\beta\gamma}$	$-\frac{n}{\beta\gamma}$	(45) $X+nY+mZ=0$, $X+\beta\gamma W=0$
n	1	l	$-\frac{l}{\gamma\alpha}$	0	$\frac{n}{\gamma\alpha}$	(16) $Y+lZ+nX=0$, $Y+\gamma\alpha W=0$
m	l	1	$\frac{l}{\alpha\beta}$	$-\frac{m}{\alpha\beta}$	0	(23) $Z+mX+lY=0$, $Z+\alpha\beta W=0$
1	$\frac{1}{n}$	m	0	$-\frac{m}{\alpha\delta}$	$\frac{1}{n\alpha\delta}$	(46) $X+n^{-1}Y+mZ=0$, $X-\alpha\delta W=0$
n	1	$\frac{1}{l}$	$\frac{1}{l\beta\delta}$	0	$-\frac{n}{\beta\delta}$	(26) $Y+l^{-1}Z+nX=0$, $Y-\beta\delta W=0$
$\frac{1}{m}$	l	1	$-\frac{l}{\gamma\delta}$	$\frac{1}{m\gamma\delta}$	0	(24) $Z+m^{-1}X+lY=0$, $Z-\gamma\delta W=0$
1	n	$\frac{1}{m}$	0	$-\frac{1}{m\alpha\delta}$	$\frac{n}{\alpha\delta}$	(35) $X+nY+m^{-1}Z=0$, $X-\alpha\delta W=0$
$\frac{1}{n}$	1	l	$\frac{l}{\beta\delta}$	0	$-\frac{1}{n\beta\delta}$	(15) $Y+lZ+n^{-1}X=0$, $Y-\beta\delta W=0$
m	$\frac{1}{l}$	1	$-\frac{1}{l\gamma\delta}$	$\frac{m}{\gamma\delta}$	0	(13) $Z+mX+l^{-1}Y=0$, $Z-\gamma\delta W=0$
1	$\frac{1}{n}$	$\frac{1}{m}$	0	$\frac{1}{m\beta\gamma}$	$-\frac{1}{n\beta\gamma}$	(36) $X+n^{-1}Y+m^{-1}Z=0$, $X+\beta\gamma W=0$
$\frac{1}{n}$	1	$\frac{1}{l}$	$-\frac{1}{l\gamma\alpha}$	0	$\frac{1}{n\gamma\alpha}$	(25) $Y+l^{-1}Z+n^{-1}X=0$, $Y+\gamma\alpha W=0$
$\frac{1}{m}$	$\frac{1}{l}$	1	$\frac{1}{l\alpha\beta}$	$-\frac{1}{m\gamma\alpha}$	0	(14) $Z+m^{-1}X+l^{-1}Y=0$, $Z+\alpha\beta W=0$
1	0	0	0	0	0	(12) $X=0$, $W=0$
0	1	0	0	0	0	(34) $Y=0$, $W=0$
0	0	1	0	0	0	(56) $Z=0$, $W=0$

51. The six nodal rays are not, the fifteen mere lines are facultative. Hence

$$b'=p'=15\,;\ t'=15.$$

52. Resuming the equation $W(a, b, c, f, g, h)(X, Y, Z)^3+2kXYZ=0$, the equation of the Hessian surface is found to be

$$\begin{aligned}&KW^2(a, b, c, f, g, h)(X, Y, Z)^3\\ &+2kW\{(a, b, c, f, g, h)(X, Y, Z)^2(FX+GY+HZ)-3KXYZ\}\\ &-k^2\{a^2X^4+b^2Y^4+c^2Z^4-2bcY^2Z^2-2caZ^2X^2-2abX^2Y^2\\ &-4XYZ[(af+gh)X+(bg+hf)Y+(ch+fg)Z]\}=0,\end{aligned}$$

where

$$(A, B, C, F, G, H)=(bc-f^2,\ ca-g^2,\ ab-h^2,\ gh-af,\ hf-bg,\ fg-ch),$$
$$K=abc-af^2-bg^2-ch^2+2fgh.$$

The Hessian and the cubic intersect in an indecomposable curve, which is the spinode curve; that is, spinode curve is a complete intersection 3×4; $d'=12$.

The equations of the spinode curve may be written in the simplified form

$$\begin{aligned}&W(a, b, c, f, g, h)(X, Y, Z)^3+2kXYZ=0,\\ &-8KXYZW\\ &+8kXYZ(afX+bgY+chZ)\\ &-k^2\{a^2X^4+b^2Y^4+c^2Z^4-2bcY^2Z^2-2caZ^2X^2-2abX^2Y^2\}=0\,;\end{aligned}$$

and it appears hereby that the node C_2 is a sixfold point on the curve, the tangents of the curve in fact coinciding with the six rays.

Each of the 15 lines touches the spinode curve twice; in fact, for the line 12 we have $X=0$, $W=0$; and substituting in the equations of the spinode curve, we have $(bY^2-cZ^2)=0$; that is, we have the two points of contact $X=0$, $W=0$, $Y\sqrt{b}=\pm Z\sqrt{c}$. Hence $\beta'=30$.

Reciprocal Surface.

53. The equation is found by equating to zero the discriminant of the ternary cubic function

$$(Xx+Yy+Zz)(a, b, c, f, g, h)(X, Y, Z)^3-2kwXYZ,$$

viz. the discriminant contains the factor w^3 which is to be thrown out, thus reducing the order to $n'=10$.

The ternary cubic, multiplying by 3 to avoid fractions, is

$$X^3\ ,\ Y^3\ ,\ Z^3\ ,\ 3Y^2Z\ ,\ 3Z^2X\ ,\ 3X^2Y\ ,\ 3YZ^2\ ,\ 3ZX^2\ ,\ 3XY^2\ ,\ 6XYZ,$$
$$3ax,\ 3by,\ 3cz,\ bz+2fy,\ cx+2gz,\ ay+2hx,\ cy+2fz,\ az+2gx,\ bx+2hy,\ fx+gy+hz-kw.$$

Write as before (A, B, C, F, G, H) for the inverse coefficients $(A=bc-f^2$, &c.), and $K=abc-af^2-bg^2-ch^2+2fgh$; and moreover

$$\Phi=(A, B, C, F, G, H)(x, y, z)^3,$$
$$P=Ax+Hy+Gz,$$

$$Q = Hx + By + Fz,$$
$$R = Gx - Fy + Cz,$$

$$t = fx + gy + hz,$$
$$U = afyz + bgzx + chxy,$$
$$V = 2Kxyz - aPyz - bQzx - cRxy$$
$$= -aHy^2z - bFz^2x - cGx^2y$$
$$-aGyz^2 - bHzx^2 - cFxy^2$$
$$+(-abc - af^2 - bg^2 - ch^2 + 4fgh)xyz,$$
$$W = (A, B, C, F, G, H)(ayz, bzx, cxy)^2,$$
$$L = k^2w^2 - 2ktw - \Phi,$$
$$M = kwU + V,$$
$$N = 2kabc\, xyzw + W:$$

54. Then the invariants of the ternary cubic are

$$S = L^2 - 12kwM,$$
$$T = L^3 - 18kwLM - 54k^2w^2N;$$

and the required equation of the reciprocal surface is

$$\frac{1}{108w^2}\{(L^2 - 12kwM)^3 - (L^3 - 18kwLM - 54k^2w^2N)^2\} = 0,$$

viz. this is

$$\begin{aligned}
0 = {} & L^3N && = (k^2w^2 - 2ktw - \Phi)^3(2kabc\, xyzw + W) \\
& + L^2M^3 && + (k^2w^2 - 2ktw - \Phi)^3(kwU + V)^2 \\
& - 18kwLMN && - 18kw(k^2w^2 - 2ktw - \Phi)(kwU + V)(2kabc\, xyzw + W) \\
& - 16kwM^3 && - 16kw(kwU + V)^3 \\
& - 27k^2w^2N^2 && - 27k^2w^2(2kabc\, xyzw + W)^2,
\end{aligned}$$

which, arranged in powers of kw, is as follows; viz. we have

$$\text{Coeff. } (kw)^3 = \quad 2abc\, xyz,$$

$$(kw)^6 = \quad 2abc\, xyz(-6t) + W + U^2,$$

$$(kw)^5 = \quad 2abc\, xyz(-3\Phi + 12t^2) + W(-6t) + U^2(-4t) + 2UV - 36abc\, xyzU,$$

$$
\begin{aligned}
(kw)^4 = \;& 2abc\,xyz(12t\Phi - 8t^3) + W(-3\Phi + 12t^2) \\
& + U^2(-2\Phi + 4t^2) + 2UV(-4t) + V^2 \\
& - 36abc\,xyzV - 18UW + 72abc\,xyztU \\
& - 16U^3 \\
& - 108a^2b^2c^2x^2y^2z^2 , \\
(kw)^3 = \;& 2abc\,xyz(3\Phi^2 - 12t^2\Phi) + W(12t\Phi - 8t^3) \\
& + U^2 4t\Phi + 2UV(-2\Phi + 4t^2) + V^2(-4t) \\
& - 18VW + 72abc\,xyztV + 36tUW - 36abc\,xyz\Phi U \\
& - 48U^2V \\
& - 108abc\,xyzW, \\
\text{„ } (kw)^2 = \;& 2abc\,xyz(-6t\Phi^2) + W(3\Phi^2 - 12t^2\Phi) \\
& + U^2\Phi^2 + 2UV(4t\Phi) + V^2(-2\Phi + 4t^2) \\
& + 36tVW - 36abc\,xyz\Phi V - 18\Phi UW \\
& - 48UV^2 \\
& - 27W^2, \\
\text{„ } (kw)^1 = \;& 2abc\,xyz(-\Phi^3) + W(-6t\Phi)^2 \\
& + 2UV\Phi^2 + V^2(4t\Phi) \\
& - 18\Phi VW \\
& - 16V^3, \\
\text{„ } (kw)^0 = \;& W(-\Phi^3) \\
& + V^2\Phi^2 ;
\end{aligned}
$$

but I have not carried the ultimate reduction further than in SCHLÄFLI, viz. I give only the terms in $(kw)^7$, $(kw)^6$, $(kw)^5$, and $(kw)^0$.

55. I present the result as follows; the coefficients deducible from those which precede, by mere cyclical permutations of the letters a, b, c and f, g, h, are indicated by („).

$$0 = (kw)^7 . 2abc\,xyz$$

$+(kw)^6 .$

y^2z^2	z^2x^2	x^2y^2	x^2yz	xy^2z	xyz^2
a^2bc+1	„	„	$abcf-14$ $gcbh+4$	„	„

$+(kw)^5 .$

y^3z^2	y^2z^3	z^3x^2	z^2x^3	x^3y^2	x^2y^3	x^3yz	xy^3z	xyz^3	xy^2z^2	x^2yz^2	x^2y^2z
a^2bcg-6 a^2cfh+2	a^2bch-6 a^2bfg+2	„	„	„	„	ab^2c^2-6 $abcf^2+42$ b^2cg^2+2 bc^2h^2+2 $bcfgh-24$	„	„	$a^2bcf-32$ $abcfgh+64$ $abfg^2-24$ $acfh^2-24$ af^2gh+8	„	„

$\vdots$

$+(kw)^0 . -\mathrm{K}[(\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{F}, \mathrm{G}, \mathrm{H})(x, y, z)^7](cy^2 - 2fyz + bz^2)(az^2 - 2gzx + bx^2)(bx^2 - 2hxy + ay^2).$

56. In explanation of the discussion of the reciprocal surface, it is convenient to remark that we have

Node C_1, $X=0$, $Y=0$, $Z=0$.	Reciprocal plane is $w=0$.
Tangent cone is	Conic of contact is
$(a, b, c, f, g, h)(X, Y, Z)^2=0$.	$(A, B, C, F, G, H)(x, y, z)^2=0$, $w=0$.
Nodal rays are sections of cone by planes $X=0, Y=0, Z=0$ respectively, viz. equations of the rays are	Lines are tangents of this conic from points $(y=0, z=0)$, $(z=0, x=0)$, $(x=0, y=0)$ respectively, viz. equations are
$X=0$, $bY^2+2fYZ+cZ^2=0$,	$w=0$, $cy^2-2fyz+bz^2=0$,
$Y=0$, $cZ^2+2gZX+aX^2=0$,	$w=0$, $az^2-2gzx+cx^2=0$,
$Z=0$, $aX^2+2hXY+bY^2=0$.	$w=0$, $bx^2-2hxy+ay^2=0$.

57. The equation shows that the section by the plane $w=0$ is made up of the conic $(A, B, C, F, G, H)(x, y, z)^2=0$, twice, and of the six lines, tangent to this conic, viz. the lines

$$w=0,\ cy^2-2fyz+bz^2=0,$$
$$w=0,\ az^2-2gzx+cx^2=0,$$
$$w=0,\ bx^2-2hxy+ay^2=0,$$

each once; the lines in question (reciprocals of the nodal rays) are thus mere scrolar lines on the reciprocal surface.

58. I do not attempt to put in evidence the nodal curve of the surface; by what precedes it is made up of 15 lines, intersecting 3 together in 15 points; and if we denote the six tangents of the conic just referred to be

$$1, 2, 3, 4, 5, 6,$$

then the fifteen lines are respectively lines passing through the intersections of each pair of these tangents; viz. through the intersection of the tangents 1 and 2, we have a line 12; and so in other cases; that is, the 15 lines are 12, 13 . . . . 56. The lines 12 and 34 meet; and the lines 12, 34, 56 meet in a point; we have thus the 15 points 12 . 34 . 56, triple points of the nodal curve.

59. As regards the cuspidal curve, the equation of the surface may be written

$$(L^3-12kwM)(4M^3+3LN)-(LM+9kwN)^2$$
$$=3(L^2M^2+L^2N-18kwLMN-16kwM^3-27k^2w^2N^2)=0,$$

and we thus have

$$4M^3+3LN=0,$$
$$LM+9kwN=0,$$
$$L^3-12kwM=0,$$

or, what is the same thing,

$$\begin{Vmatrix} L, & 12M, & -9N \\ kw, & L, & M \end{Vmatrix}=0$$

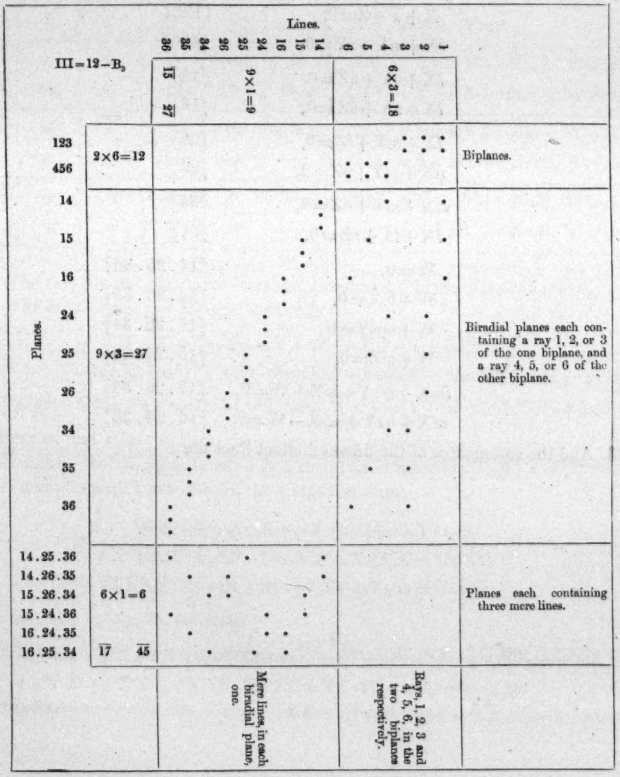
(equivalent to two equations) for the equations of the cuspidal curve. Attending to the second and third equations, the cuspidal curve may be considered as the residual intersection of the quartic and quintic surfaces $L^2-12klwM=0$, $LM+9klwN=0$, which partially intersect in the conic $w=0$, $L=0$; or say it is a curve $4\times5-2$; $d=18$.

Section III$=12-B_3$.

Equation $2W(X+Y+Z)(lX+mY+nZ)+2kXYZ=0$. Article Nos. 60 to 72.

60. The system of lines and planes is at once deduced from that belonging to II$=12-C_2$, by supposing the tangent cone to reduce itself to the pair of biplanes; 3 of the planes (a) of II$=12-C_2$ thus coming to coincide with the one biplane, and three of them with the other biplane.

61. The diagram is



62. Taking $X+Y+Z=0$ for the biplane that contains the rays 1, 2, 3, and $lX+mY+nZ=0$ for that which contains the rays 4, 5, 6, we may take $X=0$, $Y=0$, $Z=0$ for the equations of the planes [14], [25], [36] respectively; and then writing for shortness

$$m-n,\ n-l,\ l-m=\lambda,\ \mu,\ \nu,$$

and assuming, as we may do, $k=\lambda\mu\nu$, so that the equation of the surface is

$$W(X+Y+Z)(lX+mY+nZ)+(m-n)(n-l)(l-m)XYZ=0,$$

the equations of the 17 distinct planes are

$X=0.$	[14]
$Y=0,$	[25]
$Z=0,$	[36]
$X+Y+Z=0,$	[123]
$lX+mY+nZ=0,$	[456]
$lX+nY+nZ=0,$	[15]
$lX+nY+nZ=0,$	[16]
$lX+mY+lZ=0,$	[25]
$nX+mY+nZ=0,$	[26]
$mX+mY+nZ=0,$	[35]
$lX+lY+nZ=0,$	[36]
$W=0,$	[14.25.36]
$W+lX=0,$	[14.26.35]
$W+m\mu Y=0,$	[16.25.34]
$W+n\nu Z=0,$	[15.24.36]
$lmX+mnY+nlZ+W=0,$	[15.26.34]
$nlX+lmY+mnZ-W=0,$	[16.24.35]

63. And the coordinates of the fifteen distinct lines are

(a)	(b)	(c)	(f)	(g)	(h)	whence equations may be written
0	0	0	0	-1	1	(1) $X=0, Y+Z=0$
0	0	0	1	0	-1	(2) $Y=0, Z+X=0$
0	0	0	-1	1	0	(3) $Z=0, X+Y=0$
0	0	0	0	$-n$	m	(4) $X=0, mY+nZ=0$
0	0	0	n	0	$-l$	(5) $Y=0, nZ+lX=0$
0	0	0	$-m$	l	0	(6) $Z=0, lX+mY=0$
1	0	0	0	0	0	(14) $X=0, W=0$
0	1	0	0	0	0	(25) $Y=0, W=0$
0	0	1	0	0	0	(36) $Z=0, W=0$
l	n	n	$n^2\nu$	$-nl\nu$	0	(15) $lX+nY+nZ=0, W+n\nu Z=0$
l	m	m	$-m^2\mu$	0	$lm\mu$	(16) $lX+mY+mZ=0, W+m\mu Y=0$
l	m	l	0	$l^2\lambda$	$-lm\mu$	(26) $lX+mY+lZ=0, W+l\lambda X=0$
n	m	n	$mn\nu$	$-n^2\nu$	0	(24) $nX+mY+nZ=0, W+n\nu Z=0$
m	m	n	$-mn\mu$	0	$m^2\mu$	(34) $mX+mY+nZ=0, W+m\mu Y=0$
l	l	n	0	$nl\lambda$	$-l^2\lambda$	(35) $lX+lY+nZ=0, W+l\lambda X=0$

64. The rays are not, the mere lines are, facultative; hence $b'=\xi'=9:t'=6$.

65. The equation of the Hessian surface is

$$-W(X+Y+Z)(lX+mY+nZ)(\mu\nu X+\nu\lambda Y+\lambda\mu Z)$$
$$-k(l^2X^4+m^2Y^4+n^2Z^4-2mnY^2Z^2-2nlZ^2X^2-2lmX^2Y^2)$$
$$+kXYZ\{(l^2+3lm+3ln+mn)X+(m^2+3mn+3ml+nl)Y+(n^2+3nl+3nm+lm)Z\}=0.$$

The Hessian and cubic surfaces intersect in an indecomposable curve, which is the spinode curve; that is, spinode curve is a complete intersection 3×4; $\sigma'=12$.

The equations may be written in the simplified form

$$W(X+Y+Z)(lX+mY+nZ)+kXYZ=0,$$
$$l^2X^4+m^2Y^4+n^2Z^4-2mnY^2Z^2-2nlZ^2X^2-2lmX^2Y^2$$
$$-4XYZ\{l(m+n)X+m(n+l)Y+n(l+m)Z\}=0.$$

We may also obtain the equation

$$k^2(X+Y+Z)(lX+mY+nZ)(lX^2+mY^2+nZ^2-\overline{m+n}YZ-\overline{n+l}ZX-\overline{l+m}XY)$$
$$+\lambda^2Y^2Z^2+\mu^2Z^2X^2+\nu^2X^2Y^2-2XYZ(\mu\nu X+\nu\lambda Y+\lambda\mu Z)=0,$$

which shows that there is at B, an eightfold point, the tangents being given by

$$(X+Y+Z)(lX+mY+nZ)=0,$$
$$(\lambda^2,\ \mu^2,\ \nu^2,\ -\mu\nu,\ -\nu\lambda,\ -\lambda\mu)(YZ,\ ZX,\ XY)^2=0.$$

Each of the facultative lines is a double tangent of the spinode curve; whence $\beta'=18$.

Reciprocal Surface.

66. The equation may be deduced from that for $II=12-C_2$, viz. writing

$$(a,\ b,\ c,\ f,\ g,\ h)(X,\ Y,\ Z)^2=2(X+Y+Z)(lX+mY+nZ),$$

that is

$$(a,\ b,\ c,\ f,\ g,\ h)=(2l,\ 2m,\ 2n,\ m+n,\ n+l,\ l+m),$$

we have

$$(A,\ B,\ C,\ F,\ G,\ H)=-(\lambda^2,\ \mu^2,\ \nu^2,\ \mu\nu,\ \nu\lambda,\ \lambda\mu);\ K=0.$$

Writing also

$$\lambda,\ \mu,\ \nu=m-n,\ n-l,\ l-m \text{ as before,}$$

$$\lambda x+\mu y+\nu z=\sigma,$$
$$lmn\,xyz=\theta,$$
$$l(m+n)yz+m(n+l)zx+n(l+m)xy=\upsilon,$$
$$l\lambda\,yz+m\mu z\quad x+\quad n\nu xy=\psi,$$
$$(m+n)x+\quad (n+l)y+(l+m)\quad z=t,$$

we have

$$U=2\upsilon,\ V=2\sigma\psi,\ W=-4\psi^2,$$

and then

$$L=k^2w^2-2ktw+\sigma^2,\ M=2(kw\upsilon+\sigma\psi),\ N=4(4lmnkxyzw-\psi^2);$$

so that the equation is

$$\begin{aligned}
0=L^2N\quad &=\quad 4(k^2w^2-2kwt+\sigma^2)^2(4kw\theta-\psi^2)\\
+L^2M^2\quad &+\quad 4(k^2w^2-2kwt+\sigma^2)^2(kw\upsilon+\sigma\psi)^2\\
-18kwLMN\quad &-144kw(k^2w^2-2kwt+\sigma^2)(kw\upsilon+\sigma\psi)(kw\theta-\psi^2)\\
-16kwM^3\quad &-128kw(kw\upsilon+\sigma\psi)^3\\
-27k^2w^2N^2\quad &-432k^2w^2(kw\theta-\psi^2);
\end{aligned}$$

or reducing the first two terms so as to throw out from the whole equation the factor kw, the equation is

$$\begin{aligned}
&4L^2\{\theta L+(\upsilon^2-\psi^2)kw+2\psi(t\psi+\upsilon\sigma)\}\\
&-18LMN-16M^3-27kwN^2=0,
\end{aligned}$$

or, what is the same thing, it is

$$\begin{aligned}
&(k^2w^2-2kwt+\sigma^2)^2\{k^2w^2\theta+kw(-2t\theta+\upsilon^2-\psi^2)+\sigma^2\theta+2\sigma\upsilon\psi+2t\psi^2\}\\
&-36(k^2w^2-2kwt+\sigma^2)(kw\upsilon+\sigma\psi)(4kw\theta-\psi^2)\\
&-32(kw\upsilon+\sigma\psi)^3\\
&-108kw(4kw\theta-\psi^2)^2=0.
\end{aligned}$$

67. This is

$$\begin{aligned}
&(kw)^6 . \quad \theta \\
&+(kw)^5 . -\psi^3 - 6t\theta + v^3 \\
&+(kw)^4 . \quad \sigma^3 . 3\theta + \sigma\psi . 2v + \psi^3 . 6t + 12t^2\theta - 4tv^3 - 144\theta v \\
&+(kw)^3 . -2\sigma^3\psi^3 + \sigma^3(2v^3 - 10t\theta) + \sigma\psi(-8tv - 144\theta) + \psi^3(-12t^3 + 36v) \\
&\qquad -8t^2\theta + 4t^2v^3 + 288tv\theta - 32v^3 - 1728\theta^2 \\
&+(kw)^2 . \quad \sigma^4 3\theta + \sigma^3\psi . 4v + \sigma^3\psi^3 . 12t + \sigma\psi^3 . 37 \\
&\qquad +\sigma^2(12t^2\theta - 4tv^3 - 144\theta v) + \sigma\psi(+8t^2v + 288t\theta - 96v^3) + \psi^3(8t^3 - 72tv + 864\theta) \\
&+(kw) . -\sigma^4\psi^3 + \sigma^4(-6t\theta + v^3) + \sigma^3\psi(-8tv - 144\theta) \\
&\qquad +\sigma^2\psi^2(-8t - 90v) + \sigma\psi^3 . -72 + \psi^4 . -108 \\
&+(kw)^0\sigma^3(\theta,\ 2v,\ 2t,\ 4\overline{\chi}\sigma,\ \psi)^3 = 0,
\end{aligned}$$

which, reducing the last term, is

$$\begin{aligned}
&(kw)^6 lmnxyz \\
&\quad \vdots \\
&-4\sigma^3\lambda\mu\nu(y-z)(z-x)(x-y)(ny-mz)(lz-nx)(mx-ly) = 0.
\end{aligned}$$

68. I verify the last term in the particular case $z=0$ as follows: the coefficient of σ^3 is

$$(0,\ 2n(l+m)xy,\ 2\overline{m+n}x + 2\overline{n+l}y,\ 4\overline{\chi}\lambda x + \mu y,\ \nu xy)^3,$$

which is

$$\begin{aligned}
&= 2n^3\nu x^3y^3\{(l+m)(\lambda x + \mu y)^3, \\
&\qquad +(\overline{m+n}x + \overline{n+l}y)(\nu\lambda x + \nu\mu y) \\
&\qquad +2n\nu^3 xy\} \\
&= 2n^3\nu x^2y^3\{\overline{l+m}\lambda + \overline{m+n}\nu)\lambda x^2 \\
&\qquad +[2\overline{l+m}\lambda\mu + \nu(\overline{m+n}\mu + \overline{n+l}\lambda + 2n\nu)]xy \\
&\qquad +(\overline{l+m}\mu + \overline{n+l}\nu)\mu y^2\},
\end{aligned}$$

which, substituting for λ, μ, ν their values $m-n$, $n-l$, $l-m$, is

$$= 2n^3\nu x^2y^2 . -2\lambda\mu(x-y)(mx-ly);$$

or for $z=0$ the coefficient of σ^3 is

$$= -4\lambda\mu\nu m^3x^2y^3(x-y)(mx-ly),$$

agreeing with the general value

$$-4\lambda\mu\nu(y-z)(z-x)(x-y)(ny-mz)(lz-nx)(lx-my).$$

69. In the discussion of the equation it is convenient to write down the relations of the two surfaces, thus:

Cubic surface.	Reciprocal surface.
$B_{\prime\prime}$ $\quad X=0, Y=0, Z=0$	Plane $w=0$,
Biplanes $\quad X+Y+Z=0$	Points in $w=0$, viz.
$\quad lX+mY+nZ=0,$	$\quad x=y=z$ and $x:y:z=l:m:n,$
intersecting in edge.	in line $(m-n)x+(n-l)y+(l-m)z=0,$
	that is, $\lambda x+\mu y+\nu z=0$, or $\sigma=0$.
Rays in first biplane,	Lines in plane $w=0$, and through first
$\quad X=0, Y+Z=0;\ Y=0, Z+X=0,$	point, viz.
$\quad Z=0, X+Y=0;$	$\quad y-z=0, z-x=0, x-y=0;$
rays in second biplane,	lines through second point, viz.
$X=0, mY+nZ=0;\ Y=0, nZ+lX=0,$	$\quad ny-mz=0, nz-lx=0, lx-my=0.$
$Z=0, lX+mY=0.$	

70. The equation puts in evidence the section by the plane $w=0$, viz. this is the line $\sigma=0$ (reciprocal of the edge) three times, and the six lines (reciprocals of the rays) each once. Observe that the edge is *not* a line on the cubic; but its reciprocal is a line, and that an oscular line on the reciprocal surface; the six lines (reciprocals of the rays) are mere scrolar lines on the reciprocal surface; they pass, three of them, through the point $x=y=z$, and the other three through the point $x:y:z=l:m:n$; that is, they are six tangents of the point-pair (reciprocal of the pair of biplanes) formed by these two points.

71. I do not attempt to put in evidence the nodal curve on the surface; by what precedes it consists of 9 lines, reciprocals of the mere lines. If we denote by 1, 2, 3 and 4, 5, 6 the lines which pass through the points $x=0, y=0, z=0$ and through the point $x:y:z=l:m:n$ respectively, then these intersect in the nine points 14, 15, 16, 24, 25, 26, 34, 35, 36; and through each of these there passes a nodal line which may be represented by the same symbol; that is, we have the nodal lines 14, ... 36. Two lines such as 14, 25 meet; and three lines such as 14, 25, 36 meet in a point; we have thus the six points 14 . 25 . 36 &c. triple points on the nodal curve; as before, $b'=9$, $t'=6$.

72. The cuspidal curve is given by the equations

$$\left\| \begin{matrix} k^2w^2-2kwt+\sigma^2, & 24(kwv+\sigma\psi), & -36(4lmnk\,xyzw-\psi^2 \\ kw\quad , & k^2w^2-2kwt+\sigma^2, & 2(kwv+\sigma\psi) \end{matrix} \right\| =0.$$

Writing down the two equations,

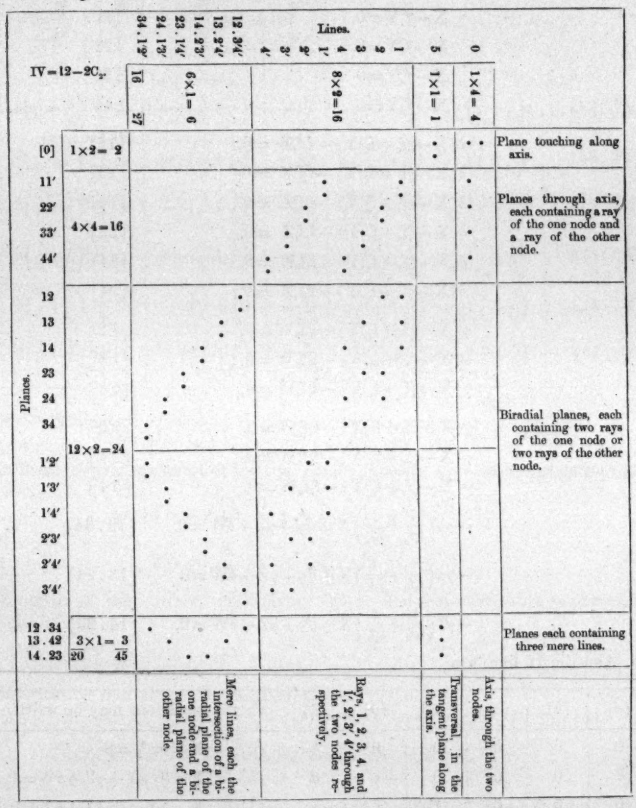
$$(k^2w^2-2kwt+\sigma^2)^2-24kw(kwv+\sigma\psi)=0,$$
$$(k^2w^2-2kwt+\sigma^2)(kwv+\sigma\psi)+18w(lmnk\,xyzw-\psi^2)=0,$$

these are respectively of the orders 4 and 5; but they intersect in the line $w=0, \sigma=0$ taken four times, or say, the cuspidal curve is a partial intersection $4\cdot5-4$; $c'=16$.

Section IV=12−2C$_3$.

Equation $WXZ+Y^2(\gamma Z+\delta W)+(a, b, c, d\chi X, Y)^3=0$. Article Nos. 73 to 84.

73. The diagram of the lines is


Lines.
IV=12−2C3.
Planes.
Axis, through the two nodes.
Transversal in the tangent plane along the axis
Rays 1, 2, 3, 4, and 1′, 2′, 3′, 4′ through the two nodes respectively.
More lines, each the intersection of a bitangential plane of the one node and a bitangential plane of the other node.

74. Writing $X(a, b, c, d)(X, Y)^3 - \gamma\delta Y^4 = -\gamma\delta(f_1X-Y)(f_2X-Y)(f_3X-Y)(f_4X-Y)$, the 20 planes are

$$X=0, \qquad [\ 0]$$
$$X-f_1Y=0, \qquad [11']$$
$$X-f_2Y=0, \qquad [22']$$
$$X-f_3Y=0, \qquad [33']$$
$$X-f_4Y=0, \qquad [44']$$

$$\delta\{X-(f_1+f_2)Y\}-f_1f_2Z=0, \qquad [12]$$
$$\delta\{X-(f_1+f_3)Y\}-f_1f_3Z=0, \qquad [13]$$
$$\delta\{X-(f_1+f_4)Y\}-f_1f_4Z=0, \qquad [14]$$
$$\delta\{X-(f_2+f_3)Y\}-f_2f_3Z=0, \qquad [23]$$
$$\delta\{X-(f_2+f_4)Y\}-f_2f_4Z=0, \qquad [24]$$
$$\delta\{X-(f_3+f_4)Y\}-f_3f_4Z=0, \qquad [34]$$

$$\gamma\{X-(f_1+f_2)Y\}-f_1f_2W=0, \qquad [1'2']$$
$$\gamma\{X-(f_1+f_3)Y\}-f_1f_3W=0, \qquad [1'3']$$
$$\gamma\{X-(f_1+f_4)Y\}-f_1f_4W=0, \qquad [1'4']$$

$$\gamma\{X-(f_2+f_3)Y\}-f_2f_3W=0, \qquad [2'3']$$
$$\gamma\{X-(f_2+f_4)Y\}-f_2f_4W=0, \qquad [2'4']$$
$$\gamma\{X-(f_3+f_4)Y\}-f_3f_4W=0, \qquad [3'4']$$

$$-\gamma\delta\left(\frac{1}{f_1f_2}+\frac{1}{f_3f_4}\right)X+dy+\gamma Z+\delta W=0, \qquad [12.34]$$

$$-\gamma\delta\left(\frac{1}{f_1f_3}+\frac{1}{f_2f_4}\right)X+dy+\gamma Z+\delta W=0, \qquad [13.24]$$

$$-\gamma\delta\left(\frac{1}{f_1f_4}+\frac{1}{f_2f_3}\right)X+dy+\gamma Z+\delta W=0, \qquad [14.23]$$

75. And the 12 lines are

(a)	(b)	(c)	(f)	(g)	(h)	whence equations may be written	
0	0	0	0	0	1	(0)	$X=0,\ Y=0$
δ	0	0	0	$-\gamma$	d	(5)	$X=0,\ dY+\gamma Z+\delta W=0$
0	0	0	f_1^2	f_1	$-\delta$	(1)	$X=f_1Y=0,\ \delta Y+f_1Z=0$
0	0	0	f_2^2	f_2	$-\delta$	(2)	" "
0	0	0	f_3^2	f_3	$-\delta$	(3)	" "
0	0	0	f_4^2	f_4	$-\delta$	(4)	" "

(a)	(b)	(c)	(f)	(g)	(h)			
f_1	$-f_1^2$	0	0	0	γ	(1′)	$X-f_1Y=0,\ \gamma Y+f_1W=0$	
f_2	$-f_2^2$	0	0	0	γ	(2′)	,,	,,
f_3	$-f_3^2$	0	0	0	γ	(3′)	,,	,,
f_4	$-f_4^2$	0	0	0	γ	(4′)	,,	,,
$-\frac{\delta}{f_3f_4}$	$\delta\left(\frac{1}{f_1}+\frac{1}{f_2}\right)$	1	$-\gamma\left(\frac{1}{f_3}+\frac{1}{f_4}\right)$	$-\frac{\gamma}{f_3f_4}$	$\frac{1}{f_3f_4}\left(\frac{1}{f_1}+\frac{1}{f_2}\right)-\frac{1}{f_1f_2}\left(\frac{1}{f_3}+\frac{1}{f_4}\right)$			(12.3′4′)*
$-\frac{\delta}{f_2f_3}$	$\delta\left(\frac{1}{f_1}+\frac{1}{f_3}\right)$	1	$-\gamma\left(\frac{1}{f_2}+\frac{1}{f_4}\right)$	$-\frac{\gamma}{f_2f_4}$	$\frac{1}{f_2f_4}\left(\frac{1}{f_1}+\frac{1}{f_3}\right)-\frac{1}{f_1f_3}\left(\frac{1}{f_2}+\frac{1}{f_4}\right)$			(13.2′4′) ,,
$-\frac{\delta}{f_1f_4}$	$\delta\left(\frac{1}{f_1}+\frac{1}{f_4}\right)$	1	$-\gamma\left(\frac{1}{f_2}+\frac{1}{f_3}\right)$	$-\frac{\gamma}{f_2f_3}$	$\frac{1}{f_2f_3}\left(\frac{1}{f_1}+\frac{1}{f_4}\right)-\frac{1}{f_1f_4}\left(\frac{1}{f_2}+\frac{1}{f_3}\right)$			(14.2′3′) ,,
$-\frac{\delta}{f_2f_3}$	$\delta\left(\frac{1}{f_2}+\frac{1}{f_3}\right)$	1	$-\gamma\left(\frac{1}{f_1}+\frac{1}{f_4}\right)$	$-\frac{\gamma}{f_1f_4}$	$\frac{1}{f_1f_4}\left(\frac{1}{f_2}+\frac{1}{f_3}\right)-\frac{1}{f_2f_3}\left(\frac{1}{f_1}+\frac{1}{f_4}\right)$			(23.1′4′) ,,
$-\frac{\delta}{f_2f_4}$	$\delta\left(\frac{1}{f_2}+\frac{1}{f_4}\right)$	1	$-\gamma\left(\frac{1}{f_1}+\frac{1}{f_3}\right)$	$-\frac{\gamma}{f_1f_3}$	$\frac{1}{f_1f_3}\left(\frac{1}{f_2}+\frac{1}{f_4}\right)-\frac{1}{f_2f_4}\left(\frac{1}{f_1}+\frac{1}{f_3}\right)$			(24.1′3′) ,,
$-\frac{\delta}{f_3f_4}$	$\delta\left(\frac{1}{f_3}+\frac{1}{f_4}\right)$	1	$-\gamma\left(\frac{1}{f_1}+\frac{1}{f_2}\right)$	$-\frac{\gamma}{f_1f_2}$	$\frac{1}{f_1f_2}\left(\frac{1}{f_3}+\frac{1}{f_4}\right)-\frac{1}{f_3f_4}\left(\frac{1}{f_1}+\frac{1}{f_2}\right)$			(34.1′2′) ,,

*equations are

$$\delta\{X-(f_1+f_2)Y\}-f_1f_2Z=0,\quad \gamma\{X-(f_3+f_4)Y\}-f_3f_4W=0.$$

76. To verify the equations of the line $12.3'4'$, observe that the two equations give

$$\gamma Z+\delta W=\gamma\delta\left\{X\left(\frac{1}{f_1f_2}+\frac{1}{f_3'f_4'}\right)-Y\left(\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}+\frac{1}{f_4}\right)\right\},$$

$$ZW=\frac{\gamma\delta}{f_1f_2f_3'f_4'}(X-\overline{f_1+f_2}Y)(X-\overline{f_3+f_4}Y):$$

the equation of the surface, multiplying by X and observing that $-\gamma\delta=af_1f_2f_3f_4$, becomes

$$X^2ZW+XY^3(\gamma Z+\delta W)+\gamma\delta Y^4-\frac{\gamma\delta}{f_1f_2f_3f_4}(X-f_1Y)(X-f_2Y)(X-f_3Y)(X-f_4Y)=0;$$

and substituting the values just obtained, this is

$$X^2(X-\overline{f_1+f_2}Y)(X-\overline{f_3+f_4}Y)+XY^3(X\overline{f_1f_2+f_3f_4}-Y\overline{f_1f_2f_3+f_1f_2f_4+f_1f_3f_4+f_2f_3f_4})$$
$$+f_1f_2f_3f_4Y^4-(X-f_1Y)(X-f_2Y)(X-f_3Y)(X-f_4Y)=0,$$

which is in fact an identity.

77. The facultative lines are the transversal and the six mere lines; $b'=g'=7$; $t'=3$.

78. The equation of the Hessian surface is found to be

$$(\gamma Z+\delta W)XZW+Y^3(\gamma Z-\delta W)^2+3(cX+dY)XZW+12\gamma\delta XY^3(aX+bY)$$
$$-(\gamma Z+\delta W)(3aX^3+9bX^2Y+6cXY^2)$$
$$-9X^2\{(ac-b^2)X^2+(ad-bc)XY+(bd-c^2)Y^2\}=0$$

79. Combining with the foregoing the equation of the surface

$$XZW+Y^2(\gamma Z+\delta W)+(a, b, c, d\text{)(}X, Y)^3=0,$$

it appears that these have along the line $X=0$, $Y=0$ the common tangent plane $X=0$, or, what is the same thing, that they meet in the line $X=0$, $Y=0$ (the axis) twice, and in a residual curve of the tenth order, which is the spinode curve; the equations may be presented in the somewhat more simple form

$$XZW+Y^2(\gamma Z+\delta W)+(a, b, c, d\text{)(}X, Y)^3=0,$$

$$-4\gamma\delta Y^2ZW-4(\gamma Z+\delta W)(a, b, c, d\text{)(}X, Y)^3+12\gamma\delta XY^2(aX+\delta Y)$$

$$+X^4(-12ac+9b^2)-3d(4aX^3Y+6bX^2Y^2+4cXY^3+dY^4)=0,$$

which, however, still contain the line $X=0$, $Y=0$ twice. The spinode curve, as just mentioned, is of the tenth order; that is, we have $\sigma'=10$.

Each of the 6 mere lines is a double tangent to the spinode curve, but the transversal is only a single tangent: to show this, observe that the equations of the transversal are $X=0$, $\gamma Z+\delta W+dY=0$; substituting in the equations of the curve the first equation, that of the cubic surface is of course satisfied identically; for the second equation, writing $X=0$, this becomes $Y^2\{-4\gamma\delta ZW-4dY(\gamma Z+\delta W)-3d^2Y^2\}=0$; or writing herein $dY=-(\gamma Z+\delta W)$, it becomes $Y^2(\gamma Z-\delta W)^2=0$. The value $Y^2=0$ gives $X=0$, $Y=0$, $\gamma Z+\delta W=0$, viz. this is a point on the axis $X=0$, $Y=0$ not belonging to the spinode curve; the value $(\gamma Z-\delta W)^2=0$ gives a point of contact $X=0$, $\gamma Z+\delta W+dY=0$, $\gamma Z-\delta W=0$; and the transversal is thus a single tangent. Hence the number of contacts is $2.6+1$, $=13$; that is, we have $\beta'=13$.

Reciprocal Surface.

80. The equation is found by equating to zero the discriminant of the binary quartic

$$\{xX^2+yXY-(\delta z+\gamma w)Y^2\}+4Zw\{X(a, b, c, d\text{)(}X, Y)^3-\gamma\delta Y^4\},$$

or say this is $(*\text{)(}X, Y)^4$, where the coefficients are

$$\begin{aligned} &6x^2 &&+24azw,\\ &3xy &&+18bzw,\\ &y^2-2(\delta z+\gamma w)x &&+12czw,\\ &-3(\delta z+\gamma w)y &&+6dzw,\\ &6(\delta z-\gamma w)^2. \end{aligned}$$

81. Forming the invariants, these are

$$\tfrac{1}{3}I=\Lambda^2+\ 24Uzw+144\mu z^2w^2$$

$$-J=\Lambda^3+36\Lambda Uzw+216Vz^2w^2+864\nu z^3w^3,$$

where

$$\Lambda = y^3 + 4(\delta z + \gamma w)x,$$

$$U = 2\gamma\delta x^3 + 2a(\delta z - \gamma w)^3 + 3by(\delta z + \gamma w) + c[y^3 - 2(\delta z + \gamma w)x] - dxy,$$

$$\begin{aligned} V = &(-8ac + 9b^2)(\delta z - \gamma w)^3 \\ &+ (2c^2 - bd)[y^3 - 2(\delta z + \gamma w)x] \\ &+ (-4ad + 6bc)y(\delta z + \gamma w) \\ &- 2cd\,xy \\ &+ d^2x^2 \\ &+ 4\gamma\delta(2cx^3 - 3bxy + ay^2), \end{aligned}$$

$$\mu = c^3 - bd,$$

$$\nu = ad^2 - 2bcd + 2c^3,$$

and the equation is

$$\frac{1}{432x^2w^2}\{(\Lambda^3 + 24Uzw + 144\mu z^2w^2)^3 - (\Lambda^3 + 36\Lambda Uzw + 216Vz^3w^2 + 864\nu z^3w^3)^2\} = 0.$$

Or, expanding, this is

$$\begin{aligned} &\Lambda^4\mu - \Lambda^3 V + \Lambda^2 U^2 \\ + \quad &4zw(\quad -\Lambda^3\nu + 12\Lambda^2 U\mu - 9\Lambda UV + 8U^3) \\ + \quad &36z^2w^2(\qquad 4\Lambda^2\mu^2 - 4\Lambda U\nu + 16U^2\mu - 3V^2) \\ + \quad &864z^3w^3(\qquad\qquad 4U\mu^2 - V\nu) \\ + &1728z^4w^4(\qquad\qquad 4\mu^3 - \nu^2) = 0, \end{aligned}$$

where observe that the value of

$$4\mu^3 - \nu^2, \ = 4(bd - c^2)^3 - (ad^2 - 3bcd + 2c^3)^2 \text{ is } = -d^2(a^2d^3 + 4ac^3 + 4b^3d - 3b^2c^2 - 6abcd).$$

82. It is convenient to modify the form of the equation as follows; write

$$U_1 = U + 8a\gamma\delta zw, \ V_1 = V + (-8ac + 9b^2)\gamma\delta zw,$$

so that

$$\Lambda = y^3 + 4(\delta z + \gamma w)x,$$

$$U_1 = -2\gamma\delta x^3 + 2a(\delta z + \gamma w)^3 + 3by(\delta z + \gamma w) + c[y^3 - 2(\delta z + \gamma w)x] - dxy,$$

$$\begin{aligned} V_1 = &(-8ac + 9b^2)(\delta z + \gamma w)^3 \\ &+ (2c^2 - bd)[y^3 - 2(\delta z + \gamma w)x] \\ &+ (-4ad + 6bc)y(\delta z + \gamma w) \\ &- 2cdxy \\ &+ d^2x^2 \\ &+ 4\gamma\delta(2cx^3 - 3bxy + ay^2), \end{aligned}$$

$$\mu = c^3 - bd,$$

$$\nu = ad^2 - 2bcd + 2c^3,$$

Λ, U_1, V_1 being, it will be observed, functions of x, y, $\delta z+\gamma w$. The transformed equation is

$$\Lambda^2(\Lambda^2\mu-\Lambda V_1+U_1^2)+\Omega zw=0,$$

where the term Ω may be calculated without difficulty: the first term of this is

$$=\{y^2+4(\delta z+\gamma w)x\}^2\,.\,4\gamma^2\delta^2[x+f_1y-f_2^2(dz+\gamma w)]\,..\,[x+f_4y-f_4^2(\delta z+\gamma w)],$$

the developed expressions of $\frac{1}{4}(\Lambda^2\mu-\Lambda V_1+U_1^2)$ and of $\gamma^2\delta^2$ into the product of the linear factors being in fact each

$$\begin{aligned}
&=x^4\,.\,\gamma^2\delta^2+x^3y\,.\,d\gamma\delta+x^2y^2\,.\,-3c\gamma\delta+xy^3\,.\,3b\gamma\delta+y^4\,.\,-a\gamma\delta\\
&\quad+[x^3(-d^2-6c\gamma\delta)+x^2y(3cd+9b\gamma\delta)+xy^2(-3bd-4a\gamma\delta)+y^3\,.\,ad](\delta z+\gamma w)\\
&\quad+[x^2(9c^2-6bd-2a\gamma)+xy(3ad-9bc)+y^2\,.\,3ac](\delta z+\gamma w)^2\\
&\quad+[x(6ac-9b^2)+y\,.\,3ab](\delta z+\gamma w)^3\\
&\quad+a^2\delta^4\,.\,(\delta z+\gamma w)^4.
\end{aligned}$$

The form puts in evidence the section by the plane $w=0$, which is the reciprocal of the node D, viz. this is a conic (the reciprocal of the tangent cone) twice, and four lines, the reciprocals of the nodal rays, each once. And similarly for the section by the plane $z=0$.

83. The nodal curve is made up of the lines which are the reciprocals of the six mere lines and the transversal; viz. we have three pairs of lines and a seventh line, the lines of each pair intersecting at a point of the seventh line, and these three points being the triple points of the nodal curve; $t'=3$ as before.

84. The equations of the cuspidal curve are at once reduced to the form

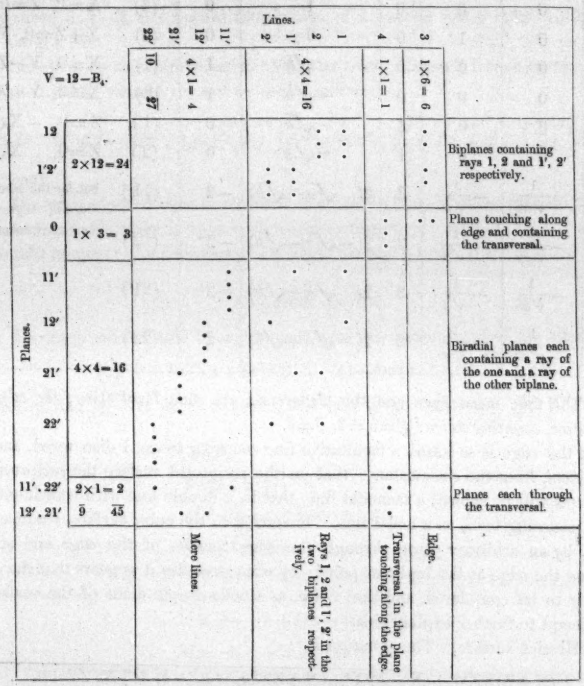
$$\begin{aligned}
&\Lambda^2+24Uzw+144\mu z^2w^2=0,\\
&\Lambda U+(18V-12\mu\Lambda)zw+72vz^2w^2=0,
\end{aligned}$$

which are two quartic surfaces having in common the conics $z=0$, $\Lambda=0$, and $w=0$, $\Lambda=0$; or we may say that the cuspidal curve is a curve $4\,.\,4-2-2$; that is $c'=12$.

Section $V=12-B_4$.

Equation $WXZ+(X+Z)(Y^2-aX^2-bZ^2)=0$. Article Nos. 85 to 94.

85. The diagram of the lines and planes is


Lines.
$V=12-B_4$.
$2\times12=24$
$1\times3=3$
$4\times4=16$
$2\times1=2$
$1\times1=1$
$1\times6=6$
Planes.
Biplanes containing rays 1, 2 and 1′, 2′ respectively.
Plane touching along edge and containing the transversal.
Biradial planes each containing a ray of the one and a ray of the other biplane.
Planes each through the transversal.
More lines.
Rays 1, 2 and 1′, 2′ in the two biplanes respectively.
Transversal in the plane touching along the edge.
Edge.

86. The planes are

$$X=0, \quad [12]$$
$$Z=0, \quad [1'2']$$
$$X+Z=0, \quad [0]$$
$$-X\sqrt{a}+Y-Z\sqrt{b}=0, \quad [11']$$
$$X\sqrt{a}+Y-Z\sqrt{b}=0, \quad [12']$$
$$-X\sqrt{a}+Y+Z\sqrt{b}=0, \quad [21']$$
$$X\sqrt{a}+Y+Z\sqrt{b}=0, \quad [22']$$
$$\overline{ab}(X+Z)+W=0, \quad [11'.22']$$
$$-2\sqrt{ab}(X+Z)+W=0, \quad [12'.21'].$$

87. And the lines are

a	b	c	f	g	h	equations may be written	
0	0	0	0	1	0	(3)	$X=0,\ Z=0$
1	0	1	0	0	0	(4)	$X+Z=0,\ W=0$
0	0	0	0	$\sqrt{b}$	1	(1)	$X=0,\ Y-Z\sqrt{b}=0$
0	0	0	0	$-\sqrt{b}$	1	(2)	$X=0,\ Y+Z\sqrt{b}=0$
0	0	0	1	$\sqrt{a}$	0	(1′)	$Z=0,\ -X\sqrt{a}+Y=0$
0	0	0	1	$-\sqrt{a}$	0	(2′)	$Z=0,\ X\sqrt{a}+Y=0$
$-\frac{1}{\sqrt{b}}$	$\frac{1}{\sqrt{ab}}$	$-\frac{1}{\sqrt{a}}$	2	$2(\sqrt{a}-\sqrt{b})$	-2	(11′)	but for the other lines the coordinate expressions are the more convenient.
$-\frac{1}{\sqrt{b}}$	$-\frac{1}{\sqrt{ab}}$	$\frac{1}{\sqrt{a}}$	2	$2(-\sqrt{a}-\sqrt{b})$	-2	(12′)	
$\frac{1}{\sqrt{b}}$	$-\frac{1}{\sqrt{ab}}$	$-\frac{1}{\sqrt{a}}$	2	$2(\sqrt{a}+\sqrt{b})$	-2	(21′)	
$\frac{1}{\sqrt{b}}$	$\frac{1}{\sqrt{ab}}$	$\frac{1}{\sqrt{a}}$	2	$2(-\sqrt{a}+\sqrt{b})$	-2	(22′)	

88. The four mere lines and the transversal are each facultative; *the edge is also facultative, counting twice*; $\varphi'=b'=7$, $t'=3$.

That the edge is as stated a facultative line counting twice, I discovered, and accept, *à posteriori*, from the circumstance that on the reciprocal surface the reciprocal of the edge is (as will be shown) a tacnodal line, that is, a double line with coincident tangent planes, counting twice as a nodal line. Reverting to the cubic surface, I notice that the section by an arbitrary plane through the edge consists of the edge and of a conic touching the edge at the biplanar point; by what precedes it appears that the arbitrary plane is to be considered, and that twice, as a node-couple plane of the surface: I do not attempt to further explain this.

89. Hessian surface. The equation is

$$(X+Z)XZW+(X-Z)^2Y^2+(X+Z)(3a,\ -a,\ -b,\ 3b\,\text{\textbar}\,X,\ Z)^2=0.$$

Combining with the equation

$$XZW+(X+Z)(Y^2-aX^2-bZ^2)=0,$$

and observing that from the two equations we deduce

$$-XZY^2+(X+Z)(aX^2+bZ^2)=0,$$

it appears that the complete intersection of the Hessian and the surface is made up of the line $X=0$, $Z=0$ (the edge) twice (that is, the two surfaces touch along the edge), and of a curve of the tenth order, which is the spinode curve; $d'=10$.

The equations of the spinode curve may be presented in the form

$$\left\| \begin{matrix} XZ, & aX^2+bZ^2-Y^2, & aX^2+bZ^2 \\ X+Z, & W, & Y^2 \end{matrix} \right\| = 0;$$

it is a curve $3.4-2$, the partial intersection of a quartic and a cubic surface which touch along a line.

The binode is on the spinode curve a singular point; through it we have two branches represented in the vicinity thereof by the equations

$$\left(\frac{X}{W} = -\tfrac{1}{2}\left(\frac{Y}{W}\right)^2,\ \frac{Z}{W} = -\left(\frac{1}{2b}\right)^{\frac{1}{2}}\left(\frac{Y}{W}\right)^{\frac{3}{2}}\right) \text{ and } \left(\frac{Z}{W} = -\tfrac{1}{2}\left(\frac{Y}{W}\right)^2,\ \frac{X}{W} = -\left(\frac{1}{2a}\right)^{\frac{1}{2}}\left(\frac{Z}{W}\right)^{\frac{3}{2}}\right)$$

respectively.

90. The edge counted once is regarded as a double tangent of the spinode curve (I do not understand this, there is apparently a higher tangency); each of the four mere lines is a double tangent; the transversal is a single tangent; hence $\beta' = 2.2+2.4+1, = 13$.

Reciprocal Surface.

91. The equation is found by equating to zero the discriminant of the binary quartic

$$y^2X^2Z^2 + 4w(Xx+Zz)XZ(X+Z) + 4w^2(aX^2+bZ^2)(X+Z)^2,$$

viz. multiplying by 6 to avoid fractions, and calling the function $(*)(X, Z)^4$, the coefficients are

$$\begin{aligned} &24aw^2, \\ &6w(x+2aw), \\ &y^2+4(x+z)w+4(a+b)w^2, \\ &6w(z+2bw), \\ &24bw^2; \end{aligned}$$

and then writing

$$\begin{aligned} L &= y^2+4(x+z)w+4(a+b)w^2, \\ M &= 4(xz+2bx+azw), \\ N &= 16aby^2-bx^2-ay^2, \end{aligned}$$

we find

$$\begin{aligned} \tfrac{1}{3}I &= L^2-12w^2M, \\ -J &= L^3-18w^2LM-54w^4N, \end{aligned}$$

and then the equation is

$$\frac{1}{108w^4}\{(L^2-12w^2M)^3-(L^3-18w^2LM-54w^4N)^2\} = 0,$$

viz. it is

$$L^3N+L^2M^3-18w^2LMN-16w^2M^3-27w^4N^2 = 0.$$

92. This, completely developed, is

$$\begin{aligned}
&64w^6.ab(a+b)^3\{(a+b)y^3-(x-z)^3\}\\
&+32w^5.2ab\left\{\begin{matrix}3(a+b)[(a-2b)x+(-2a+b)z]y^2\\+(x-z)^3[(-3a+5b)x+(5a-3b)z]\end{matrix}\right\}\\
&+16w^4\left\{\begin{matrix}3ab(a^2-7ab+b^2)y^4\\+[b(9a^2+26ab-b^2)x^2-26ab(a+b)xz+a(-a^2+26ab+9b^2)z^2]y^2\\+(x-z)^2[b(-12a+b)x^2+22abxz+a(a-12b)z^2]\end{matrix}\right\}\\
&+\ 8w^3\left\{\begin{matrix}3ab[2a-b)x+(-a+2b)z]y^4\\+[b(-2a+5b)x^2+b(3a-2b)x^2z+a(-2a+3b)xz^2+a(5a-2b)z^3]y^2\\+2(x-z)^2[-2bx^3+bx^2z+axz^2-2az^3]\end{matrix}\right\}\\
&+\ 4w^2\left\{\begin{matrix}3ab(a+b)y^6\\+[b(9a-2b)x^2+8abxz+a(-2a+9b)z^2]y^4\\+2[-6bx^4+bx^3z-(a+b)x^2z^2+axz^3-6az^4]y^2\\+4x^2z^2(x-z)^2\end{matrix}\right\}\\
&+\ 2w\left\{\begin{matrix}2ab(x+z)y^6\\-[3bx^3+2bx^2z+2axz+3az^3]y^4\\+4x^2z^2(x+z)y^2\end{matrix}\right\}\\
&+\qquad y^4(ay^2-x^2)(cy^2-z^2)=0,
\end{aligned}$$

where we see that the section by the plane $w=0$ (reciprocal of B_4) is made up of the line $w=0$, $y=0$ (reciprocal of the edge) four times, and of the lines $w=0$, $ay^2-x^2=0$; $w=0$, $by^2-z^2=0$ (reciprocals of the rays) each once.

93. The surface contains the line $y=0$, $w=0$ (reciprocal of the edge); and if we attend only to the terms of the lowest order in y, w, viz.

$$x^2z^2\{16(x-z)^2w^2+8(x+z)y^2w+y^4\},$$

which terms equated to zero give

$$w=-\tfrac{1}{4}\frac{1}{(\sqrt{x}\pm\sqrt{z})^2}y^2,$$

we see that the line in question ($y=0$, $w=0$) is a tacnodal line on the surface, the tacnodal plane being $w=0$, *a fixed plane for all points of the line*: it has already been seen that this plane meets the surface in the line taken 4 times; every other plane through the line meets the surface in the line taken twice. We have in what precedes the *à posteriori* proof that in the cubic surface the edge is a facultative line to be counted twice.

94. Cuspidal curve. The equation of the surface may be written

$$(L^2-12w^2M)(4M^2+3LN)-(LM+9w^2N)^2=0,$$

and we thus have

$$4M^3 + 3LN = 0,$$
$$LM + w^3N = 0,$$
$$L^3 - 12w^3M = 0,$$

or, what is the same thing,

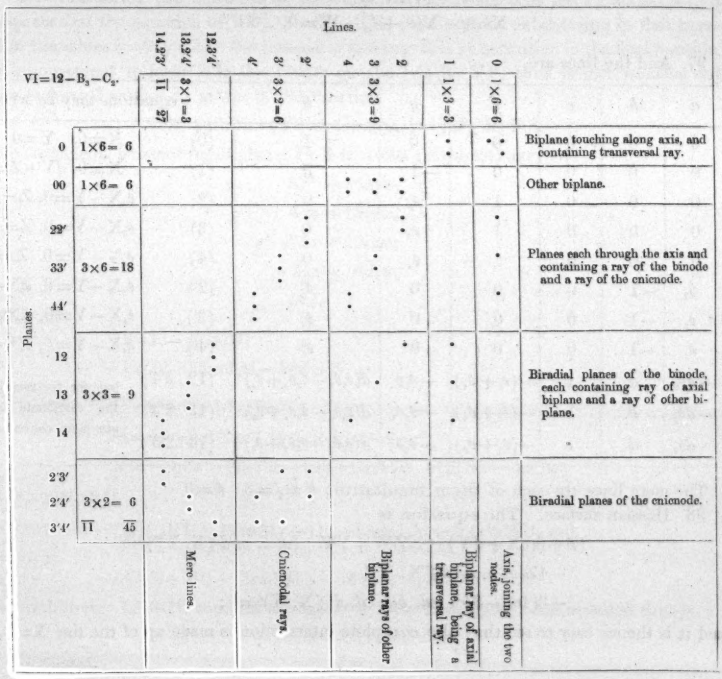
$$\left\| \begin{matrix} L, & 12M, & -9N \\ w^3, & L, & M \end{matrix} \right\| = 0$$

for the equation of the cuspidal curve. Attending to the second and third equations, these are quartics having in common $w^3=0, L=0$, that is, the line $y=0, w=0$ four times; or the cuspidal curve is a partial intersection $4\times4-4 : c'=12$.

Section $VI=12-B_3-C_2$.

Equation $WXZ+Y^2Z+(a, b, c, d\XX, Y)^3=0$. Article Nos. 95 to 102.

95. The diagram of the lines and planes is



96. Writing $(a, b, c, d)(X, Y)^3 = -d(\theta_2 X - Y)(\theta_3 X - Y)(\theta_4 X - Y)$, the planes are

$$\begin{aligned}
X &= 0, && [0]\\
Z &= 0, && [00]\\
\theta_2 X - Y &= 0, && [22']\\
\theta_3 X - Y &= 0, && [33']\\
\theta_4 X - Y &= 0, && [44']\\
d(\theta_2 X - Y) - Z &= 0, && [12]\\
d(\theta_3 X - Y) - Z &= 0, && [13]\\
d(\theta_4 X - Y) - Z &= 0, && [14]\\
X\theta_2\theta_3 - Y(\theta_2 + \theta_3) - W &= 0, && [2'3']\\
X\theta_3\theta_4 - Y(\theta_3 + \theta_4) - W &= 0, && [2'4']\\
X\theta_3\theta_4 - Y(\theta_3 + \theta_4) - W &= 0, && [3'4']
\end{aligned}$$

97. And the lines are

a	b	c	f	g	h	equations may be written	
0	0	0	0	0	1	(0)	$X=0,\ Y=0$
0	0	0	0	-1	d	(1)	$X=0,\ dY+Z=0$
0	0	0	1	θ_2	0	(2)	$\theta_2 X - Y = 0,\ Z=0$
0	0	0	1	θ_3	0	(3)	$\theta_3 X - Y = 0,\ Z=0$
0	0	0	1	θ_4	0	(4)	$\theta_4 X - Y = 0,\ Z=0$
θ_2	-1	0	0	0	θ_2^2	(2')	$\theta_2 X - Y = 0,\ \theta_2^2 X + W = 0$
θ_3	-1	0	0	0	θ_3^2	(3')	$\theta_3 X - Y = 0,\ \theta_3^2 X + W = 0$
θ_4	-1	0	0	0	θ_4^2	(4')	$\theta_4 X - Y = 0,\ \theta_4^2 X + W = 0$
$-d\theta_2$	d	1	$-(\theta_3+\theta_4)$	$-\theta_3\theta_4$	$d(\theta_3\theta_4 - \theta_2\overline{\theta_3+\theta_4})$	(12 . 3'4')	but for the remaining lines the coordinate expressions are more convenient.
$-d\theta_3$	d	1	$-(\theta_2+\theta_4)$	$-\theta_2\theta_4$	$d(\theta_2\theta_4 - \theta_3\overline{\theta_2+\theta_4})$	(13 . 2'4')	
$-d\theta_4$	d	1	$-(\theta_2+\theta_3)$	$-\theta_2\theta_3$	$d(\theta_2\theta_3 - \theta_4\overline{\theta_2+\theta_3})$	(14 . 2'3')	

The mere lines are each of them facultative; $b' = g' = 3$; $t' = 0$.

98. Hessian surface. The equation is

$$\begin{aligned}
&\{Z + 3(cX + dY)\}\{XZW + Y^2 Z + (a, b, c, d)(X, Y)^3\}\\
&\quad - 4Z(a, b, c, d)(X, Y)^3\\
&\quad - 3(4ac - 3b^2, ad, bd, cd, d^2)(X, Y)^4 = 0;
\end{aligned}$$

and it is thence easy to see that the complete intersection is made up of the line $X=0$,

$Y=0$ (the axis) three times, and of a curve of the ninth order, which is the spinode curve; $\sigma'=9$.

99. The equations of the spinode curve may be written in the simplified form

$$XZW+Y^3Z+(a, b, c, d)(X, Y)^3=0,$$

$$4Z(a, b, c, d)(X, Y)^3+3(4ac-3b^2, ad, bd, cd, d^2)(X, Y)^4=0,$$

the line $X=0$, $Y=0$ here appearing as a triple line on the second surface; the curve is a partial intersection, $3\times 4-3$.

The node C_2 is a triple point on the curve, the tangents being the nodal rays.

The node B_3 is a quintuple point, one tangent being $X=0$, $3dY+4Z=0$, and the other tangents being given by $Z=0$, $(4ac-3b^2, ad, bd, cd, d^2)(X, Y)^4=0$.

Each of the facultative lines is a double tangent to the curve, or we have $\beta'=6$.

Reciprocal Surface.

100. Comparing the equation of the cubic surface with that for $IV=12-2C_2$, it appears that the equation of $VI=12-B_3-C_2$ is obtained by substituting in that equation the values $\delta=0$, $\gamma=1$. But instead of making this substitution in the final formula, it is convenient to make it in the binary quartic $(*)(X, Y)^4$, thus in fact working out the reciprocal surface by means of the function

$$(xX^2+yXY-wY^2)^2+4zw(a, b, c, d)(X, Y)^3,$$

the coefficients whereof (multiplying by 6 to avoid fractions) are

$$\begin{aligned} &6x^2 &&+24azw,\\ &3xy &&+18bzw,\\ y^2&-2xw &&+12czw,\\ &-3yw &&+6dzw,\\ &6w^2. \end{aligned}$$

We find

$$\tfrac{1}{3}I=L^2-12zwM,$$

$$-J=L^3-18zwLM-54z^2w^2N,$$

where

$$L=y^2+6(x+3cz)w,$$

$$M=2dxy+6(2cx-by+2bdz)w-4aw^2,$$

$$N=-4d^2x^2-8d(3bx-2ay+2adz)w-12(3b^2-4ac)w^2.$$

The equation is

$$\frac{1}{108z^2w^2}\{(L^2-12zwM)^3-(L^3-18zwLM-54z^2w^2N)^2\}=0,$$

viz. it is

$$L^3(LN+M^2)-18zwLMN-16zwM^3-27z^2w^2N^2=0,$$

where however $LN+M^2$ contains the factor w, $=wP$ suppose; the equation thus is

$$L^3P-18zLMN-16zM^3-27z^2wN^2=0.$$

Write

$$A=4x+12cz,$$
$$B=6cx-3by+6bdz-2aw,$$
$$C=6bdx-4ady+4ad^2z+3(3b^2-4ac)w,$$

and therefore

$$L=y^3+Aw,$$
$$M=2dxy+2Bw,$$
$$N=-4d^2x^3-4Cw,$$

then we have

$$P=\frac{1}{w}\{-(y^3+Aw)(4d^2x^3+4Cw)+(2dxy+2Bw)^2\}$$
$$=-4\{Cy^3-2Bdxy+Ad^2x^3+w(AC-B^2)\},$$

or the equation is

$$4L^3\{Cy^3-2Bdxy+Ad^2x^3+w(AC-B^2)\}$$
$$+18zLMN+16zM^3+27z^2wN^2 \qquad =0.$$

101. Consider the section by the plane $w=0$, we have $L=y^3$, $M=2dxy$, $N=-4d^2x^3$, and the equation becomes $4y^4(Cy^3-2Bdxy+Ad^2x^3)+(128-144=)-16d^2x^3y^3z=0$; which substituting for A, B, C the values

$$A=4x+12cz,$$
$$B=6cx-3by+6bdz,$$
$$C=6bdx-4ady+4ad^2z,$$

becomes $16dy^3(y-dz)(dx^3-3cx^2y+3cxy^2-ay^3)=0$; which is in fact the line $w=0$, $y=0$ (reciprocal of the edge) three times, and the lines $w=0$, $(y-dz)(d, -c, b, -a)(x, y)^3=0$ (reciprocals of the biplanar rays) each once. Observe that the edge $(X=0, Z=0)$ is not a line of the cubic surface, but the reciprocal line $y=0, w=0$ presents itself as an oscular line of the reciprocal surface.

102. The equations of the cuspidal curve are in the first instance obtained in the form

$$\begin{Vmatrix} L, & M, & 3N \\ 12zw, & L, & -4M \end{Vmatrix}=0.$$

Consider the two equations

$$L^2-12zwM=0,$$
$$LM+9zwN=0,$$

each of the fourth order, but which are satisfied by $zw=0$, $L=0$; that is, by $(w=0, y^3=0)$, $(z=0, y^3+4xw=0)$. The line $(w=0, y=0)$ however presents itself in the intersection of the two surfaces, not twice only, but 4 times. To show this, observe that the line in question is a nodal line on the surface $L^2-12zwM=0$; in fact, attending only to the terms of the second order in y, w, we find

$$\{(4x+12cz)^2-144cxz-144bdz^2\}w^2-24dxzyw=0,$$

giving the two sheets

$$\{(4x+12cz)^2-144cxz-144bdz^2\}w-24dxzy=0 \text{ and } w=0;$$

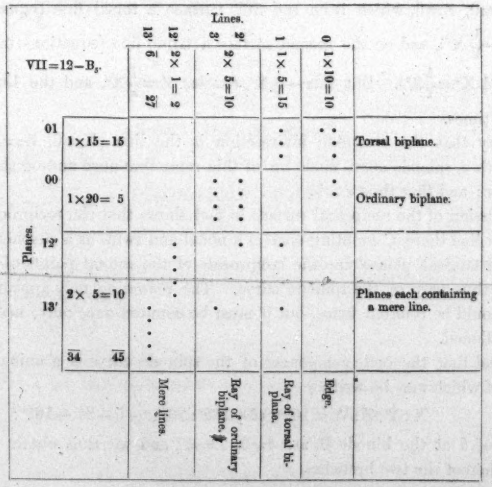
in regard to the last-mentioned sheet the form in the vicinity thereof is given by $w=Ay^3$, viz. we have approximately $L=y^2$, $M=2dxy$, and thence $y^4-12z.Ay^3.2dxy=0$, that is, $A=\frac{1}{24dxz}$ or $w=\frac{1}{24dxz}y^3$; the line is thus a flecnodal line on the surface $L^2-12zwM=0$.

Next as regards the surface $LM+9zwN=0$; the line $y=0, w=0$ is a simple line on the surface, the terms of the lowest order being $9zw(-4d^2x^2)=0$; that is, we have $w=0$, and for a next approximation $w=Ay^3$, viz. $L=y^2$, $M=-2dxy$, $N=-4d^2x^2$, and therefore $-2dxy^3+9z.Ay^3(-4d^2x^2)=0$, that is, $A=-\frac{1}{18dxz}$, or $w=-\frac{1}{18dxz}y^3$; there is thus a three-fold intersection with one sheet and a simple intersection with the other sheet of the surface $L^2-12zwM=0$. The surfaces intersect, as has been mentioned in the conic $z=0, y^2+4xw=0$; or we have the line $y=0, w=0$ four times, the conic once, and a residual cuspidal curve of the order $4.4-4-2, =10$; that is, $c'=10$.

Section VII$=12-B_7$.

Equation $WXZ+Y^2Z+YX^2-Z^3=0$. Article Nos. 103 to 116.

103. The diagram of lines and planes* is



* The marginal symbols in the preceding diagrams constitute a real notation of the lines and planes; but here, and still more so in some of the following diagrams, they are mere marks of reference, showing which are the lines and planes to which the several equations respectively belong.

104. The planes are

		The lines are		
$Z=0$,	[10]	$X=0$,	$Z=0$,	(0)
$X=0$,	[00]	$Y=0$,	$Z=0$,	(1)
$Y+Z=0$,	[12′]	$X=0$,	$Y+Z=0$,	(2′)
$Y-Z=0$,	[13′]	$X=0$,	$Y-Z=0$,	(3′)
		$X-W=0$,	$Y+Z=0$,	(12′)
		$X+W=0$,	$Y-Z=0$,	(13′).

105. The two mere lines are facultative, and the edge is also facultative; $\varrho'=b'=3$; $t'=0$.

106. Hessian surface. The equation is

$$Z(WXZ+Y^2Z+YX^2-Z^2)$$
$$-4X^3YZ+X^4+4Z^4=0.$$

The complete intersection with the surface is thus given by the equations

$$WXZ+Y^2Z+YX^2-Z^3=0,$$
$$-4X^3YZ+X^4+4Z^4=0,$$

which is made up of the line $X=0$, $Z=0$ (the edge) four times and a curve of the eighth order. To see this, observe that the last-mentioned surfaces have in common the line $X=0$, $Z=0$, which is on the first surface a torsal line (equation in vicinity being $Z=-\frac{1}{Y}X^3$), and on the second surface a triple line (equations in vicinity being $Z=\frac{1}{Y}X^3$ and $X^3=\frac{1}{Y}Z^3$). But $Z=-\frac{1}{Y}X^3$ *touches* $Z=\frac{1}{Y}X^3$, and the line counts thus $(2+2=)$ 4 times.

107. I say that the complete intersection is the line $(X=0, Z=0)$ three times together with a spinode curve made up of this same line once and of the curve of the eighth order; and that thus $\sigma'=9$.

The discussion of the reciprocal surface in fact shows that the reciprocal of the edge is a singular line thereof, counting once as a nodal and twice as a cuspidal line thereof; the cuspidal tangent planes are the reciprocals of the several points of the edge, and the edge is thus part of the spinode curve. The reasoning may appear to show that the edge should be counted twice, but it must be counted once only, making the order $=9$ as mentioned.

108. I find that the octic component of the spinode curve is a unicursal curve, the equations of which may be written

$$X:Y:Z:W=16\theta^8:4\theta+16\theta^5:16\theta^3:-5-8\theta^4-16\theta^8;$$

the values of θ at the binode B_3 are $\theta=0$, $\theta=\infty$, and we thus obtain in the neighbourhood thereof the two branches

$$\frac{Y}{W}=-5\left(\frac{Z}{W}\right)^3,\ \frac{X}{W}=\tfrac{25}{4}\left(\frac{Z}{W}\right)^5 \text{ and } \frac{X}{W}=\left(\frac{Z}{W}\right)^{\frac{1}{3}},\ \frac{Y}{W}=-\left(\frac{Z}{W}\right)^2.$$

109. Each of the lines $(X-W=0,\ Y+Z=0)$ and $(X+W=0,\ Y-Z=0)$ is a double tangent of the spinode octic; in fact for the first of these lines we have

$$16\theta^6+8\theta^4+16\theta^3+5=0,\quad 16\theta^6+16\theta^5+4\theta=0,$$

that is,

$$(2\theta^3+1)^2(4\theta^4-4\theta^3+5)=0,\quad 4\theta(2\theta^3+1)^3=0,$$

so that the line touches at the two points given by $2\theta^3+1=0$; and similarly the other line touches at the two points given by $2\theta^3-1=0$.

The edge $X=0$, $Z=0$ has apparently a higher contact with the spinode octic, viz. the equations $X=0$, $Z=0$ are satisfied by $\theta=0$ twice, $\theta=\infty$ five times; but it must be reckoned only as a double tangent. Hence $\beta'=2\,.\,2+2$, $=6$.

Reciprocal Surface.

110. The equation is obtained by equating to zero the discriminant of the binary quartic

$$X^3(yZ-wX)^3+4wZ^3(wZ^3+zZX+xX^3),$$

viz. calling this $(*)(X, Z)^4$, the coefficients (multiplying by 6) are

$$(6w^3,\ -3yw,\ y^2+4xw,\ 6zw,\ 24w^2);$$

and then writing

$$\begin{aligned} L &= \quad y^2+4xw,\\ M &= -2yz-4w^2,\\ N &= -4z^2+16xw, \end{aligned}$$

we have

$$\begin{aligned} \tfrac{1}{8}I &= L^2-12w^2M,\\ -J &= L^3-18w^2LM-54w^4N, \end{aligned}$$

and the equation is, as in former cases,

$$L^2(LN+M^2)-18w^2LMN-16w^3M^3-27w^4N^2=0;$$

but $LN+M^2$ and therefore the whole equation divides by w, and we thus obtain

$$16L^2(-xz^2+y^2x+w(yz+4x^2)+w^2)-18wLMN-16wM^3-27w^2N^2=0;$$

or, completely developed, this is

$$\begin{aligned} &w^7\,.\,64\\ &+w^6\,.\,32\ (\ 3yz-4x^2)\\ &+w^4\,.\,16x(\ 5y^2+9z^2)\\ &+w^3\,.\quad (\ y^4+30y^2z^2+160yzx^2-27z^4+64x^4)\\ &+w^2\,.\ 4x(11y^2z+12y^2x^2-9yz^3-4z^2x^2)\\ &+w\ .\ y^2(\ y^2z+12y^2x^2-yz^3-8z^2x^2)\\ &+\quad y^4x(\ y^2-z^2)=0. \end{aligned}$$

111. To transform the equation so as to put in evidence the nodal curve, I collect the terms according to their degrees in (y, z) and (x, w); viz. the equation thus becomes

$$\begin{aligned}
&64x^4w^3-128x^3w^5+64w^7\\
&+z^2(-16x^3w^3+144xw^4)\\
&+zy(\ 160x^3w^3+\ 96w^5\ )\\
&+y^2(\ \ 48x^3w^3+\ 80xw^4)\\
&+z^4.-27w^3\\
&+z^3y\ .-36xw^3\\
&+z^2y^2.-8x^3w+30w^3\\
&+zy^3.\ 44xw^3\\
&+y^4.\ 12x^3w+w^3\\
&+z^3y^3.-w\\
&+z^2y^4.-x\\
&+zy^5.\ w\\
&+y^6.\ x\qquad\qquad =0.
\end{aligned}$$

And if for a moment we write $z=\alpha+\gamma$, $y=\alpha-\gamma$ and collect, ultimately replacing α, γ by their values $\frac{1}{2}(z+y)$, $\frac{1}{2}(z-y)$, the equation can be expressed in the form

$$\begin{aligned}
&64w^3(x^2-w^2)^2\\
&+\ 8w^3(z\ +y\ )^2(x\ +w\ )^2(x+3w)\\
&+\ 8w^3(z\ -y\ )^2(x\ -w\ )^2(x-3w\ )\\
&-32w^3(z^2-y^2)\,(x^2-w^2)x\\
&+\ \tfrac{1}{4}w\,(z\ +y\ )^4(x\ +w\ )^2\\
&-\ \ w\,(z\ +y\ )^2(z-y)(x+w)(3x+7w)\\
&+\ \tfrac{1}{2}w\,(z^2-y^2)^2(11x^2-27w^2)\\
&-\ \ w\,(z\ +y\ )\,(z-y)^2(x-w)(3x-7w)\\
&+\ \tfrac{1}{4}w\,(z\ -y\ )^4(x-w)^2\\
&-\ \ y^2(z^2-y^2)\,(zw+xy)=0,
\end{aligned}$$

and observing that we have

$$\begin{aligned}
zw+xy&=-z(x-w)+x(z+y)\\
&=\ \ z(x+w)-x(z-y),
\end{aligned}$$

we see that every term of the equation is at least of the second order in $z+y$ and $x-w$ conjointly; and also at least of the second order in $z-y$ and $x+w$ conjointly; that is, the surface has the nodal lines $(z+y=0,\ x-w=0)$ and $(z-y=0,\ x+w=0)$, which are the reciprocals of the lines $12'$ and $13'$ respectively. The nodal curve is made up of

these two lines and of the line $y=0, w=0$ (reciprocal of edge), as will presently appear; so that we have $b'=3$.

112. The equations of the cuspidal curve are

$$\begin{aligned} L^3-12w^2M&=0,\\ LM+\ 9w^3N&=0,\\ 4M^3+\ 3LN\ &=0. \end{aligned}$$

Attending to the two equations

$$L^3-12w^2M=y^4+8y^3xw+16x^2w^2+24yzw^2+48w^4=0,$$
$$LM+9w^3N=y^3z+2y^3w^2+4xyzw+(8-72=)-64xw^3+18z^2w^2=0,$$

these surfaces are each of the order 4, and the order of their intersection is $=16$. But the two surfaces contain in common the line ($y=0$, $w=0$) 7 times; in fact on the first surface this is a cusp-nodal line $4xw+y^2+Ay^{\frac{5}{2}}=0$; and on the second surface it is a nodal line $w(4xy+18zw)=0$; the sheet $w=0$ is more accurately $4xw+y^2+By^3\ldots=0$; whence in the intersection with the first surface the line counts 5 times in respect of the first sheet and 2 times in respect of the second sheet; together $(5+2=)7$ times, and the residual curve is of the order $(16-7=)9$.

113. I say that the cuspidal curve is made up of this curve of the 9th order, and of the line $y=0$, $w=0$ (reciprocal of the edge) once; so that $c'=10$. In fact, considering the line in question $y=0$, $w=0$ in relation to the surface, the equation of the surface (attending only to the lowest terms in y, w) may be written

$$-xz^2(y^2+4xw)^2+w(-y^3z^2)+w^2(-36xyz^2)+\&c.=0,$$

giving in the vicinity of the line

$$4xw+y^2=Ay^{\frac{5}{2}},$$

and then

$$-xz^2A^2+\frac{z^2}{x}\left(\tfrac{1}{4}-\tfrac{36}{16}\right)=0,$$

that is, $A^2=-2\frac{z}{x^2}$ or $4xw+y^2=\sqrt{-2}.\frac{z^{\frac{1}{2}}}{x}y^{\frac{5}{2}}$; wherefore the line is a cusp-nodal line, counting once as a nodal and once as a cuspidal line; and so giving the foregoing results $b'=3$, $c'=10$.

114. I revert to the equation which exhibits the nodal lines $(x-w=0, y+z=0)$, $(x+w=0, y-z=0)$ for the purpose of showing that they have respectively no pinch-points; that is, that in regard to each of them we have $j'=0$. In fact for the first of these lines, neglecting the terms which contain $x-w$, $y+z$ conjointly in an order above the second, the equation may be written

$$\begin{aligned} &64w^2(x+w)^2 \qquad\quad (x-w)^2\\ &+\ 8w^2(x+w)^2(x+3w)(z+y)^2\\ &+\ 8w^2(z-y)^2(x-3w)(x-w)^2 \end{aligned}$$

$$\begin{aligned}
&-32w^3(z-y)(x+w)x \qquad (x-w)(z+y)\\
&+\tfrac{1}{2}w\ (z-y)^3(11x^3-27w^3)(z+y)^3\\
&-\ w\ (z-y)^3(3x-7w) \quad (x-w)(z+y)\\
&+\tfrac{1}{4}w\ (z-y)^4 \qquad (x-w)^3\\
&+y^3z\ (z-y) \qquad (x-w)(z+y)\\
&-y^3x\ (z-y) \qquad (z+y)^3=0,
\end{aligned}$$

viz. this is

$$(A, B, C)(x-w, z+y)^3=0,$$

where, collecting the terms and reducing the values by means of the equations $x-w=0$, $z+y=0$, or say by writing $x=w$, $-y=z$, we have

$$\begin{aligned}
A=\ & 64w^3(x+w)^3 && = 256w^6\\
&+\ 8w^3(z-y)^3(x-3w) && -\ 64w^3z^3\\
&+\ \tfrac{1}{4}w^3(z-y)^4 && +\ 4wz^4\\
& && = 4w(z^3-8w^3)^2,\\
B=\ & -32(z-y)(x+w)xw^3 && = -128w^4z\\
&-w(z-y)^3(3x-7w) && +\ 32w^3z^3\\
&+y^3z(z-y) && -\ 2z^5\\
& && = -2z(z^3-8w^3)^2,\\
C=\ & 8w^3(x+w^3)(x+3w) && = 128w^5\\
&+\tfrac{1}{2}w(z-y)^3(11x^3-27w^3) && -\ 32w^3z^2\\
&-xy^3(z-y) && +\ 2wz^4\\
&-xy^3(z-y) && +\ 2wz^4\\
& && = 2w(z^3-8w^3)^2.
\end{aligned}$$

Hence the condition $4AC-B^2=0$ of a pinch-point is $(z^3-8w^3)^5=0$, so that the pinch-points (if any) would be at the points $x-w=0$, $y+z=0$, $z^3-8w^3=0$; or say at $x, y, z, w=1, -2\sqrt{2}, 2\sqrt{2}, 1$. But these values give L, M, N=12, 12, −16; values which satisfy the equations $L^3-12w^3M=0$, $LM+9w^3N=0$, $4M^3+3LN=0$, and as the points in question are obviously not on the line $y=0$, $w=0$, they lie on the ninthic component of the cuspidal curve, being in fact points β', and not pinch-points.

The line $y=0$, $w=0$ *quà* nodal line would have every point a pinch-point, but being part of the cuspidal curve, no point thereof is regarded as a pinch-point; that is, in regard to this line also we have $j'=0$. And therefore for the entire nodal curve $j'=0$.

115. The cuspidal ninthic curve is a unicursal curve, the equations of which can be very readily obtained by considering it as the reciprocal of the spinode torse; we in fact have

$$x : y : z : w = ZW+2XY : 2YZ+X^2 : WX+Y^2-3Z^2 : ZX,$$

or substituting for X, Y, Z, W their values ($=16\theta^3$, $4\theta+16\theta^6$, $16\theta^3$, $-5-8\theta^4-16\theta^8$) and omitting a common factor $16\theta^3$, we find for the cuspidal curve

$$x : y : z : w = 3\theta+24\theta^5-16\theta^9 : 24\theta^3+32\theta^6 : -4-48\theta^4 : 16\theta^3$$

(values which verify the equation $Xx+Yy+Zz+Ww=0$); the spinode curve being thus of the order $=9$ as mentioned.

For $\theta=\infty$ we have the singular point ($y=0, z=0, w=0$), (reciprocal of torsal biplane), and in the vicinity thereof $x : y : z : w = 1 : -2\theta^{-3} : 3\theta^{-5} : -\theta^{-6}$, therefore

$$\left(\frac{y}{x}\right)^2 = -4\frac{w}{x}, \quad \left(\frac{y}{x}\right)^5 = -\frac{32}{27}\left(\frac{z}{x}\right)^3.$$

For $\theta=0$ we have the singular point $x=0$, $y=0$, $w=0$ (reciprocal of the other biplane), and in the vicinity thereof $x : y : z : w = -\frac{3}{4}\theta : -6\theta^3 : 1 : -4\theta^6$, therefore

$$\frac{y}{z} = -\frac{32}{3}\left(\frac{x}{z}\right)^3, \quad \frac{w}{z} = \frac{256}{27}\left(\frac{x}{z}\right)^3.$$

116. The section of the surface by the plane $z=0$ is an interesting curve. Writing $z=0$ in the equation of the surface, I find that the resulting equation may be written

$$(64w^3,\ 144xw^2,\ w^3+76x^2w+xy^2)(w^3+27x^3,\ y^2-32xw)^2=0,$$

where observe that

$$\begin{aligned} &64w^3(w^3+76x^2w+xy^2)-(72xw^2)^2 \\ &\quad =64w^2[w(w^3+27x^3)+x(y^2-32xw)]; \end{aligned}$$

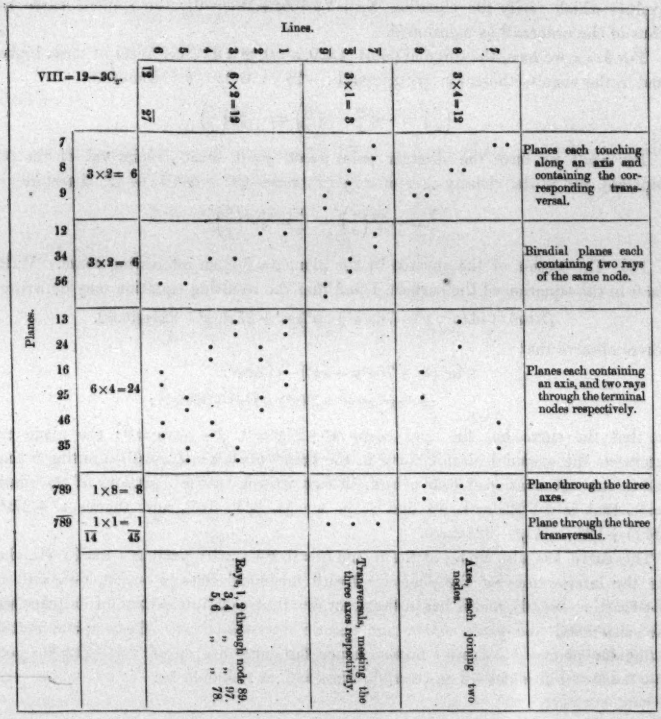
so that the curve has the four cusps $w^3+27x^3=0$, $y^2-32xw=0$; the plane $z=0$ intersects the cuspidal ninthic curve in the point ($y=0, z=0, w=0$) counting 5 times, and in the last-mentioned four points: in fact, writing in the equations of the ninthic $z=0$, that is $1+12\theta^4=0$, we find x, y, $w=\frac{8}{3}\theta$, $\frac{64}{3}\theta^3$, $16\theta^3$, and thence $w^3+27x^3=\frac{64}{3}\theta^3(1+12\theta^4)=0$, $y^2-32xw=0$.

The curve has also nodes at the points ($y=0, x+w=0$; $y=0, x-w=0$), viz. these are the intersections of the plane $z=0$ with the nodal lines ($y-z=0, x+w=0$) and ($y+z=0, x-w=0$), and it has at the point ($y=0, w=0$) (intersection of its plane with the cusp-nodal line $y=0$, $w=0$, and quintic intersection with the cuspidal ninthic) a singular point$=2$ cusps$+7$ nodes; hence the curve has cusps $=(4+2=)6$; nodes $(2+7=)9$; or 2 nodes$+3$ cusps$=36$; class $=6$, as it should be.

Section VIII=12−3C$_\gamma$.

Equation $Y^3+Y(X+Z+W)+4aXZW=0$. Article Nos. 117 to 125.

117. The diagram of the lines and planes is


Lines.
VIII=12−3C$_\gamma$.
6×2=12
3×1=3
3×4=12
Planes.
3×2= 6
3×2= 6
6×4=24
1×8= 8
1×1= 1
Planes each touching along an axis and containing the corresponding transversal.
Biradial planes each containing two rays of the same node.
Planes each containing an axis, and two rays through the terminal nodes respectively.
Plane through the three axes.
Plane through the three transversals.
Rays, 1, 2 through node 89.
3, 4 " " 97.
5, 6 " " 78.
Transversals, meeting the three axes respectively.
Axes, each joining two nodes.

118. Take m_1, m_2 as the roots of the equation $(m-1)^2=4am$, so that $m_1+m_2=2+4a$, $m_1m_2=1$, then the planes are

$X=0,$	[7]
$Y=0,$	[8]
$Z=0,$	[9]
$Y+Z+X=0,$	[12]
$Y+X+W=0,$	[34]
$Y+Z+W=0,$	[56]
$Y=(m_1-1)X,$	[13]
$Y=(m_2-1)X,$	[24]
$Y=(m_2-1)Z,$	[16]
$Y=(m_1-1)Z,$	[25]
$Y=(m_1-1)W,$	[46]
$Y=(m_2-1)W,$	[35]
$Y=0,$	[789]
$Y+X+Z+W=0,$	[789]

119. And the lines are

a	b	c	f	g	h	equations may be written
0	0	0	0	0	1	(7) $X=0, Y=0$
0	0	0	1	0	0	(8) $Z=0, Y=0$
0	1	0	0	0	0	(9) $W=0, Y=0$
1	1	1	0	0	0	($\bar{7}$) $Y+Z+X=0, W=0$
0	0	1	−1	1	0	($\bar{8}$) $Y+X+W=0, Z=0$
1	0	0	0	−1	1	($\bar{9}$) $Y+Z+W=0, X=0$
0	0	0	$\frac{1}{m_1-1}$	1	$\frac{1}{m_2-1}$	(1) $Y=(m_1-1)X=(m_2-1)Z$
0	0	0	$\frac{1}{m_2-1}$	1	$\frac{1}{m_1-1}$	(2) $Y=(m_2-1)X=(m_1-1)Z$
−1	$\frac{1}{m_2-1}$	0	0	0	$\frac{1}{m_2-1}$	(3) $Y=(m_2-1)W=(m_1-1)X$
−1	$\frac{1}{m_2-1}$	0	0	0	$\frac{1}{m_1-1}$	(4) $Y=(m_1-1)W=(m_2-1)X$
0	$-\frac{1}{m_1-1}$	1	$\frac{1}{m_2-1}$	0	0	(5) $Y=(m_1-1)Z=(m_2-1)W$
0	$-\frac{1}{m_2-1}$	1	$\frac{}{m_1-}$	0	0	(6) $Y=(m_2-1)Z=(m_1-1)W$

120. The three transversals are each facultative; $\varrho'=b'=3$; $t'=0$.

121. Hessian surface. The equation is

$$4a\mathrm{XZW}(3\mathrm{Y}+\mathrm{X}+\mathrm{Z}+\mathrm{W})+\mathrm{Y}^2(\mathrm{X}^2+\mathrm{Z}^2+\mathrm{W}^2-2\mathrm{XZ}-2\mathrm{XW}-2\mathrm{ZW})=0.$$

The complete intersection with the cubic surface is made up of the lines (Y=0, X=0), (Y=0, Z=0), (Y=0, W=0) (the axes) each twice, and of a sextic curve which is the spinode curve; $\sigma'=6$.

The spinode curve is a complete intersection 2×3; the equations may in fact be written

$$\mathrm{Y}^3+\mathrm{Y}^2(\mathrm{X}+\mathrm{Z}+\mathrm{W})+4a\mathrm{XZW}=0,$$
$$3\mathrm{Y}^2+4\mathrm{Y}(\mathrm{X}+\mathrm{Z}+\mathrm{W})+4(\mathrm{XZ}+\mathrm{XW}+\mathrm{ZW})=0;$$

the nodes D, C, A are nodes (double points) of the curve, the tangents at each node being the nodal rays.

Each of the transversals is a single tangent of the spinode curve; in fact for the transversal Y+Z+X=0, W=0, these equations of course satisfy the equation of the cubic surface; and substituting in the equation of the Hessian, we have $\mathrm{Y}^2(\mathrm{X}-\mathrm{Z})^2=0$. But Y+Z+X=0, W=0, Y=0 is a point on the axis W=0, Y=0, not belonging to the spinode curve; we have only the point of contact Y+X+Z=0, W=0, X−Z=0. Hence $\beta'=3$.

Reciprocal Surface.

122. The equation is found by means of the binary cubic,

$$a\mathrm{T}(\mathrm{T}-y\mathrm{U})^2+(\mathrm{T}-x\mathrm{U})(\mathrm{T}-z\mathrm{U})(\mathrm{T}-w\mathrm{U}),$$

viz. writing for shortness

$$\beta=x+z+w,$$
$$\gamma=xz+xw+zw,$$
$$\delta=xzw;$$

this is a binary cubic $(*\mathrm{XT}, \mathrm{U})^3$, the coefficients whereof are

$$3(a+1),\ -2ay-\beta,\ ay^2+\gamma,\ -3\delta,$$

and the equation is hence found to be

$$\begin{aligned}&4a^3y^3(y^3-\beta y^2+\gamma y-\delta)\\ &+a^2\{(12\gamma-\beta^2)y^4-(8\beta\gamma+36\delta)y^3+(30\beta\delta+8\gamma^2)y^2-36\gamma\delta y+27\delta^2\}\\ &+2a\{(6\gamma^3-\beta^2\gamma-9\beta\delta)y^3+(12\beta^2\delta-2\beta\gamma^2-18\gamma\delta)y+2\gamma^3+27\delta^2-9\beta\gamma\delta\}\\ &-(\beta^2\gamma^2+18\beta\gamma\delta-4\beta^3\delta-4\gamma^3-27\delta^2)=0;\end{aligned}$$

or substituting for β, γ, δ in the first and last lines their values

$$(=x+z+w,\ xz+xw+zw,\ xzw),$$

this is

$$\begin{aligned}&4a^3y^3(y-x)(y-z)(y-w)\\&+a^2\{(12\gamma-\beta^2)y^4-(8\beta\gamma+36\delta)y^3+(30\beta\delta+8\gamma^2)y^2-36\gamma\delta y+27\delta^2\}\\&+2a\{(6\gamma^2-\beta^2\gamma-9\beta\delta)y^3+(12\beta^2\delta-2\beta\gamma^2-18\gamma\delta)y+2\gamma^3+27\delta^2-9\beta\gamma\delta\}\\&-(x-z)^2(x-w)^2(z-w)^2=0.\end{aligned}$$

123. The nodal curve is made up of the lines $(y=x=z)$, $(y=x=w)$, $(y=z=w)$, reciprocals of the three transversals.

To show this I remark that, writing

$$\begin{aligned}\beta'&=(x-y)+(z-y)+(w-y),\\\gamma'&=(x-y)(z-y)+(x-y)(w-y)+(z-y)(w-y),\\\delta'&=(x-y)(z-y)(w-y),\end{aligned}$$

the equation of the surface may be written

$$\begin{aligned}&4a^3y^3(y-x)(y-z)(y-w)\\&+a^2\{y^2(12\beta'\delta'-\gamma'^2)+x\,.\,18\gamma'\delta'+27\delta'^2\}\\&+2a\{y(-6\beta'^2\delta'+2\beta'\gamma'^2+9\gamma'\delta')+2\gamma'^3+27\delta'^2-9\beta'\gamma'\delta'\}\\&-(x-z)^2(x-w)^2(z-w)^2=0,\end{aligned}$$

whence observing that γ' is of the order 1 and δ' of the order 2 in $(x-y)$, $(z-y)$ conjointly, each term of the equation is at least of the second order in $(x-y)$, $(z-y)$ conjointly; or we have $y=x=z$, a nodal line; and similarly the other two lines are nodal lines.

124. The foregoing transformed equation is most readily obtained by reverting to the cubic in T, U, viz. writing $p=x-y$, $r=z-y$, $s=w-y$, and therefore $x=y+p$, $z=y+r$, $w=y+s$, the cubic function (putting therein $\mathrm{T}=\mathrm{V}+y\mathrm{U}$) becomes

$$a(\mathrm{V}+y\mathrm{U})\mathrm{V}^3+(\mathrm{V}-p\mathrm{U})(\mathrm{V}-r\mathrm{U})(\mathrm{V}-s\mathrm{U});$$

writing $\beta', \gamma', \delta'=p+r+s, pr+ps+rs, prs$, the coefficients are $(3(a+1), ay-\beta', \gamma', -3\delta')$, and the equation of the surface is thus obtained in the form

$$\begin{aligned}&27(a+1)^3\delta'^2\\&+18(a+1)(ay-\beta')\gamma'\delta'\\&+\ 4(a+1)\gamma'^3\\&-\ 4(ay-\beta')^3\delta'\\&-\ (ay-\beta')^2\gamma'^2=0,\end{aligned}$$

which, arranging in powers of a, and reversing the sign, is the foregoing transformed result.

125. The cuspidal curve is given by the equations

$$\begin{Vmatrix} 3(a+1), & -2ay-\beta, & ay^2+\gamma \\ -2ay-\beta, & ay^2+\gamma, & -\delta \end{Vmatrix} =0,$$

or say by the equations

$$3(a+1)(ay^3+\gamma)-(2ay+\beta)^3=0,$$

that is

$$a(a-3)y^3+4a\beta y-3(a+1)\gamma=0,$$

and

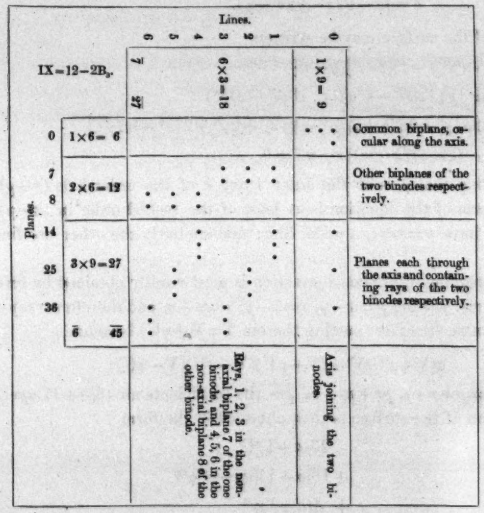
$$-3(a+1)\delta+(2ay+\beta)(ay^3+\gamma)=0,$$

consequently $e=6$. It is to be added that the cuspidal curve is a complete intersection, 2×3.

Section $IX=12-2B_3$.

Equation $WXZ+(a, b, c, d)(X, Y)^3=0$. Article Nos. 126 to 136.

126. The diagram of the lines and planes is


Lines.
6 5 4 3 2 1 0
IX=12−2B3.
7 27
6×3=18
1×9= 9
Planes.
0 1×6= 6
Common biplane, ocular along the axis.
7 8 2×6=12
Other biplanes of the two binodes respectively.
14 25 36 3×9=27
Planes each through the axis and containing rays of the two binodes respectively.
6 45
Rays, 1, 2, 3 in the non-axial biplane 7 of the one binode, and 4, 5, 6 in the non-axial biplane 8 of the other binode.
Axis joining the two binodes.

127. Writing $(a, b, c, d)(X, Y)^3=-d(f_1X-Y)(f_2X-Y)(f_3X-Y)$, the planes are

$$X=0, \quad [0]$$
$$Z=0, \quad [7]$$
$$W=0, \quad [8]$$
$$f_1X-Y=0, \quad [14]$$
$$f_2X-Y=0, \quad [25]$$
$$f_3X-Y=0, \quad [36];$$

and the lines are

$$X=0,\ Y=0, \quad (0)$$
$$f_1X-Y=0,\ Z=0, \quad (1)$$
$$f_2X-Y=0,\ Z=0, \quad (2)$$
$$f_3X-Y=0,\ Z=0, \quad (3)$$
$$f_1X-Y=0,\ W=0, \quad (4)$$
$$f_2X-Y=0,\ W=0, \quad (5)$$
$$f_3X-Y=0,\ W=0, \quad (6)$$

128. There is no facultative line; $\xi'=b'=0$, $t'=0$; and hence also $\beta'=0$.

129. Hessian surface. The equation is

$$X\{ZW(cX+dY)-3X(ac-b^2, ad-bc, bd-c^2)(X, Y)^2\}=0,$$

so that the Hessian breaks up into the plane $X=0$ (axial or common biplane) and into a cubic surface.

The complete intersection of the Hessian with the cubic surface is made up of the line $X=0$, $Y=0$ (the axis) four times; and of a system of four conics, which is the spinode curve; $d'=8$.

In fact combining the equations

$$WXZ+(a, b, c, d)(X, Y)^3=0$$

and

$$ZW(cX+dY)-3X(ac-b^2, ad-bc, bd-c^2)(X, Y)^2=0,$$

these intersect in the axis once, and in a curve of the eighth order which breaks up into four conics; for we can from the two equations deduce

$$(a, b, c, d)(X, Y)^3(cX+dY)+3X^2(ac-b^2, ad-bc, bd-c^2)(X, Y)^2=0,$$

that is,

$$(4ac-3b^2, ad, bd, cd, d^2)(X, Y)^4=0,$$

a system of four planes each intersecting the cubic $XZW+(a, b, c, d)(X, Y)^3=0$ in the axis and a conic; whence, as above, spinode curve is four conics.

It is easy to see that the tangent planes along any conic on the surface pass through a point, and form therefore a quadric cone; hence in particular the spinode torse is made up of the quadric cones which touch the surface along the four conics respectively.

Reciprocal Surface.

130. The equation is obtained by means of the binary cubic

$$X(xX+yY)^3+4zw(a, b, c, d)(X, Y)^3,$$

viz. calling this $(*)(X, Y)^3$ the coefficients are

$$(3x^2+12azw, 2xy+12bzw, y^2+12czw, 12dzw).$$

The equation is found to be

$$\begin{aligned}&432(a^3d^3-6abcd+4ac^3+4b^3d-3b^2c^2)x^3w^3\\&+216[(ad^2-3bcd+2c^3)x^2+(-2acd+4b^2d-2bc^2)xy+(-abd+2ac^2-b^2c)y^2]z^2w^2\\&+9[3d^2x^4-12cdx^3y+(10bd+8c^2)x^2y^2-(4ad+8bc)xy^3+(4ac-b^2)y^4]zw\\&-y^3(dx^3-3cx^2y+3bxy^2-ay^3)=0.\end{aligned}$$

The section by the plane $w=0$ (reciprocal of $B_3=D$) is the line $w=0$, $y=0$ (reciprocal of edge) three times, and the lines $w=0$, $dx^3-3cx^2y+3bxy^2-ay^3=0$ (reciprocals of the biplanar rays). And similarly for the section by the plane $z=0$ (reciprocal of $B_3=C$).

The section by the plane $y=0$ is made up of the lines $(y=0, z=0)$, $(y=0, w=0)$ each once, and of two conics, $y=0$,

$$\begin{aligned}&16(a^3d^3-6abcd+4ac^3+4b^3d-3b^2c^2)z^3w^3\\&+\ 8(ad^2-3bcd+2c^3)x^2zw\\&+\ d^2x^4=0.\end{aligned}$$

131. There is not any nodal curve; $b'=0$.

132. Cuspidal curve. The equations may be written

$$\left\| \begin{matrix} 3x^3+12azw, & 2xy+12bzw, & y^3+12czw \\ 2xy+12bzw, & y^3+12czw, & 12dzw \end{matrix} \right\| =0.$$

Forming the equations

$$\begin{aligned}&(bd-c^2)\,.\,144z^2w^2+2(dxy-cy^2)\,.\,12zw-y^4=0,\\&(ad-bc)\,.\,144z^2w^2+(3dx^2-2cxy-by^2)\,.\,12zw-2xy^3=0,\end{aligned}$$

these are two quartic surfaces having in common the lines $(y=0, w=0)$, $(y=0, z=0)$: attending to the line $(y=0, z=0)$, this is on the second surface, an oscular line, $z=\frac{1}{18dxw}y^3$; on the first surface it is a nodal line, the one tangent plane being $6(bd-c^2)w.z+dxz.y=0$, the other tangent plane being $z=0$, but the line being in regard to this sheet an oscular line, $z=\frac{1}{24dxw}y^3$. Hence in the intersection of the two surfaces the line counts $(1+3=)4$ times; similarly the line $y=0$, $w=0$ counts $(1+3=)4$ times; and there is a residual intersection of the order $(16-4-4=)8$, which is the cuspidal curve; $c'=8$.

133. The cuspidal curve is a system of 4 conics; in fact from the preceding equations written in the forms

$$\begin{aligned}&(bd-c^2,\ 2(dxy-cy^2),\ -y^4)(12zw,\ 1)^2=0,\\&(ad-bc,\ 3dx^2-2cxy-by^2,\ -2xy^3)(12zw,\ 1)^2=0,\end{aligned}$$

eliminating zw, we obtain

$$\left.\begin{array}{l} 3(bd-c^2), \\ 2(-ad^2-3bcd+4c^3), \\ 6(acd+b^2d-2bc^2), \\ 6(bc-ad)b, \\ a^2d-b^3, \end{array}\right\} (x, y)^4=0,$$

which shows that the cuspidal curve lies in 4 planes, and it hence consists of four conics; these are of course the reciprocals of the quadric cones which touch the cubic surface along the four conics which make up the spinode curve.

134. The equation of the surface, attending only to the terms of the second order in y, z, w, is $27d^2x^4zw=0$; it thus appears that the point $y=0, z=0, w=0$ (reciprocal of the plane $X=0$) (which is oscular along the axis joining the two binodes, or BB-axis) is a binode on the reciprocal surface, the biplanes being $z=0, w=0$, viz. these are the planes reciprocal to the binodes ($X=0, Y=0, W=0$) and ($X=0, Y=0, Z=0$) of the cubic surface; we have thus $B'=1$.

It is proper to remark that the binode $y=0, z=0, w=0$ is not on the cuspidal curve, as its being so would probably imply a higher singularity.

135. A simple case, presenting the same singularities as the general one, is when $a=d$, $b=c=0$: to diminish the numerical coefficients assume $a=d=\frac{1}{12}$, the cubic surface is thus $12XZW+X^3+Y^3=0$, and the equation of the reciprocal surface, multiplying it by 4, becomes

$$\begin{array}{l} z^3w^3 \\ +6x^2z^2w^2 \\ +(9x^4-12x^3y)zw \\ -4y^3(x^3-y^3)=0, \end{array}$$

viz. this is the surface

$$\begin{array}{l} 4y^6 \\ -4y^3x(x^3+3zw) \\ +zw(3x^2+zw)^2=0 \end{array}$$

considered in the Memoir "On the Theory of Reciprocal Surfaces." The cuspidal curve is, as there shown, composed of the four conics $y=0, 3x^2+zw=0$ and $y^2-2x^3=0$, $x^2-zw=0$; and it is there shown that the two points ($x=0, y=0, z=0$), ($x=0, y=0, w=0$), each reckoned 8 times, are to be considered as off-points of the reciprocal surface.

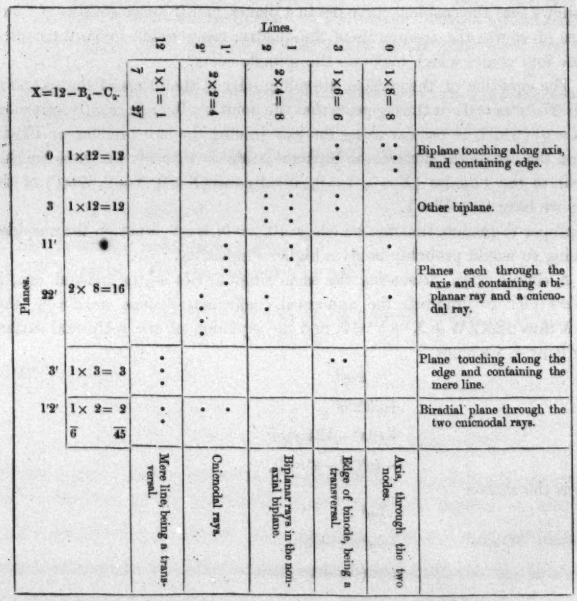
136. The like investigation applies to the general surface, and we have thus $\theta'=16$; the points in question are still the points ($x=0, y=0, z=0$), ($x=0, y=0, w=0$); viz. these are the points of intersection of the surface by the line ($x=0, y=0$), which points are also the common points of intersection of the four conics which compose the cuspidal curve, that is, they are quadruple points on the cuspidal curve; it does not appear

that the points are on this account, viz. *quà* quadruple points of the cuspidal curve, off-points of the surface, nor does this even show that the points should be reckoned each 8 times. As already remarked, the singularity requires a more complete investigation.

Section $X=12-B_4-C_2$.

Equation $WXZ+(X+Z)(Y^2-X^2)=0$. Article Nos. 137 to 143.

137. The diagram of the lines and planes is



138. The planes are and the lines are

$X=0$,	[0]	$X=0$, $Y=0$,	(0)
$Z=0$,	[3]	$X=0$, $Z=0$,	(3)
$X-Y=0$,	[11′]	$X-Y=0$, $Z=0$,	(1)
$X+Y=0$,	[22′]	$X+Y=0$, $Z=0$,	(2)
$W=0$,	[3′]	$X-Y=0$, $W=0$,	(1′)
$X+Z=0$,	[12′]	$X+Y=0$, $W=0$,	(2′)
		$X+Z=0$, $W=0$,	(12)

139. The facultative lines are the edge counting twice, and the mere line;

$$\varepsilon=b'=3;\ t'=1.$$

140. Hessian surface. The equation is

$$X(X+Z)(ZW+3X^2-XZ)+Y^2(X-Z)^2=0.$$

The complete intersection with the surface consists of the line ($X=0$, $Y=0$), the axis, 4 times; the line ($X=0$, $Z=0$), the edge, 2 times; and a sextic curve, which is the spinode curve; $d'=6$.

Writing the equations of the surface and the Hessian in the form

$$X(ZW+Y^2)-X^3+Z(Y^2-X^2)=0,$$

$$X(X+Z)(ZW+Y^2)+(Z-3X)\{-X^3+Z(Y^2-X^2)\}=0,$$

we see that the equations of the spinode curve may be written

$$ZW+Y^2=0,$$

$$-X^3+Z(Y^2-X^2)=0,$$

viz. the curve is a complete intersection, 2×3.

There is at B_4 a triple point $\frac{Y}{W}=-\left(\frac{Z}{W}\right)^2$, $\frac{X}{W}=-\left(\frac{Z}{W}\right)^3$; and at C_2 a double point, the tangents coinciding with the nodal rays $W=0$, $Y^2-X^2=0$.

The edge and the mere line are each of them single tangents of the spinode curve. But the edge counting twice in the nodal curve, its contact with the spinode curve will also count twice, that is, we have $\beta'=2.1+1$, $=3$.

Reciprocal Surface.

141. The equation is obtained by means of the binary cubic

$$4w^2X(X+Z)^2+4wZ(X+Z)(xX+zZ)+y^2XZ^2;$$

or calling this $(*)(X, Z)^3$, the coefficients are

$$(12w^2,\ 8w^2+4wx,\ 4w^2+4wx+4wz+y^2,\ 12wz),$$

and thence the equation is found to be

$$\begin{aligned}
&16w^4[y^2-(x-z)^2]\\
&+16w^3[2x-5z)y^2-2(x-2z)(x-z)^2]\\
&+\ 8w^2[y^4+(x^2-xz+6z^2)y^2-2x^2(x-z)^2]\\
&+\ 4w[(2x+3z)y^4-2x^2(x+z)y^2],\\
&+\ y^4(y^2-x^2)=0,
\end{aligned}$$

where the section by the plane $w=0$ (reciprocal of binode) is $y^4(y^2-x^2)=0$, viz. this is the line $w=0$, $y=0$ (reciprocal of the edge) four times, and the lines $w=0$, $y^2-x^2=0$ (reciprocals of the biplanar rays).

The section by the plane $z=0$ is found to be $(y^2-x^2)(y^2+4xw+4w^2)^2=0$, viz. this

is the two lines $z=0$, $y^2-x^2=0$ (reciprocals of the nodal rays), and the conic $z=0$, $y^2+4xw+4w^2=0$ (reciprocal of the nodal cone $WX+Y^2-X^2=0$) twice.

142. Nodal curve. The equation shows that the line $y=0$, $x-z=0$ (reciprocal of the line $W=0$, $X+Z=0$) is a nodal line on the surface.

It also shows that the line $y=0$, $w=0$ (reciprocal of the edge) is a tacnodal line ($=2$ nodal lines) on the surface; in fact attending only to the lowest terms in y, w, we have

$$-x^2[16(x-z)^2w^2+8(x+z)wy^2+y^4]=0,$$

that is,

$$4(x-z)w+\frac{\sqrt{x}\pm\sqrt{z}}{\sqrt{x}\mp\sqrt{z}}y^2=0,$$

two values, $w=Ay^2$, $w=By^2$, which indicates a tacnodal line.

The nodal curve is thus made up of the line $y=0$, $x-z=0$ once, and the line $y=0$, $w=0$ twice; $b'=3$.

143. Cuspidal curve. The equations

$$\left\| \begin{matrix} 12w^2 & 8w^3+4wx, & 4w^2+4wx+4wz+y^2 \\ 8w^3+4wx, & 4w^2+4wx+4wz+y^2, & 12wz \end{matrix} \right\|=0$$

give

$$(4w+2x)^2-3(4w^2+4wx+4wz+y^2)=0,$$
$$-36w^2z+(2w+x)(4w^2+4wx+4wz+y^2)=0,$$

or, as these are more simply written,

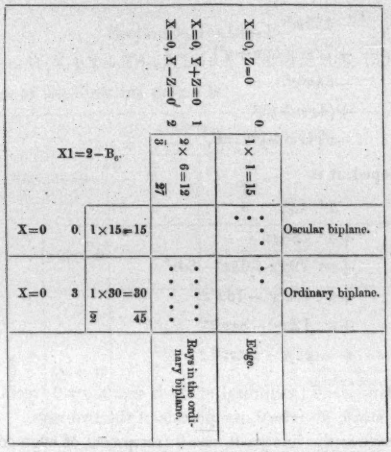
$$4w^2+4wx-12wz+4x^2-3y^2=0,$$
$$8w^3+12w^2x-28w^2z+w(4x^2+4xz+2y^2)+xy^2=0,$$

so that the cuspidal curve is a complete intersection 2×3; $c'=6$.

Section $XI=12-B_6$.

Equation $WXZ+Y^2Z+X^3-Z^3=0$. Article Nos. 144 to 149.

144. The diagram of the lines and planes is



where the equations of the lines and planes are shown in the margins of the diagram.

145. The edge is a facultative line counting 3 times; this will appear from the discussion of the reciprocal surface. Therefore $\xi'=b'=3$; $t'=1$.

146. Hessian surface. This is

$$Z(WXZ+Y^2Z-3X^3-3Z^3)=0,$$

breaking up into $Z=0$, the oscular biplane, and into a cubic surface (itself a surface $XI=12-B_6$). The complete intersection with the cubic surface is made up of the line $X=0$, $Z=0$ (the edge) six times, and of a residual sextic ($=3$ conics), which is the spinode curve; $d'=6$.

The equations of the sextic are in fact $XZ+Y^2=0$, $X^3+Z^3=0$, so that this consists of three conics, each in a plane passing through the edge.

The edge touches each of the three conics at the point $X=0$, $Z=0$, $Y=0$; but it must be reckoned as a single tangent of the spinode curve, and then counting it three times, $\beta'=3$.

Reciprocal Surface.

147. The equation is obtained by means of the binary cubic

$$(12w^2,\ 4zw,\ y^2+4xw,\ -12w^2X)(Z,\ X)^3,$$

viz. it is

$$\begin{aligned}&432w^6\\ &+72w^3z(4xw+y^2)\\ &-64w^3z^3\\ &+(4xw+y^2)^3\\ &-z^2(4xw+y^2)^2=0,\end{aligned}$$

or, completely developed, it is

$$\begin{aligned}&w^6.\ 432\\ &+w^4.\ 288xz\\ &+w^3.\ 72y^2z+64x^3-64z^3\\ &+w^2.\ 48x^2y^2-16x^3z^2\\ &+w\ .\ 12xy^4-8xy^2z^2\\ &+\quad y^4(y^2-z^2)=0;\end{aligned}$$

the section by the plane $w=0$ (reciprocal of B_6) is $w=0$, $y=0$ (reciprocal of edge) four times, together with $w=0$, $y^2-z^2=0$, reciprocals of the two rays.

148. The nodal curve is the line $y=0$, $w=0$ (reciprocal of edge counting as 3 lines); $b'=3$. In fact the form of the surface in the vicinity is given by $w=-\frac{1}{4x}y^2\pm\frac{1}{4}\sqrt{\frac{z}{x^3}}\,y^3$, viz. there are two sheets osculating along the line in question, that is intersecting in this line taken 3 times.

149. For the cuspidal curve we have

$$\begin{Vmatrix} 12w^2,\ 4zw, & y^2+4xw \\ 4zw,\ y^2+4xw, & -12w^2 \end{Vmatrix}=0,$$

giving

$$\begin{aligned}12xw+3y^2\ &-4z^2=0,\\ 36w^2\ &+4wxz+y^2z=0;\end{aligned}$$

or multiplying the first by $3z$ and subtracting the second, we have $108w^3+4z^3=0$. Hence the equations are

$$\begin{aligned}z^3+27w^2&=0,\\ 12xw+3y^2-4z^2&=0,\end{aligned}$$

viz. the cuspidal curve is made up of three conics lying in planes through the line $z=0$, $w=0$.

The curve may be put in evidence by writing the equation of the surface in the form

$$(3y^2+5z^2+12xw,\ 24z,\ 16(3y^2-4z^2+12xw,\ z^2+27w^2)^2=0,$$

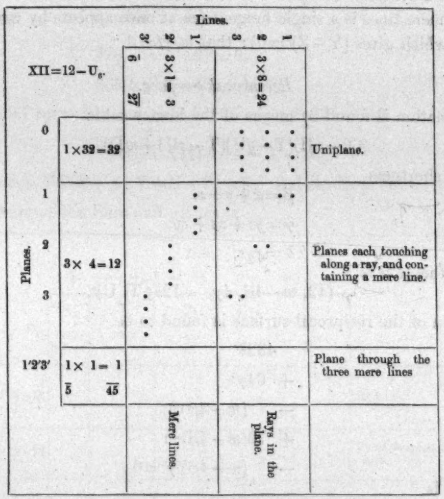
where

$$16(3y^2+5z^2+12xw)-144z^2=16(3y^2-4z^2+12xw).$$

Section XII$=12-U_6$.

Equation $W(X+Y+Z)^2+XYZ=0$. Article Nos. 150 to 156.

150. The diagram of the lines and planes is



151. The planes are

		The lines are	
$X+Y+Z=0$,	[0]	$X=0,\ Y+Z=0,$	(1)
$X=0,$	[1]	$Y=0,\ Z+X=0,$	(2)
$Y=0,$	[2]	$Z=0,\ X+Y=0,$	(3)
$Z=0,$	[3]	$X=0,\ W=0,$	(1′)
$W=0,$	[′2′3′]	$Y=0,\ W=0,$	(2′)
		$Z=0,\ W=0,$	3′)

152. The three mere lines are each facultative: $\varrho'=b'=3$; $t'=1$.

153. Hessian surface. The equation is

$$(X+Y+Z)^2(X^2+Y^2+Z^2-2YZ-2ZX-2XY)=0,$$

viz. the surface consists of the uniplane $X+Y+Z=0$ twice, and of a quadric cone having its vertex at U_6, and touching each of the planes $X=0$, $Y=0$, $Z=0$.

The complete intersection with the cubic surface is made up of the rays each twice and of a residual sextic, which is the spinode curve; $\sigma'=6$.

The equations of the spinode curve are

$$W(X+Y+Z)^2+XYZ=0,$$
$$X^2+Y^2+Z^2-2YZ-2ZX-2XY=0,$$

viz. the curve is a complete intersection, 2×3.

Each of the mere lines is a single tangent (as at once appears by writing for instance $W=0$, $X=0$, which gives $(Y-Z)^2=0$); that is, $\beta'=3$.

Reciprocal Surface.

154. The equation is found by means of the binary cubic

$$4(T-xU)(T-yU)(T-zU)+wT^2U,$$

viz. writing for shortness

$$\beta=x+y+z,$$
$$\gamma=yz+zx+xy,$$
$$\delta=xyz,$$

then the cubic function is

$$(12,\ w-4\beta,\ 4\gamma,\ -12\delta)(T,\ U)^3,$$

and the equation of the reciprocal surface is found to be

$$\begin{aligned} &432\delta^2\\ &+\ 64\gamma^3\\ &-\quad (w-4\beta)^3\delta\\ &+\ 72(w-4\beta)\gamma\delta\\ &-\quad (w-4\beta)^2\gamma^2=0; \end{aligned}$$

expanding, this is

$$\begin{aligned} &w^3\ .-\delta\\ &+w^2\ .12\beta\delta-\gamma^2\\ &+8w\ .-6\beta^2\delta+\beta\gamma^2+9\gamma\delta\\ &+\quad 16(4\beta^3\delta-\beta^2\gamma^2-18\beta\gamma\delta+4\gamma^3+27\delta^2)=0; \end{aligned}$$

or substituting for β, γ, δ in the first and last lines, this is

$$\begin{aligned} &w^3\ .-xyz\\ &+w^2\ .(12\beta\delta-\gamma^2)\\ &+8w\ .-6\beta^2\delta+\beta\gamma^2+9\gamma\delta\\ &+\quad 16(y-z)^2(z-x)^2(x-y)^2=0 \end{aligned}$$

(where β, γ, $\delta=x+y+z$, $yz+zx+xy$, xyz). The section by the plane $w=0$ (reciprocal

of the unode) is made up of the lines $w=0, y-z=0$; $w=0, z-x=0$; $w=0, x-y=0$ (reciprocals of the rays) each twice.

155. The nodal curve is at once seen to consist of the lines $(y=0, z=0)$, $(z=0, x=0)$, $(x=0, y=0)$, reciprocals of the facultative lines; in fact, in regard to (y, z) conjointly γ is of the order 1, and δ is of the order 2; hence every term of the equation is of the order 2 in y, z; and the like as to the other two lines: $b'=3$ as above.

156. For the cuspidal curve we have

$$\left\| \begin{matrix} 12\ , & w-4\beta, & 4\gamma \\ w-4\beta, & 4\gamma\ , & -12\delta \end{matrix} \right\| = 0,$$

or say

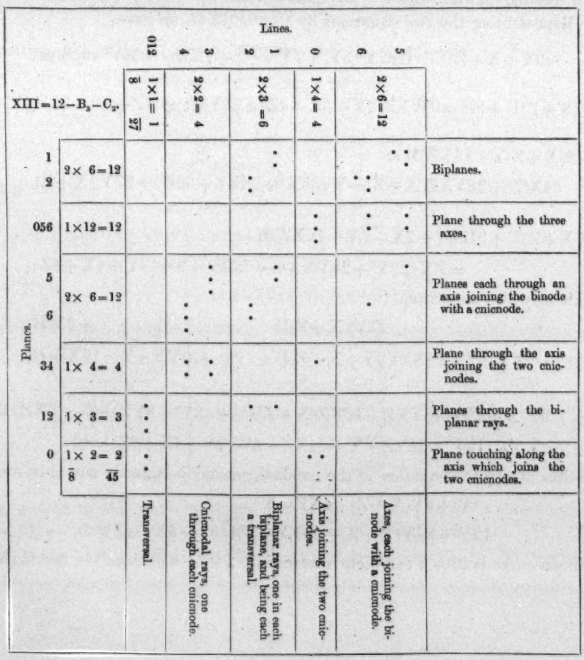
$$48\gamma-(w-4\beta)^2=0,$$
$$36\delta+\gamma(w-4\beta)=0,$$

whence the cuspidal curve is a complete intersection 2×3; $c'=6$.

Section XIII$=12-B_3-2C_2$.

Equation $WXZ+Y^2(Y+X+Z)=0$. Article Nos. 157 to 164.

157. The diagram of the lines and planes is



158. The planes are

		The lines are	
$X=0$,	[1]	$X=0$, $Y=0$,	(5)
$Z=0$,	[2]	$Z=0$, $Y=0$,	(6)
$Y=0$,	[056]	$Y=0$, $W=0$,	(0)
$Y+X=0$,	[5]	$X=0$, $Y+Z=0$,	(1)
$Y+Z=0$,	[6]	$Z=0$, $Y+X=0$,	(2)
$Y-W=0$,	[34]	$W=Y=-Z$,	(3)
$X+Y+Z=0$,	[12]	$W=Y=-X$,	(4)
$W=0$,	[0]	$W=0$; $X+Y+Z=0$,	(012)

159. The transversal is facultative; $\xi'=\eta'=1$, $\zeta'=0$.

160. The Hessian surface is

$$WXZ(3Y+X+Z)+Y^2(Z-X)^2=0.$$

The complete intersection with the surface is made up of the line $Y=0$, $X=0$ (CB-axis) 3 times; the line $Y=0$, $Z=0$ (CB-axis) 3 times; line $Y=0$, $W=0$ (CC-axis) twice, and of a residual quartic, which is the spinode curve; $\sigma'=4$.

161. Representing the two equations by $U=0$, $H=0$, we have

$$(3Y+X+Z)U-H=Y^2(3Y^2+4Y\overline{X+Z}+4XZ), \quad =MY^2 \text{ suppose},$$

and

$$27(X+Z)U+9H=9WXZ(3Y+4X+4Z)+36Y^2(X^2+XZ+Z^2)+27Y^2(X+Z);$$

but

$$(-9(X+Z)Y+16XZ)M=$$
$$64X^2Z^2+28YXZ(X+Z)-Y^2(36X^2+28XZ+36Z^2)-27Y^2(X+Z),$$

whence

$$27(X+Z)U+9H+(-9\overline{X+Z}Y+16XZ)M$$
$$=ZX\{12Y^2+28Y\overline{X+Z}+64XZ+9W(3Y+4X+4Z)\};$$

or, as this may also be written,

$$27Y^2(X+Z)U \qquad +9Y^2H$$
$$+(-9Y\overline{X+Z}+16XZ)(3Y+X+Z)U \qquad +(9Y\overline{X+Z}-16XZ)H,$$

that is,

$$\{-9Y(X+Z)^2+48YXZ+16XZ(X+Z)\}U+\{9Y^2+9Y\overline{X+Z}-16XZ\}H$$
$$=Y^2ZX\{12Y^2+28Y\overline{Z+X}+64XZ+9W(3Y+4X+4Z)\}=0;$$

and we thus obtain the equation of the residual quartic, or spinode curve, in the form

$$3Y^2+4Y(X+Z)+4XZ=0,$$
$$12Y^2+28Y(X+Z)+64XZ+9W(3Y+4X+4Z)=0.$$

The spinode curve is thus a complete intersection, 2×2; and since the first surface is a

cone having its vertex on the second surface, we see moreover that the spinode curve is a nodal quadriquadric. Instead of the last equation we may write more simply

$$4Y(X+Z)+16XZ+3W(3Y+4X+4Z)=0.$$

The equations of the transversal are $W=0$, $X+Y+Z=0$, and substituting in the equations of the spinode curve we obtain from each equation $(X-Z)^3=0$, that is, the transversal is a single tangent of the spinode curve; $\beta'=1$.

Reciprocal Surface.

162. The equation of the cubic is derived from that belonging to $VI=12-B_3-C_2$ by writing therein $a=b=0$, $c=\frac{1}{3}$, $d=1$. Making this change in the formulæ for the reciprocal surface of the case just referred to, we have

$$\begin{aligned} L &= y^3+4(x+z)w, \\ M &= 2x(y+2w), \\ N &= -4x^3, \\ P &= 16x^3(y+w-x-z); \end{aligned}$$

and substituting in the equation

$$L^2P+8zM^3-9zLMN-27z^3wN^3=0,$$

the equation divides by x^3; or throwing this out, the equation is

$$\begin{aligned} &(y^3+4xw+4zw)^2(y+w-x-z) \\ &\quad -8xz(y+2w)^3 \\ &\quad +9xz(y^3+4xw+4zw)(y+2w) \\ &\quad -27x^2z^2w=0; \end{aligned}$$

reducing, this is

$$\begin{aligned} &w^3 \cdot 16(x-z)^2 \\ &+w^2\left\{\begin{array}{l} y^3(x+z) \\ +2y(x^2-4xz+z^2) \\ +(x+z)(2x-z)(-x+2z) \end{array}\right\} \\ &+w\left\{\begin{array}{l} y^4 \\ +8y^2(x+z) \\ -2y^2(4x^2+23xz+4z^2) \\ +36xyz(x+z) \\ -27x^2z^2 \end{array}\right\} \\ &+ y^2(y-x)(y-z)=0. \end{aligned}$$

The section by the plane $w=0$ (reciprocal of B_3) is $w=0$, $y=0$ (the edge) 3 times; and $w=0$, $y-x=0$; $w=0$, $y-z=0$ (reciprocals of the CB-axes).

163. Nodal curve. This is the line $y=x=z$; wherefore $b'=1$. To put the line in

evidence, write for a moment $x=y+\alpha$, $z=y+\gamma$, then the equation is readily converted into

$$\begin{aligned} & w^3 \,.\, 16(\alpha-\gamma)^3 \\ +w^2 & \left\{\begin{matrix} -y(\alpha^3-4\alpha\gamma+\gamma^3) \\ +(\alpha+\gamma)(2\alpha-\gamma)(-\alpha+2\gamma) \end{matrix}\right\} \\ +w & \left\{\begin{matrix} y^2(\alpha^3-10\alpha\gamma+\gamma^3) \\ -18y\alpha\gamma(\alpha+\gamma) \\ -27\alpha^2\gamma^2 \end{matrix}\right\} \\ & + y^3\alpha\gamma=0, \end{aligned}$$

which, each term being of the second order in α, $\gamma(=x-y, z-y)$ respectively, exhibits the nodal line in question.

164. Cuspidal curve. Multiplying by 27, the equation may be written

$$(7y-3x-3z-5w,\ -y+6w,\ -w^2)(y^2+16yw-12\overline{x+z}w+16w^2,$$
$$-20y^2+24y\overline{x+z}-27xz-8yw+16w^2)^2=0,$$

where

$$4w(7y-3x-3z-5w)+(-y+6w)^2=y^2+16yw-12(x+z)w+16w^2;$$

and we have thus in evidence the cuspidal curve,

$$y^2+16yw-12(x+z)w+16w^2=0,$$
$$-20y^2+24y(x+z)-27xz-8yw+16w^2=0,$$

which is a complete intersection, 2×2, or quadriquadric curve; $c'=4$.

Section XIV$=12-B_2-C_2$.

Equation $WXZ+Y^2Z+YX^2=0$. Article Nos. 165 to 171.

165. The diagram of the lines and planes is

XIV$=12-B_2-C_2$.			W=0, Y=0	Z=0, Y=0	X=0, Z=0	X=0, Y=0	Lines are
			3	2	1	0	
			$1\times 2=\frac{2}{27}$	$1\times 5=5$	$1\times 10=10$	$1\times 10=10$	
Planes are							
$Z=0$	12	$1\times 15=15$		• • •	• • •		Torsal biplane.
$X=0$	01	$1\times 20=20$			• •	• • • • •	Ordinary biplane.
$Y=0$	023	$1\times 10=10$ $\quad 5\quad\quad 45$	• • • • •	• •		•	Plane through axis and the two rays.
			Cuinodal ray.	Biplanar ray in torsal biplane.	Edge.	Axis.	

where the equations of the planes and lines are shown in the margins.

166. The edge is a facultative line, as will appear from the discussion of the reciprocal surface: $g'=b'=1$; $t'=0$.

167. Hessian surface. The equation is

$$WXZ^2+Y^2Z^2-3X^2YZ+X^4=0.$$

The complete intersection with the surface is made up of the line $X=0$, $Y=0$ (the axis) 5 times, the line $X=0$, $Z=0$ (the edge) 4 times, and a skew cubic, the equations of which may be written

$$\begin{Vmatrix} X, & Y, & W \\ 4Z, & X, & -5Y \end{Vmatrix} =0.$$

In fact from the equations $U=0$, $H=0$ we deduce $H-ZU=X^2(X^2-4YZ)=0$; and if in $U=0$ we write $X^2=4YZ$, it becomes $Z(XW+5Y^2)=0$; and then in $5U=0$, writing $5Y^2=-XW$, we have

$$5WXZ+Z(-XW)+5X^2Y=X(5XY+4ZW)=0.$$

168. I say that the spinode curve is made up of the edge $X=0$, $Z=0$ once, and of the cubic curve; and therefore $\sigma'=4$.

In fact in the reciprocal surface the cuspidal curve is made up of the skew cubic, and of a line the reciprocal of the axis, being a cusp-nodal line, and so counting once as part of the cuspidal curve: the pencil of planes through the line is thus part of the cuspidal torse; and reverting to the original cubic surface, we have the axis as part of the spinode curve: I assume that it counts *once*.

The edge is a single tangent of the spinode curve; $\beta'=1$.

Reciprocal Surface.

169. The equation is obtained by means of the binary cubic

$$4wZ^2(Xx+Zz)+X(YZ-wX)^2,$$

or, as this may be written,

$$(3w^2,\ -2yw,\ y^2+4xw,\ 12zw)(X,\ Z)^3.$$

The equation is in the first instance obtained in the form

$$\begin{aligned} & 108w^6z^2 \\ - & \ 32w^4y^3z \\ + & \ 36w^4yz(y^2+4xw) \\ + & \ w^2(y^2+4xw)^2 \\ - & \ w^2y^2(y^2+4xw)^2=0; \end{aligned}$$

but the last two terms being together $=4w^2x(y^2+4xw)^2$, the whole divides by $4w^2$, and it then becomes

$$\begin{aligned} & 27w^2z^2 \\ - & \ 8wy^3z \\ + & \ 9wyz(y^2+4xw) \\ + & \ x(y^2+4xw)^2=0; \end{aligned}$$

or, expanding, it is

$$\begin{aligned} & w^2\,.\,27z^2 \\ + & w^2\,.\,36xyz+16x^3 \\ + & w\ .\ y^3z+\ 8x^2y^2 \\ + & \ xy^4=0. \end{aligned}$$

The section by the plane $w=0$ (reciprocal of B_5) is $w=0$, $y=0$ (reciprocal of edge) 4 times, together with $w=0$, $x=0$ (reciprocal of biplanar ray).

The section by the plane $z=0$ (reciprocal of C_3) is $x(y^2+4xw)^2=0$, viz. this is $z=0$, $y^2+4xw=0$ (reciprocal of nodal cone) twice, together with $z=0$, $x=0$ (reciprocal of nodal ray).

170. Nodal curve. This is the line $w=0$, $y=0$, reciprocal of edge. The equation in the vicinity is $y=-\frac{1}{4x}w\pm\sqrt{-\frac{z}{8x^4}}w^{\frac{5}{2}}$, showing that the line is a cusp-nodal line counting once in the nodal and once in the cuspidal curve: wherefore $b'=1$.

171. Cuspidal curve. The equation of the surface may be written

$$(x, -y, 3w)(12xw-y^2, 9zw+8xy)^2=0,$$

where $4x \cdot 3w-y^2=12xw-y^2$. This exhibits the cuspidal curve $12xw-y^2=0$, $9zw+8xy=0$, breaking up into the line $w=0$, $y=0$ (reciprocal of edge) and a skew cubic; the line is really part of the cuspidal curve, or $c'=4$.

The equations of the cuspidal cubic may be written in the more complete form

$$\left\| \begin{matrix} 12x, & y, & z \\ y, & w, & -8x \end{matrix} \right\| =0.$$

Section $XV=12-U_7$.

Equation $WX^2+XZ^2+Y^2Z=0$. Article Nos. 172 to 176.

172. The diagram of the lines and planes is

$XV=12-U_7$.			$Z=0, W=0$	$X=0, Z=0$	$X=0, Y=0$	Lines are
			3	2	1	
			$1\times 1=1$	$1\times 10=10$	$1\times 16=16$	
			3 27			
Planes are $X=0$	21	$1\times 40=40$		• • • •	• • • • • •	Uniplane.
$Z=0$	23	$1\times 5=5$	• • • • •	• •		Plane touching along the single ray.
		3 45				
			Mere line.	Single ray.	Torsal ray.	

where the equations of the lines and planes are shown in the margins.

173. The mere line is facultative: $g'=b'=1$; $t'=0$.

174. The Hessian surface is

$$X^2(XZ-Y^2)=0,$$

viz. this is the uniplane $X=0$ twice, and a quadric cone having its vertex at U_7.

The complete intersection with the surface is made up of $X=0$, $Y=0$ (torsal ray)

6 times; $X=0$, $Z=0$ (single ray) 2 times; and of a residual quartic, which is the spinode curve; $\sigma'=4$.

The equations of the spinode curve are $XZ-Y^2=0$, $XW+2Z^2=0$; the first surface is a cone having its vertex on the second surface; and the curve is thus a nodal quadri-quadric.

The mere line is a single tangent of the spinode curve; $\beta'=1$.

Reciprocal Surface.

175. The equation is obtained by means of the binary cubic

$$(-3y^3,\ 2yz,\ 4xw,\ 6yw)(X,\ Y)^3,$$

viz. throwing out the factor y, the equation is

$$w^3(-64x^3)+w(-16x^2z^3+72xy^2z+27y^4)+16y^2z^3=0.$$

The section by the plane $w=0$ (reciprocal of U_7) is $w=0$, $z=0$ (reciprocal of torsal ray) three times, and $w=0$, $y=0$ (reciprocal of single ray) twice.

Nodal curve. This is the line $x=0$, $y=0$, reciprocal of the mere line: $b'=1$.

Cuspidal curve. The equation of the surface may be written

$$(64x,\ -16z,\ -3w)(z^3+3xw,\ 9y^2+4zx)^3=0,$$

where

$$4.64x(-3w)-256z^3=-256(z^3+3xw).$$

This exhibits the cuspidal curve $z^3+3xw=0$, $9y^2+4zx=0$, where the surfaces are each of them cones; the vertex of the second cone is on the first cone, and the two cones have at this point a common tangent plane; the curve is thus a cuspidal quadriquadric.

176. [The equation

$$(64x,\ -16z,\ -3w)(z^3+3xw,\ 9y^2+4zx)^3=0$$

resembles that of a quintic torse, viz. the equation of a quintic torse is

$$(\ \ x,\ -4z,\ \ 8w)(z^3-2wx,\ \ y^2-2zx)^3=0,$$

which equation, writing $9y$ for y, $-2x$ for x, and $\frac{3}{4}w$ for w, becomes

$$(-2x,\ -4z,\ \ 6w)(z^3+3xw,\ 9y^2+4zx)^3=0,$$

or, what is the same thing,

$$(\ \ x,\ \ \ 2z,\ -3w)(z^3+3xw,\ 9y^2+4zx)^3=0;$$

and developing, this is

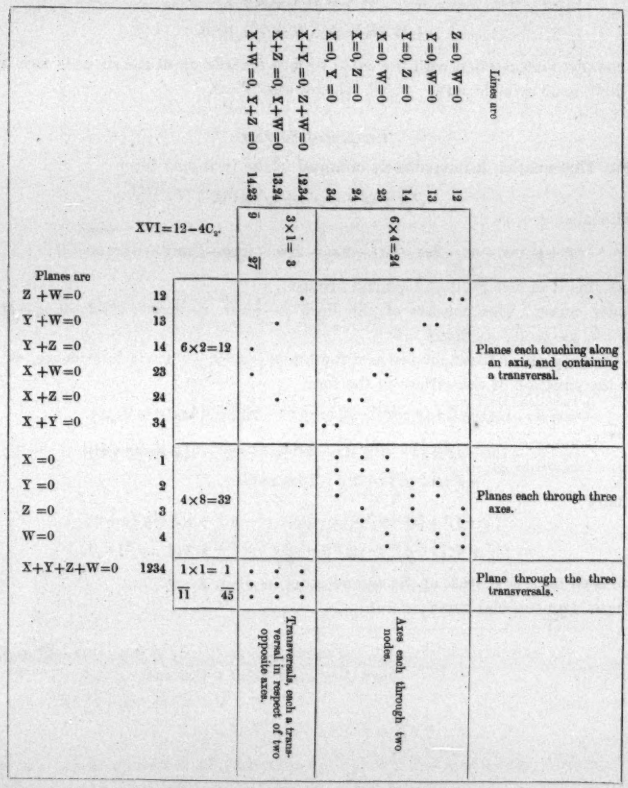
$$\begin{aligned} &x^3.\ w^3\\ &+x^2.\ -2z^3w\\ &+x.\ -18y^2zw+z^4\\ &\quad -27y^4w+2y^2z^3=0, \end{aligned}$$

which, however, differs from the equation of the reciprocal surface, not only in the numerical coefficients, but by the presence of a term xz^4.]

Section XVI=12−4C$_2$.

Equation W(XY+XZ+YZ)+XYZ=0. Article No. 177 to 180.

177. The diagram of the lines and planes is



where the equations of the lines and planes are shown in the margins.

178. The transversals are each facultative: $\varrho'=b'=3$; $t'=1$.

179. Hessian surface. The equation is

$$4XYZW-(X+Y+Z+W)(WXY+WXZ+WYZ+XYZ)=0,$$

or, what is the same thing,

$$\begin{aligned} &X^2(YZ+YW+ZW)\\ +&Y^2(ZW+ZX+WX)\\ +&Z^2(WX+WY+XY)\\ +&W^2(XY+XZ+YZ)=0. \end{aligned}$$

The complete intersection with the cubic surface is made up of the six axes each twice, and there is no spinode curve; $\sigma'=0$, whence also $\beta'=0$.

Reciprocal Surface.

180. The equation is immediately obtained in the irrational form

$$\sqrt{x}+\sqrt{y}+\sqrt{z}+\sqrt{w}=0,$$

or rationalizing, it is

$$(x^2+y^2+z^2+w^2-2yz-2zx-2xy-2xw-2yw-2zw)^2-64xyzw=0;$$

so that this is in fact STEINER'S quartic surface.

Nodal curve. This consists of the lines $x-y=0$, $z-w=0$; $x-z=0$, $y-w=0$; $x-w=0$, $y-z=0$; so that $b'=3$.

To put any one of these, for instance the line $x-y=0$, $z-w=0$, in evidence, we may write the equation of the surface in the form

$$[(x-y)^2+(z-w)^2-2(x+y)(z+w)]^2-64xyzw=0,$$

that is

$$\begin{aligned} &\{(x-y)^2+(z-w)^2\}\{(x-y)^2+(z-w)^2-4(x+y)(z+w)\}\\ &\quad+4[(x+y)^2(z+w)^2-16xyzw]=0, \end{aligned}$$

or finally

$$\begin{aligned} &\{(x-y)^2+(z-w)^2\}\{(x-y)^2+(z-w)^2-4(x+y)(z+w)\}\\ &\quad+4\{(x-y)^2(z-w)^2+4xy(z-w)^2+4zw(x-y)^2\}=0, \end{aligned}$$

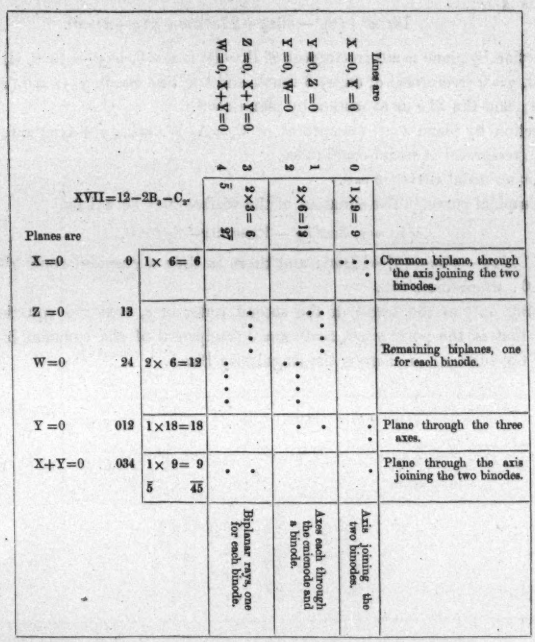
where each term is at least of the second order in $x-y$, $z-w$.

There is no cuspidal curve; $c'=0$.

Section XVII=12−2B$_2$−C$_2$.

Equation WXZ+XY^2+Y^3=0. Article Nos. 181 to 185.

181. The diagram of the lines and planes is



where the equations of the lines and planes are shown in the margins.

182. There is no facultative line; $\beta'=\varepsilon'=0$, $\tau'=0$.

183. The Hessian surface is

$$X(WXZ+3YZW+XY^2)=0,$$

viz. this breaks up into X=0 (the common biplane), and into a cubic surface.

The complete intersection with the cubic surface is made up of X=0, Y=0 (BB-axis) 4 times, of Y=0, Z=0 and Y=0, W=0 (CB-axes) each three times; and of a residual conic, which is the spinode curve; $\sigma'=2$. The equations of the spinode curve are Y^2−3ZW=0, 4X+3Y=0; viz. it lies in a plane passing through the BB-axis; since there is no facultative line, $\beta'=0$.

Reciprocal Surface.

184. The equation is found to be

$$(y^2+4zw)^2-xy^2-36xyzw+27x^2zw=0,$$

or say this is

$$16z^2w^2+(8y^2-36xy+27x^2)zw+y^2(y-x)=0.$$

The section by plane $w=0$ (reciprocal of $B_3=D$) is $w=0$, $y^2(y-x)=0$, viz. this is the line $w=0$, $y=0$ (reciprocal of edge) 3 times, and the line $w=0$, $y-x=0$ (reciprocal of ray) once; and the like as to section by plane $z=0$.

The section by plane $x=0$ (reciprocal of $C_3=A$) is $x=0$, $(y^2+4zw)^2=0$, viz. this is the conic (reciprocal of nodal cone) twice.

There is no nodal curve; $b'=0$.

185. Cuspidal curve. The equation of the surface may be written

$$(1,\ -y,\ 3zw,\ y^3-12zw,\ 9x-8y)^2=0,$$

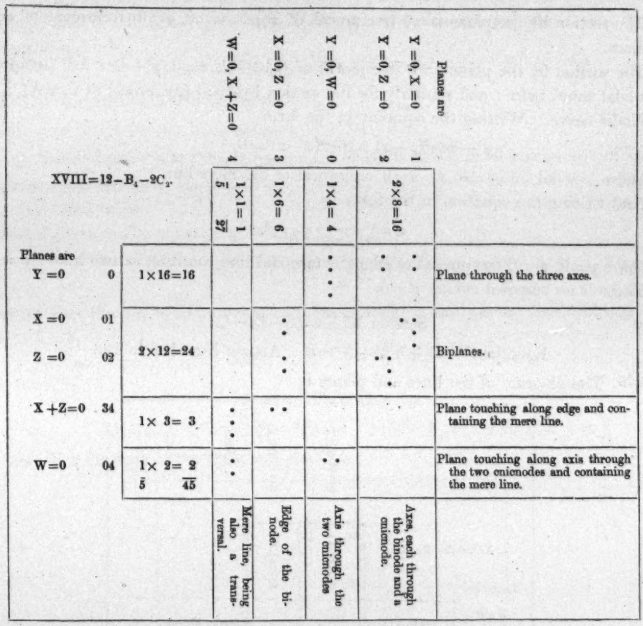
where $4.1.3zw-y^2=-(y^3-12zw)$; and there is thus a cuspidal conic $y^2-12zw=0$, $9x-8y=0$: wherefore $c'=2$.

Attending only to the terms of the second order in y, z, w, the equation becomes $x^2zw=0$; that is, the point $y=0$, $z=0$, $w=0$ (reciprocal of the common biplane) is a binode of the surface; or there is the singularity $B'=1$.

Section XVIII=12−B_4−$2C_2$.

Equation $WXZ+Y^2(X+Z)=0$. Article Nos. 186 to 189.

186. The diagram of the lines and planes are



where the equations of the lines and planes are shown in the margins.

187. The mere line is facultative; the edge is also facultative counting twice (this will appear from the discussion of the reciprocal surface): $b'=c'=3$, $t'=1$.

188. The Hessian surface is

$$(X+Z)WXZ+(X-Z)^2Y^2=0.$$

The complete intersection with the cubic surface is Y=0, Z=0 and Y=0, X=0 (the CB-axes) each 4 times; Y=0, W=0 (BB-axis) twice; and X=0, Z=0 (the edge) twice. There is no spinode curve, $\sigma'=0$; wherefore also $\beta'=0$.

Reciprocal Surface.

189. The equation is obtained from the binary quadric $4w(X+Z)(Xx+Zz)+y^2XZ$, or say

$$(8wx,\ 4w(x+z)+y^2,\ 8wz)(X,\ Z)^2.$$

The equation is thus

$$(y^3+4wx+4wz)^2-64w^2xz=0,$$

or in an irrational form

$$y+2\sqrt{wx}+2\sqrt{wz}=0.$$

The section by the plane $w=0$ (reciprocal of B_4) is $w=0$, $y=0$ (reciprocal of edge) 4 times.

The section by the plane $z=0$ (reciprocal of $C_2=C$) is $z=0$, $y^3+4wx=0$ (reciprocal of nodal cone) twice; and similarly for the section by $x=0$ (reciprocal of $C_2=A$).

Nodal curve. Writing the equation in the form

$$y^4+8wy^2(z+x)+16w^2(x-z)^2=0,$$

we have a nodal line $y=0$, $x-z=0$, reciprocal of the mere line:

And writing the equation in the form

$$w=\frac{1}{4(\sqrt{x}\pm\sqrt{z})^2}y^2,$$

we have $y=0$, $w=0$ (reciprocal of edge), a tacnodal line counting as two lines; $b'=3$.

There is no cuspidal curve; $c'=0$.

Section $XIX=12-B_6-C_2$.

Equation $WXZ+Y^2Z+X^3=0$. Article Nos. 190 to 193.

190. The diagram of the lines and planes is

$XIX=12-B_6-C_2$		ξ 27	$1\times15=15$	$1\times12=12$	
Planes are			$X=0, Z=0$	$X=0, Y=0$	Lines are
$Z=0$	$1\times15=15$		• • • • •		Osculur biplane.
$X=0$	$1\times30=30$ ξ 45		• •	• • • • • •	Ordinary biplane.
			Edge of the binode.	Axis joining the binode and the uninode.	

where the equations of the lines and planes are shown in the margins.

191. The axis is a facultative line counting 3 times (as will appear from the reciprocal surface); $\xi'=b'=3$, $t'=1$.

192. The Hessian surface is

$$Z(WXZ+Y^2Z-3X^3)=0,$$

viz. this is the oscular biplane $Z=0$ and a cubic surface.

The complete intersection with the cubic surface is made up of $X=0$, $Z=0$ (the edge) 6 times, and $X=0$, $Y=0$ (the axis) 6 times. There is no spinode curve, $\sigma'=0$; whence also $\beta'=0$.

Reciprocal Surface.

193. The equation is at once found to be

$$64zw^3+(y^2+4xw)^2=0.$$

The section by the plane $w=0$ (reciprocal of B_0) is $w=0$, $y=0$ (reciprocal of edge) 4 times. The section by the plane $z=0$ (reciprocal of C_3) is $z=0$, $y^2+4xw=0$ (reciprocal of nodal cone) twice.

Nodal curve. The equation gives

$$w=-\frac{1}{4x}y^2\pm\frac{i\sqrt{z}}{x^{\frac{3}{2}}}y^3+\text{ \&c.},$$

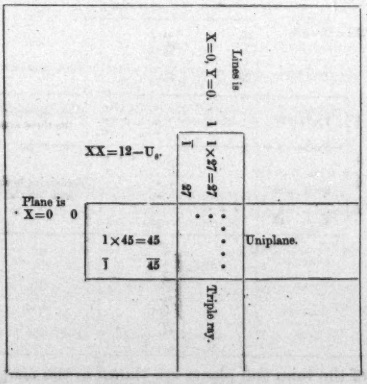
showing that the line $w=0$, $y=0$ (reciprocal of edge) is an oscnodal line counting as 3 lines; $b'=3$.

There is no cuspidal curve; $c'=0$.

Section $XX=12-U_3$.

Equation $X^2W+XZ^2+Y^3=0$. Article Nos. 194 to 197.

194. The diagram of the lines and planes is



where the equations of the line and plane are shown in the margins.

195. There is no facultative line; $b'=g'=0$, $t'=0$.

196. The Hessian surface is $X^2Y=0$, viz. this is the uniplane $X=0$, 3 times, and the plane $Y=0$ through the ray. The complete intersection with the cubic surface is made

up of $X=0$, $Y=0$ (the ray) 10 times, and of a residual conic, which is the spinode curve; $\sigma'=2$.

The equations of the spinode conic are $Y=0, XW+Z^2=0$, viz. the plane of the conic passes through the ray. Since there is no facultative line, $\beta'=0$.

Reciprocal Surface.

197. The equation is at once found to be

$$27(x^2+4xw)^2-64w^3y=0.$$

The section by the plane $w=0$ (reciprocal of the Unode) is $w=0$, $z=0$ (reciprocal of ray 4 times.

There is no nodal curve; $b'=0$. But there is a cuspidal conic, $y=0$, $z^2+4xw=0$.

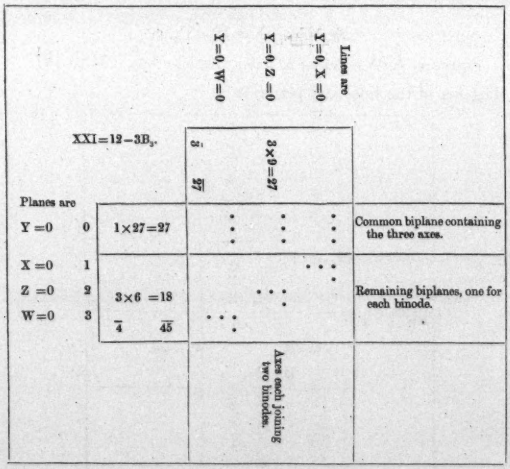
The point $y=0, z=0, w=0$ (reciprocal of the uniplane $X=0$) is a point which must be considered as uniting the singularities $B'=1$, $\chi'=2$.

I give in an Annex a further investigation in reference to this case of the cubic surface.

Section $XXI=12-3B_2$.

Equation $WXZ+Y^3=0$. Article Nos. 198 to 201.

198. The diagram of the lines and planes is



where the equations of the lines and planes are shown in the margins.

199. There is no facultative line; $\xi'=b'=0$, $\tau'=0$.

200. The Hessian surface is $XYZW=0$, the common biplane and the other biplanes each once. The complete intersection with the surface consists of the axes each 4 times; there is no spinode curve, $\sigma'=0$; whence also $\beta'=0$.

Reciprocal Surface.

201. This is $27xzw-y^3=0$, viz. it is a cubic surface of the form XXI$=12-3B_2$. There is no nodal curve, $b'=0$, and no cuspidal curve, $c'=0$. Moreover $B'=3$.

Synopsis for the foregoing sections. Article No. 202.

202. I annex the following synopsis, for the several cases, of the facultative lines (or node-couple curve) and of the spinode curve of the cubic surface; also of the nodal curve and the cuspidal curve of the reciprocal surface. It is to be observed that in designating a curve, for instance, as $18=4\times5-2$, this means that it is a curve of the order 18, the partial intersection of a quartic surface and a quintic surface, but without any explanation of the nature of the common curve 2 which causes the reduction, viz. without explaining whether this is a conic or a pair of lines, and so in other cases; this may be seen by reference to the proper section of the Memoir.

	Facultative lines.	Nodal curve.	Spinode curve.	Cuspidal curve.
I$=12$	27	27	$12=3\times4$	$24=6\times4$
II$=12-C_2$	15	15	$12=3\times4$	$18=4\times5-2$
III$=12-B_2$	9	9	$12=3\times4$	$16=4\times4-4$
IV$=12-2C_2$	7	7	$10=3\times4-2$	$12=4\times4-2-2$
V$=12-B_4$	7=5+edge twice	7=5+rec. of edge twice, rec. of edge tacnodal	$10=3\times4-2$	$12=4\times4-4$
VI$=12-B_2-C_2$	3	3	$9=3\times4-2$	$10=4\times4-4-2$
VII$=12-B_3$	3=2+edge	3=2+rec. of edge, rec. of edge is cuspnodal	9=edge+unicursal 8-thic	10=rec. of edge+unicursal 9-thic, rec. of edge is cuspnodal
VIII$=12-3C_2$	3	3	$6=2\times3$	$6=2\times3$
IX$=12-2B_2$	none	none	8=4 conics	8=4 conics
X$=12-B_4-C_2$	3=1+edge twice	3=1+rec. of edge twice, rec. of edge is tacnodal	$6=2\times3$	$6=2\times3$
XI$=12-B_3$	3=edge 3 times	3=rec. of edge 3 times, rec. of edge is osculnodal	6=3 conics	6=3 conics
XII$=12-U_3$	3	3	$6=2\times3$	$6=2\times3$
XIII$=12-B_2-2C_2$	1	1	$4=2\times2$, nodal quadriquadric	$4=2\times2$ quadriquadric
XIV$=12-B_2-C_2$	1=edge	1=rec. of edge, rec. of edge is cuspnodal	4=3+edge	4=3+ rec. of edge, rec. of edge is cuspnodal
XV$=12-U_7$	1	1	$4=2\times2$, nodal quadriquadric	$4=2\times2$ cuspidal quadriquadric
XVI$=12-4C_2$	3	3	none	none
XVII$=12-2C_2-C_2$	none	none	2= conic	2= conic
XVIII$=12-B_4-C_2$	3=1+ edge twice	1+ rec. of edge twice, rec. of edge tacnodal	none	none
XIX$=12-B_3-C_2$	3= axis 3 times	3= rec. of axis 3 times, rec. of axis osculnodal	none	none
XX$=12-U_3$	none	none	2= conic	2= conic
XXI$=12-3B_2$	none	none	none	none

I pass now to the two cases of cubic scrolls.

Section XXII$=$S(1, 1). Equation $X^2W+Y^2Z=0$. Article No. 203.

203. As this is a scroll there is here no question of the 27 lines and 45 planes; there is a nodal line $X=0, Y=0$, $(b=1)$ and a single directrix line, $Z=0, W=0$.

The Hessian surface is $X^2Y^2=0$; the complete intersection with the cubic surface is made up of $X=0$, $Y=0$ (the nodal line) 8 times, and of the lines $X=0$, $Z=0$, and $Y=0$, $W=0$, each twice.

The reciprocal surface is $x^2z-y^2w=0$; viz. this is a like scroll, XXII$=$S(1, 1); $c'=0$, $b'=1$.

Section XXIII$=$S$(\overline{1, 1})$. Equation $X(XW+YZ)+Y^3=0$. Article No. 204.

204. This is also a scroll; there is a nodal line $X=0$, $Y=0$, and a single directrix line united therewith.

The Hessian surface is $X^4=0$; the complete intersection with the cubic surface is $X=0$, $Y=0$ (the nodal line) 12 times.

The reciprocal surface is $w(xw+yz)-z^3=0$; viz. this is a like scroll, XXIII$=$S$(\overline{1, 1})$; $c'=0$, $b'=1$.

Annex containing Additional Researches in regard to the case XX$=12-U_8$; *equation*

$$WX^2+XZ^2+Y^3=0.$$

Let the surface be touched by the line (a, b, c, f, g, h), that is, the line the equations whereof are

$$\begin{vmatrix} 0, & h, & -g, & a \\ -h, & 0, & f, & b \\ g, & -f, & 0, & c \\ -a, & -b, & -c, & 0 \end{vmatrix} (X, Y, Z, W)=0.$$

Writing the equation in the form $cW.cX^2+X(cZ)^2+c^3Y^3=0$, and substituting for cW, cZ their values in terms of X, Y, we have

$$(-gX+fY)cX^2+X(aX+bY)^2+c^3Y^3=0,$$

that is

$$(a^2-cg,\ 2ab+cf,\ b^2,\ c^3)(X, Y)^3=0,$$

or say

$$(3(a^2-cg),\ 2ab+cf,\ b^2,\ 3c^3)(X, Y)^3=0,$$

viz. the condition of contact is obtained by equating to zero the discriminant of the cubic function. We have thus

$$\begin{aligned} &27c^4(a^2-cg)^2 \\ &+\ 4b^6(a^2-cg) \\ &+\ 4c^3(2ab+cf)^3 \\ &-\ b^4(2ab+cf)^2 \\ &-18b^2c^3(a^2-cg)(2ab+cf)=0, \end{aligned}$$

viz. this is

$$\begin{aligned}
&+27a^4c^2\\
&-\ 4a^3b^3\\
&+30a^3b^2cf\\
&-54a^3c^2g\\
&+36ab^3cg\\
&+24abc^2f^2\\
&+\ 4b^3h\\
&-\ 1b^2f^3\\
&+18b^2c^2fg\\
&+27c^4g^2\\
&+\ 4c^3f^3=0,
\end{aligned}$$

which is the condition in order that the line (a, b, c, f, g, h) may touch the surface $X^2W+XZ^2+Y^3=0$; and if we unite thereto the conditions that the line shall pass through a given point $(\alpha, \beta, \gamma, \delta)$, we have in effect the equation of the circumscribed cone, vertex $(\alpha, \beta, \gamma, \delta)$.

Writing (f, g, h, a, b, c) in place of (a, b, c, f, g, h), we obtain

$$\begin{aligned}
&27f^4h^2\\
&-\ 4f^3g^3\\
&+30f^3g^2ha\\
&-54f^3h^2b\\
&+36fg^3h^2b\\
&+24fgh^3a^2\\
&+\ 4g^3c\\
&-\ 1g^4a^2\\
&+18g^3h^2ab\\
&+27h^4f^2\\
&+4h^3a^2=0
\end{aligned}$$

as the condition that the line (a, b, c, f, g, h) shall touch the reciprocal surface

$$27(4xw+z^2)^2+64y^3w=0;$$

and if we consider a, b, c, f, g, h as standing for

$$\gamma y-\beta z,\ \alpha z-\gamma x,\ \beta x-\alpha y,\ \delta x-\alpha w,\ \delta y-\beta w,\ \delta z-\gamma w,$$

values which satisfy the relation

$$\left(\begin{vmatrix} 0, & h, & -g, & a \\ -h, & 0, & f, & b \\ g, & -f, & 0, & a \\ -a, & -b, & -c, & 0 \end{vmatrix}(\alpha, \beta, \gamma, \delta)\right)=0,$$

2 x 2

then the equation in (a, b, c, f, g, h) is that of the circumscribed cone, vertex $(\alpha, \beta, \gamma, \delta)$; the order being (as it should be) $d'=6$.

The cuspidal conic is $y=0$, $4xw+z^2=0$, and we at once obtain $a^2-4cg=0$ as the condition that the line (a, b, c, f, g, h) shall pass through the cuspidal cone. Hence attributing to (a, b, c, f, g, h) the foregoing values, we have

$$a^2-4cg=0$$

for the eqution of the cone, vertex $(\alpha, \beta, \gamma, \delta)$, which passes through the cuspidal conic; this is of course a quadric cone, $d'=2$. I proceed to determine the intersections of the two cones.

Representing by $\Theta=0$ the foregoing equation of the circumscribed cone, and putting for shortness

$$\mathrm{X}=27h^2(f^2-bh)-2g^2(2fg+ah),$$

I find that we have identically

$$\Theta-(f^2-bh)\mathrm{X}+(g^2-4ah^2-8fgh^2)(a^2-4cg) \\ -(32fg^2h+16agh^2)(af+bg+ch)=0:$$

whence in virtue of the relation $af+bg+ch=0$, we see that the equations $\Theta=0$, $a^2-4cg=0$, are equivalent to

$$(f^2-bh)\mathrm{X}=0,\ a^2-4cg=0,$$

or the twelve lines of intersection break up into the two systems

$$f^2-bh=0,\ a^2-4cg=0,$$

and

$$(\mathrm{X}=)\quad 27h^2(f^2-bh)-2g^2(2fg+ah)=0,\quad a^2-4cg=0.$$

To determine the lines in question, observe that we have

$$\begin{vmatrix} 0, & h, & -g, & a \\ -h, & 0, & f, & b \\ g, & -f, & 0, & c \\ -a, & -b, & -c & \end{vmatrix} (\alpha, \beta, \gamma, \delta)=0;$$

and we can by the first three of these express a, b, c linearly in terms of f, g, h; the equations $f^2-bh=0$, $a^2-4cg=0$, $27h^2(f^2-bh)-2g^2(2fg+ah)=0$ become thus homogeneous equations in (f, g, h); the equations may in fact be written

$$\delta^2(a^2-4cg)=(\gamma^2+4\alpha\delta)g^2+\beta^2h^2-2\beta\gamma gh-4\beta\delta hf=0,$$
$$\delta(f^2-bh)=\delta f^2-ah^2+\gamma hf=0,$$
$$\delta\mathrm{X}=27h^2(\delta f^2-ah^2+\gamma hf)+2g^2(\beta h^2-\gamma gh-2\delta fg)=0,$$

viz. interpreting (f, g, h) as coordinates *in plano*, the first equation represents a conic, the second a pair of lines, and the third a quartic.

We have identically

$$\{2\beta\delta f-(\gamma^2+4\alpha\delta)g+\beta\gamma h\}^2-4\beta^2\delta(\delta f^2-ah^2+\gamma hf) \\ =(\gamma^2+4\alpha\delta)\{(\gamma^2+4\alpha\delta)g^2+\beta^2h^2-2\beta\gamma gh-4\beta\delta hf\};$$

and it thus appears that the two conics touch at the points given by the equations

$$\delta f^2 - \alpha h^2 + \gamma hf = 0,$$
$$2\beta\delta f - (\gamma^2 + 4\alpha\delta)g + \beta\gamma h = 0:$$

we have moreover

$$-(\gamma^2+4\alpha\delta)(\beta h^2 - \gamma gh - 2\delta fg) = 4\beta\delta(\delta f^2 - \alpha h^2 + \gamma hf)$$
$$+(-2\delta f - \gamma h)[2\beta\delta f - (\gamma^2+4\alpha\delta)g + \beta\gamma h],$$

hence at the last-mentioned two points $-\beta h^2 + \gamma gh + 2\delta fg$ is $=0$; and the quartic $X=0$ thus passes through these two points.

The conic $(a^2-cg)=0$ and the quartic $X=0$ intersect besides (as is evident) in the point $g=0$, $h=0$ reckoning as two points, since it is a node of the quartic; and they must consequently intersect in four more points: to obtain these in the most simple manner, write for a moment

$$\Omega = -(\gamma^2+4\alpha\delta)g^2 + \beta^2h^2,$$

then we have identically

$$16\beta^2\delta g^2(\delta f^2 - \alpha h^2 + \gamma hf) - \Omega^2 = -[(\gamma^2+4\alpha\delta)g^2 + \beta^2h^2]^2 + 4\beta^2g^2(\gamma h + 2\delta f)^2$$
$$= -\{(\gamma^2+4\alpha\delta)g^2 + \beta^2h^2 - 2\beta\gamma gh - 4\beta\delta fg\}\{(\gamma^2+4\alpha\delta)g^2 + \beta^2h^2 + 2\beta\gamma gh + 4\beta\delta fg\}$$
$$= -\delta(a^2-4cg)\{(\gamma^2+4\alpha\delta)g^2 + \beta^2h^2 + 2\beta\gamma gh + 4\beta\delta fg\};$$

and moreover

$$2\beta(\beta h^2 - 2\delta fg - \gamma gh) - \Omega = (\gamma^2+4\alpha\delta)g^2 + \beta^2h^2 - 2\beta\gamma gh - 4\beta\delta fg = \delta(a^2-4cg).$$

Hence when $a^2-4cg=0$, we have

$$\delta f^2 - \alpha h^2 + \gamma hf = \frac{\Omega^2}{16\beta^2\delta g^2},\quad \beta h^2 - 2\delta fg - \gamma gh = \frac{\Omega}{2\beta};$$

and substituting these values in the equation $X=0$, it becomes

$$27h^2 \cdot \frac{\Omega^2}{16\beta^2\delta} + 2g^2 \cdot \frac{\Omega}{2\beta} = 0,$$

viz. multiplying by $16\beta^2\delta$, and omitting the factor Ω, this is

$$27h^2\Omega + 16\beta\delta g^4 = 0,$$

or finally

$$16\beta\delta g^4 - 27(\gamma^2+4\alpha\delta)g^2h^2 + 27\beta^2h^4 = 0,$$

a pencil of four lines, each passing through the point $g=0$, $h=0$, and therefore intersecting the conic

$$(\gamma^2+4\alpha\delta)g^2 + \beta^2h^2 - 2\beta\gamma gh - 4\beta\delta hf = 0$$

at that point and at one other point; and we have thus four points of intersection, which are the required four points.

Recapitulating, the conic $a^2-4cg=0$ meets the sextic $(f^2-\delta h)X=0$ in the two points

$$\begin{cases} \delta f^2 - \alpha h^2 + \gamma hf = 0, \\ 2\beta\delta f - (\gamma^2+4\alpha\delta)g + \beta\gamma h = 0 \end{cases}$$

each three times, in the point $g=0$, $h=0$ twice, and in the four points

$$\begin{cases} 16\beta\delta g^4 - 27(\gamma^3+4\alpha\delta)g^2h^2+27\beta^2h^4=0, \\ (\gamma^3+4\alpha\delta)g^3+\beta^2h^3-2\beta\gamma gh-4\beta\delta hf=0 \end{cases}$$

each once. Or reverting to the proper significations of (a, b, c, f, g, h), instead of points, we have 2 lines each three times, a line twice, and 4 lines each once; the line $g=0, h=0$, that is, $g=0$, $h=0$, $a=0$, being, it will be observed, the line $\frac{y}{\beta}=\frac{x}{\gamma}=\frac{w}{\delta}$ drawn from $(\alpha, \beta, \gamma, \delta)$ to the point $y=0$, $z=0$, $w=0$, which is the reciprocal of the uniplane $X=0$: the twelve lines are the $d'c'$ lines of intersection of the circumscribed cone d' with the cuspidal cone c', viz. $d'c'=[d'c']+3\sigma'+\chi'$; $[d'c']=4$ referring to the last-mentioned four lines; $\sigma'=2$ to the two lines; and $\chi'=2$ to the line $g=0$, $h=0$, $a=0$, which it thus appears must in the present case be reckoned twice.

VIII. *On the Osteology of the Solitaire or Didine Bird of the Island of Rodriguez*, Pezophaps solitaria (*Gmel.*). *By* Alfred Newton, *M.A., Professor of Zoology and Comparative Anatomy in the University of Cambridge, and* Edward Newton, *M.A., Auditor-General of Mauritius. Communicated by* P. L. Sclater, *M.A., Ph.D., F.R.S.*

Received May 6,—Read June 11, 1868.

Evidence as to the fact of the island of Rodriguez in the Indian Ocean having been formerly inhabited by a Didine bird specifically distinct from the true Dodo of Mauritius (*Didus ineptus*, Linn.) appears to have been first satisfactorily adduced in 1844 by the late Hugh Edwin Strickland*. It is true that, eighty years ago, Gmelin, in his edition of the 'Systema Naturæ' of Linnæus (tom. i. p. 728), described in scientific language, from the account given by Leguat† (with which he was apparently acquainted only at second hand through the intervention of Buffon‡), a second species of the genus *Didus* as inhabiting that island; but this species had been long rejected by all zoologists, except those whose labours had been confined to the work of compilation, until, as just mentioned, Strickland proved, from a renewed examination of the original and other accounts, that the species thus described, however it might have been misrepresented, had certainly once existed, and moreover that remains of it were contained in at least three collections—the Museum of the Jardin des Plantes, the Andersonian Museum of Glasgow, and the collection of the Zoological Society of London.

The opinion thus propounded was, four years later, set forth at greater length by its author, the subject of it being then declared not only specifically but generically distinct from *Didus ineptus*, and called *Pezophaps solitaria* (Gmel.)§, and since that time no one has ventured to impugn its accuracy. Thus, though formally accepting Gmelin as the first author who applied the set rules of zoological nomenclature to this species, little credit is really due to him from the fact. Leguat, in 1708, was its original describer‖,

* Proc. Zool. Soc. 1844, pp. 77–79. (Reprinted in Ann. and Mag. Nat. Hist. xiv. pp. 324–326.)

† Voyage et Avantures de François Leguat &c. Londres: 1708, 2 vols. 12mo (vol. i. pp. 98–102); A New Voyage to the East-Indies by Francis Leguat &c. London: 1708, 1 vol. 8vo (pp. 71–74).

‡ Hist. Nat. des Oiseaux, vol. i. p. 485.

§ The Dodo and its Kindred &c. By H. E. Strickland, M.A. &c., and A. G. Melville, M.D. &c. London: 1848, 4to (pp. 46–56, and 113–119).

‖ The name "*Solitaire*," or (in the contemporaneous English translation of his work) "Solitary," by which Leguat called the Didine bird of Rodriguez had in reality been previously bestowed by Carré (Voyage des Indes Orientales &c., par Mr. Carré, Paris: 1699, vol. i. p. 12) on the Didine bird of Bourbon (Réunion). As the rule of priority is not necessarily enforced with regard to vernacular names, there seems to be no reason why the Rodriguez species should not retain that by which it has become generally known. We can afford to keep the Bourbon species waiting for a common name until some particle of its remains is discovered.

if he may not be called its discoverer; for we have no knowledge of any writer who had previously distinguished it from the better-known Dodo of Mauritius*, and Strickland was the first naturalist who placed its former existence beyond all doubt.

The remains of this bird, the "Solitaire" of Rodriguez, existing in museums until within the last few years seem to have been as follows:—The Museum of the Jardin des Plantes possessed *five* bones, said to have been found in a cave in Rodriguez in 1789†; these were a femur, a tarso-metatarsal, a humerus, the median portion of a sternum, and a portion of the cranium‡. The Andersonian Museum at Glasgow contained *six*, being part of a collection made by the late Mr. Telfair, a right and left femur with the fragment of a third, a left tibia, and a right and left tarso-metatarsal§. The Zoological Society of London possessed the remaining portion of this collection‖; but the specimens (*five* in number) were for some time mislaid, and were consequently not available for the use of Messrs. Strickland and Melville when they published their work¶. In 1851 these specimens were rediscovered in the house of the Society by Mr. Bartlett, and proved to be a portion of a right humerus, a right femur, a right tibia, and a right and left tarso-metatarsal—the last two belonging to different individuals. In the description of these bones, which Mr. Bartlett published soon after**, he was induced into several grave, though perfectly excusable, errors, and in fact only referred one of the specimens (the right tarso-metatarsal) to the Solitaire, though there can now be no doubt that they all belong to that species. Nor was Strickland much more fortunate, for early in the following year (1852) he also described these same specimens††, and, misled as his predecessor had been by the same causes (of which, however, he assigned a different explanation), he equally imagined that he had before him the remains of more than one species‡‡.

Besides the *sixteen* bones just enumerated *two* others were the property of Strickland—one, a left tarso-metatarsus, found at the same time and place as those in the Museum at Paris; and another, a right tarso-metatarsus, said to have been found in 1831 by Colonel Dawkins, but in the same cave as the others§§.

* Sir Thomas Herbert, who in 1628 sailed by "Digarroys" (*i. e.* Rodriguez), mentions ('Some Years Travels into divers parts of Asia and Afrique, &c.' London: 1638, fol. p. 341) "Dodos" as being found in that island, confounding the species to which they belonged with that of Mauritius (*id. op. cit.* p. 347); but he gives no description of the former.

† Strickland, 'The Dodo &c.' p. 51.

‡ These had been exhibited by Cuvier to the French Academy of Sciences in 1830 as belonging to the true Dodo (Strickland, *ut suprà cit.* p. 52). We desire to express our very sincere thanks to M. Alphonse Milne-Edwards for his kindness in procuring for us models of all these specimens.

§ Sir William Jardine was good enough some years since to give me casts of these specimens.—A. N.

‖ Proc. Zool. Soc. 1833, p. 32.

¶ 'The Dodo &c.' p. 52.

** Proc. Zool. Soc. 1851, pp. 280–284, *Aves*, pl. xlv. Reprinted Ann. and Mag. Nat. Hist. 2nd ser. xiv. pp. 297–301.

†† Trans. Zool. Soc. vol. iv. part 6 (1859), pp. 187–196, pl. 55.

‡‡ These bones are now in the British Museum.

§§ Ann. and Mag. Nat. Hist. 2nd ser. iv. pp. 335–337. For a cast of one of these we are indebted to the kindness of Mrs. H. E. Strickland.

In addition to these *eighteen* specimens we are informed that in 1860 or 1861 a tibia, the shaft of a tarso-metatarsal, and some fragments of the shaft of a femur, all of which belonged to the Solitaire, were sent to Professor Owen by M. Bouton of the Museum at Mauritius; but the fate of these specimens is unknown to us.

Thus up to the year last mentioned not more than *twenty-one* specimens, of which many were duplicates, of the bones of the Solitaire had been brought to the notice of naturalists. They included portions of the cranium and sternum, the humerus, the femur, tibia, and tarso-metatarsal; but the portions of the cranium and sternum, and the only perfect humerus, all of which were in France, were so much encrusted with stalagmite that their characters were in some parts wholly obscured.

In November 1864 one of the authors of this paper visited Rodriguez*, and near the entrance of a cave on the south-west side of that island he picked up two bones, a perfect left tarso-metatarsal and the shaft of a left humerus. About the same time Captain Barclay found a third bone, a femur. These were sent to England, and described and figured† under the provisional name of "*Didus nazarenus*, Bartlett," the describer having fallen into the same mistake as that naturalist and Strickland had already done.

Meanwhile Mr. George Jenner, the magistrate of Rodriguez, was urged to make a more thorough search for further remains of the Solitaire in the localities whence the specimens last mentioned had been found.

Accordingly, on the 1st August 1865, a box from this gentleman reached one of the authors at Mauritius. It was found to contain upwards of *eighty-one* specimens of bones, belonging to at least *sixteen* individuals. Among these bones were a portion of the coracoid, examples of the ulna, radius, and fibula, portions of the pelvis and one of the digital phalanges, all of which had been previously unknown. But, more than this, from a series so large it became at once plain, by the presence of many specimens of intermediate size, that the theory of the existence in the island of Rodriguez of two species of Solitaire (originated by Mr. Bartlett, afterwards adopted with a modification by Strickland, and followed subsequently by one of the present authors in describing the results of the previous "find") became untenable, and this fact was declared when a few months later the specimens arrived in England, and were exhibited at a meeting of the Zoological Society‡.

Immediately on tidings of this very important discovery being made known in this country§, the British Association for the Advancement of Science, then assembled at Birmingham, at the instance of Mr. P. L. Sclater, appointed a Committee to assist one of the present authors in his researches into the Didine Birds of the Mascarene Islands, and placed a handsome sum of money at his disposal for that purpose. Whereupon Mr. Jenner was requested to order a new examination of the more promising caves

* Ibis, 1865, pp. 146–154.

† Proc. Zool. Soc. 1865, pp. 199–201, pl. viii.

‡ Ibid. pp. 715–718.

§ Report of the Thirty-fifth Meeting of the British Association; Notices and Abstracts of Miscellaneous Communications to the Sections, p. 92.

in Rodriguez*. Owing to the scarcity of labourers in that island and the irregularity of communication between it and Mauritius, nearly a year passed before anything could be done, and at last it was found necessary to send a small band of coolies to Rodriguez for the express purpose of digging in the caves there†. These men were dispatched in August 1866 and returned to Mauritius in about two months, Mr. Jenner sending with them the results of their labour, consisting of nearly 2000 bones or fragments of bones of the Solitaire, with a few others belonging to various animals. Two facts, however, it must be said are greatly to be regretted in connexion with this expedition. It had been found impossible to put the coolies employed under an intelligent superintendent, who could have carefully noted down all the circumstances under which the bones were found; and the report which Mr. Jenner was requested to draw up, giving an account of the caves visited and so forth, has unfortunately never reached us‡. Consequently nothing more is known of the specimens described in the present paper than that they were found in caves in the island of Rodriguez, but it is impossible for us to say how long they have been there buried§.

This much, by way of historical statement, is due to those who have before laboured upon the subject of which we are now about to treat as well as to those who have aided us in our attempt to recover the remains of this lost form. A few more words only are necessary before we begin to describe, as best we can, the osteology of the Solitaire of Rodriguez.

First, we have to remark that while the enormously large series of specimens at our disposal has undoubtedly enabled us to form and declare certain opinions with greater confidence than we should otherwise have felt at liberty to do, we confess to having been in some respects much embarrassed by the wealth of our materials. It is not a very difficult task to describe in great detail a single skeleton, but it is not easy to draw up a description which, while being minute and diagnostic, shall yet fit some fifty or more examples. There does not seem to be a single bone in the skeleton of *Pezophaps solitaria* which is not liable to greater or less individual variation of some kind or other. This variation in point of absolute size, in which of course it is most easily recognizable, induced Mr. Bartlett, Strickland, and one of ourselves (as has been already stated) into a serious error. But the individual variation is not at all confined to absolute size; it extends to the relative proportion of divers parts of the bones, to processes or depressions upon them such as are commonly held to be specifically characteristic, so that it is often utterly impossible to predicate any definite limits of individual modification.

* We must by no means omit to mention here that the results of Mr. George Clark's fortunate discovery of bones of the true Dodo recounted by him in 'The Ibis' (1866, pp. 141–146), though not known in England till some months after the grant of the British Association had been voted, did much to inspirit those who were interested in the matter of the resurrection of the Solitaire of Rodriguez.

† Report of the Thirty-sixth Meeting of the British Association, p. 402.

‡ Trans. Roy. Soc. Maur. N. Ser. iii. pp. 31–38.

§ Professor Steenstrup, and there cannot be a better authority, on seeing the bones found in 1865, recognized in some of them certain characters which he believed showed unfailingly that these birds had been eaten by men or dogs. No such characters are observable by us on the bones found in 1866.

This statement may seem bold, but we are convinced that it is true. With great deference to the mightiest masters of the science, we venture to submit that many of them have too often undervalued the benefit of consulting long series of specimens, and that the principle of *ex uno disce omnia* is one which must on no account be admitted to hold good in anatomy any more than in zoology. Nor is the variability of which we speak wholly dependent on age or sex*. Thus we are compelled to refrain from giving any of those precise measurements of the specimens we describe, such as are usually found in works of this nature, an omission which we trust will not be found productive of real inconvenience, owing to the number of figures by which the present paper is illustrated —figures nearly all of the natural size, and drawn most carefully by the artist, Mr. Ford.

Next we must state that Professor Owen having published an elaborate description of the osteology of *Didus ineptus*†, we have endeavoured, so far as was possible, to make our description of the osteology of the kindred bird conform with his both in general arrangement and special treatment, believing that by so doing we were but consulting the convenience of those who may wish to compare the structure of the two. Professor Owen's masterly treatise has been always at our side while examining, selecting, and describing the bones of *Pezophaps solitaria*, as has also been the equally elaborate dissertation of Dr. Melville‡ on such bones of this species as were known to him, and we have often availed ourselves of the expressions of either author for the express purpose of making the comparison the more obvious.

Here, lastly, it must be mentioned that, thanks in a great measure to the generosity of Mr. George Clark of Mahébourg, in Mauritius, we have had the advantage of studying at the same time a series of Dodos' bones (the fruits of his remarkable discovery) more extensive even than that which furnished the subject of Professor Owen's treatise, and this by the kindness of the latter we have also had an opportunity of examining. Further, we have been indebted to various other friends and naturalists for other specimens tending to illustrate the subject, to Professors Reinhardt and Fritsch for casts of Didine remains existing in Copenhagen§ and Prague respectively, to Professor Alphonse Milne-Edwards for models of the *Pezophaps*-bones in Paris, and to Sir William Jardine for casts of those in Glasgow; while we have also to express our acknowledgments to the authorities of the Royal College of Surgeons of England for their liberality in lending us the precious skeleton of *Didunculus strigirostris*‖ from

* We have certainly never made a close study of such an extensive series of the bones of any one other species as we have done of the Solitaire, and therefore we are perhaps not in a position to form a true comparison; but it does seem to us that the amount of individual variation both here and in the Dodo is extraordinarily great. The possible explanation of this fact, if it be a fact, will be discussed in our concluding observations.

† Trans. Zool. Soc. vol. vi. pp. 49–85, pls. xv.–xxiv. ‡ 'The Dodo' &c. part ii. pp. 113–119.

§ The Copenhagen Dodo's head is often spoken of as "mutilated," and by one writer as "lacking the lower jaw." Its sole imperfections are the absence of the left pterygoid, and the enlargement of the foramen magnum. The mandible is as perfect as that of the Oxford specimen, or as one in our own possession.

‖ The more valuable since it is the specimen described and figured by Professor Owen in his paper "On the Osteology of the Dodo" (*ut suprà cit.*).

their Museum, when informed by their Curator, Mr. FLOWER, of our wish to compare its structure with that of *Didus* and *Pezophaps*. Finally, our thanks are in a special manner due to Mr. JOHN WILLIS CLARK, the Superintendent of our University's Museums of Zoology and Comparative Anatomy, for the trouble he has taken in mounting for us some of the bones both of the Dodo and the Solitaire with the success commonly attained by him in reconstructing the skeletons of animals.

§ 2. *Vertebræ*.

The hundred and sixty-one vertebræ contained in the collection, few of which preserve all their processes in a perfect state, form a subject of no small perplexity, and it is beyond our power to determine precisely the number which the skeleton contained.

On the presumption that *Pezophaps* possessed the same number of dorsal (8) and cervical (12) vertebræ as is assigned to *Didus* by Professor OWEN'S draughtsman*, there is such a great numerical preponderance of specimens of the eighth, ninth, and tenth cervicals (Plate XV. figs. 24–34), counting backwards, that one is at first led to suppose that *Pezophaps* must have been endowed with more, till a diligent study of each example compels the belief that no other presumption will meet the case equally well, and an explanation of the curious fact that the specimens of each of these three vertebræ are nearly twice as numerous as most of those of the rest must be sought elsewhere. And in support of this view it may be stated that of the three anchylosed dorsal vertebræ there are present in the collection but five examples.

The penultimate free dorsal (Plate XV. figs. 56–59) intervening between that which coalesces with the pelvis and the three† anchylosed vertebræ just mentioned, is represented in the collection by nine specimens, none of which is entirely perfect. The hypapophysis is aborted; the centrum posteriorly resembles that of the next anterior vertebra; the neural canal is elongated vertically and contracted in the midst; the neural spine is directed forwards, but its outline is not to be completely traced in any of the examples we possess; near the anterior articular surface on either side is an articular somewhat reniform costal cavity; the postzygapophyses have comparatively small articular surfaces, oval, and slightly concave.

The three anchylosed dorsal vertebræ (Plate XV. figs. 51–55), the last of which is the antepenultimate dorsal, bear a great general resemblance to the same bones in *Didus*. Taking a specimen of each species of nearly the same absolute size, the greater extent of the articular surfaces, and especially the lateral expansion of the anterior centrum, in *Didus* is plainly perceptible. But though these relative proportions are preserved, some

* Professor OWEN himself does not offer any opinion as to the number of vertebræ in *Didus*. We prefer citing his draughtsman as the authority for the number assigned to the bird in the large plate illustrating his paper, because on the same plate *Didunculus* appears to be represented as possessing fourteen cervical and seven dorsal vertebræ, being altogether two *more* than we are able to count in the very specimen, now in the Museum of the Royal College of Surgeons, which served as the subject for his pencil.

† Three is the most usual number in the *Columbæ*, but in *Goura* two only are anchylosed. In one example (Plate XVI. fig. 60) of *Pezophaps* the anchylosis is incomplete.

specimens of these bones in *Pezophaps* are in every way very much bigger, and longitudinally bear to the corresponding part of their ally the proportion of ten to nine. In these larger (male?) specimens, the hypapophyses of the anterior and middle vertebræ of the anchylosed three are not generally confluent so as to inclose a full elliptical space, and in none of the specimens, whether large or small, is there any trace of such a second foramen as seems to be invariably the case in *Didus*.

The specimens of anterior dorsal vertebræ (Plate XV. figs. 42–50) are in a very mutilated condition, and it is almost impossible to institute such a comparison between their remains and the corresponding bones in *Didus* as would serve any useful purpose.

Further onward (Plate XV. figs. 24–41) a sensible though gradual elongation and attenuation of the vertebræ is manifested, which becomes perhaps the greatest about the region of the fifth, sixth, and seventh cervicals (Plate XV. figs. 13–23) on the presumption here followed. The postzygapophyses are considerably elongated as compared with *Didus* in proportion to the breadth of the vertebræ. The neural spine of the anterior cervicals in that bird is much more prominent but less expanded than in *Pezophaps* (Plate XV. figs. 6–12). From the gradual elongation just mentioned the neck of this last, when living, must have been absolutely longer than that of *Didus*, a conclusion which testifies in another point to the fidelity of Leguat's figure, of which more will be said in our concluding observations.

Of the axis the collection contains six specimens, to one of which the atlas is united, possibly by partial anchylosis (Plate XV. figs. 1–5). The former closely resembles the axis of *Didus* as figured by Professor Owen (*op. cit.* pl. xvii. figs. 12, 13); but the odontoid process projects much less, and indeed hardly at all beyond the anterior upper portion, while on the other hand the neural spine and (as in the case already mentioned of the succeeding vertebræ) postzygapophyses project considerably further backward. The same is to be observed in the postzygapophyses of the atlas as compared with *Didunculus*; and in this respect *Pezophaps* resembles *Goura*. The articulating cavity for the reception of the occipital condyles is deep, and its lateral margins approach each other inferiorly somewhat more closely than appears to be the case in either *Goura* or *Didunculus*.

§ 3. *Ribs*.

Of ribs, the collection contains (besides a vast number of fragments) nearly seventy examples, scarcely any of which are perfect, and a large portion consists of merely the upper extremities of the dorsal ribs. Of these there are about fifty-three specimens, and of the sternal ribs some fourteen. Very few of the former have their epipleural appendage preserved (Plate XVI. fig. 63), and as this seems to be the most (if not the only very) characteristic feature possessed by the ribs in birds, not much of importance can be determined in its absence. It would therefore be unnecessary to dwell at any great length on these bones, which present no very apparent distinctive characters from those of *Didus*. Perhaps on the whole the ribs of *Pezophaps* have their heads and articular tubercles proportionally somewhat thicker. This is certainly the case in the

larger (male?) specimens, and many of these are quite as stout as those ordinarily found in *Didus*. A pneumatic foramen near the base of the tubercle is generally but not invariably present, appearing to be, as in *Didus*, rather an individual character. The curvature is much the same in both birds, showing that *Pezophaps* must have been as bulky as its ally. Many of the posterior ribs bear on their outer surface the same marks of pressure or friction (Plate XVI. figs. 62, 63), caused by the movements of the thigh, as have been observed in *Didus*. There appears to have been eight pairs of dorsal and four of sternal ribs. The first probably articulated with the thirteenth vertebra, the last or eighth with the twentieth, and seems to have been sometimes coalescent at its sternal extremity with the sternal rib of the preceding or seventh (Plate XVI. fig. 64); but this feature did not always obtain, as another specimen of the latter from the opposite side, and most likely from a younger bird, shows (Plate XVI. fig. 61). The first and (with the exception of the single coalescent specimen just mentioned) eighth ribs are wanting. On the left side the sixth dorsal rib is missing, but the corresponding sternal rib is present. On the right side the contrary occurs, the sternal rib articulating with the sixth dorsal being absent. Thus the ribs represented in the collection may thus be enumerated:—

Left			Right	
Dorsal.	Sternal.		Sternal.	Dorsal.
0	—	1	—	0
×	—	2	—	×
×	—	3	—	×
×	×	4	×	×
×	×	5	×	×
0	×	6	0	×
×	×	7	×	×
0	—	8	—	½

§ 4. *Pelvis.*

The pelvis of *Pezophaps* is represented in the collection by remains which must have belonged to at least thirty-seven individuals; but by far the greater number of these are fragmentary, consisting chiefly of the coalesced sacral vertebræ with mutilated apophyses and separated from the iliac and other bones. One specimen, however, is comparatively perfect and possesses the pubic bones (Plate XVII. figs. 66, 68, XVIII. fig. 70), the form of which in *Didus* can be only conjectured. Compared with the pelvis of that bird, it is by no means so remarkable for the flatness and great breadth of the posterior half. One specimen, complete in this respect, has eighteen coalesced sacral vertebræ. Two equally perfect examples of *Didus ineptus* show only sixteen, which is probably the normal number in that species. The articular surface of the centrum of the last dorsal (Plate XVII. figs. 66, 67) is in *Pezophaps* almost exactly as in *Didus*, but the pit for the head of the rib is placed somewhat more forward in the former. The anterior outlet of the neural canal exhibits as much individual variety as it does in *Didus*, but

is generally vertically ovate and sometimes constricted midway. The buttresses of bone supporting the præzygapophyses are more slender than in *Didus*, and the præzygapophyses themselves are smaller; with their facets inclined more forward and less vertically sloped. The fore part of the neural spine, which is nearly as strong as in *Didus*, has a similar rough syndesmotic surface and a curved anterior outline, but it is not constantly confluent with the ilia, the thickened margins* of which (Plates XVI. fig. 65, XVII. fig. 68, XVIII. fig. 70), rising behind it and meeting together, appose, more or less, their curved inner (and under) edges to one another till nearly over the anterior renal cavity, when, instead of sweeping backward and forming a prominent gluteal ridge as in *Didus*, they suddenly turn downwards, then, in some specimens, as sharply forwards, and finally trend backwards towards the part over the acetabulum, soon becoming obsolete. The coalesced neural spines of the remaining sacral vertebræ, after dipping down to allow of the meeting together of the margins of the ilia as just described, rise again and form a continuous, elevated and slightly curved ridge (Plates XVI. fig. 65, XVII. figs. 68, 69, XVIII. fig. 70), which, sloping slightly downwards towards its termination, extends to the posterior extremity of the sacral region, where it presents a small lozenge-shaped, partly articular and partly syndesmotic surface to the anterior caudal vertebra, as in *Didus*, receding around the neural canal, here reduced to a very small foramen, but with the centrum projecting backwards.

The abutments of the first sacral vertebra (Plate XVII. fig. 66), which seems to be somewhat more elongated, are generally almost exactly as in *Didus*. One specimen, however, shows a remarkable deviation from the ordinary form, in having the pleurapophysis, usually a slender straight filament, very greatly thickened, and curving downwards outwardly to meet the ilia, with which it is confluent.

In the next three vertebræ there is considerable individual variation: sometimes all have their pleurapophyses thickened, sometimes the first two only, and in one specimen (that which has just been mentioned as having the first sacral abnormal) the last two. The majority of specimens show the thickening of the first two. On the whole, however, the general aspect of this part is very like what it is in *Didus*.

The fourth and fifth sacrals have their pleurapophyses occasionally developed, but though in one specimen those of the fourth are very much thickened, in none are they sufficiently produced to coalesce with the ilia. In another specimen the diapophysis of the fifth is greatly thickened on the right side so as to present the appearance of a thickened pleurapophysis, while on the left it is of the ordinary character.

The next succeeding sacral vertebræ closely resemble those of *Didus*, and are attached to the ilia by their diapophyses only.

In the seventh, eighth or ninth sacrals, however, and most generally in the eighth, the pleurapophysis reappears, just as it does in *Didus*; extending backwards and rapidly widening, it abuts against the underside of the ilium, immediately above the acetabulum, but not, as in *Didus*, behind it.

* These margins show no disposition to coalesce. In one example (Plate XVI. fig. 65), apparently of a very old bird, they are developed to excess, and then they turn down outwardly on themselves.

The next succeeding vertebra occasionally developes an imperfect pleurapophysis, but the remainder are attached to the ilia only by diapophyses. These are stouter and less lamelliform than in *Didus*, and, owing to the more convex form of the posterior portion of the pelvis, the cavities they intercept are shorter. The whole of these vertebræ are broader than in *Didus*, and this comparative widening of the centrum becomes more obvious towards the posterior extremity.

The ilium above (Plates XVI. fig. 65, XVII. fig. 68, XVIII. fig. 70) is not so distinctly divided into two parts as in *Didus*, owing to the obsoleteness of the gluteal ridge already mentioned. The anterior part is less concave, the posterior more convex, and both are narrower than in *Didus*. The anterior border of the ilium is thickened and curved with the convexity forward, but the most convex part of the curve is nearer the lower margin than in *Didus*, where it is situated about the middle. The form of the upper margin of the ilium has been previously described. The orifices of the ilioneural canals are very much smaller than in *Didus*, and often more circular. The acetabular region is much as in *Didus*, but the trochanterian surface is narrower, more prominent, and directed more forward. Immediately above this surface, where the ilia have diverged furthest from each other, they rise, and without showing any angular ridge, bend over convexly. Here they slope downward till about as far as the fourteenth sacral vertebra, approaching each other slightly, but not so much as in *Didus*. They then again diverge. The inner surface of the ilium is also thickened behind the ischiadic foramen, but in *Pezophaps* to a less degree. The outer border of the posterior part of the ilium projects pretty equally in the two forms, and the coalescence of the ilium, ischium, and pubis is much the same in each. The pubis, of which in *Didus* only a small portion has been described, is present in one specimen of *Pezophaps* (Plates XVII. figs. 66, 68, XVIII. fig. 70) in what appears to be a nearly perfect condition. It originates below rather than behind the acetabulum, and diverges, as soon as it is free, both vertically and horizontally from the ischium. It is at first trihedral with the addition of a sharp projecting ridge on its upper and outer margin. This ridge disappears as the bone passes backward, and broadens into a lamina with the upper edge thickened, and bevelled on the outer side. After passing beyond the extremity of the sacrum the pubis bends upward, slanting slightly inward, and the bevel turning to the inner side disappears. The ischium of *Pezophaps* greatly resembles that of *Didus*, so far as can be judged from the imperfect examples of the latter which exist. The ischiadic foramen of the former, however, is as broad behind as before. The posterior portion of the ischium, at the distance from the ischiadic foramen of about its own length, becomes sensibly convex. It then narrows and becomes concave, its lower margin sloping downward and outward, as if to pass and avoid the pubic style, which is here directed upward and inward in a different plane; but the extremities of both pubis and ischium are broken off, and it is not perfectly certain that they may not have met eventually, though it is not easy to suppose that they have done so. Quite enough, however, remains to show that in the posterior portions of both these bones *Pezophaps* differs entirely from *Didunculus*, or indeed any Columbine form, and we must go far to seek a similar case. The conjectural restoration of the ischium and

pubis in *Didus* by Professor OWEN'S artist (*op. cit.* pl. xv.) is probably altogether erroneous; but if it be so the error is, under the circumstances, most excusable, for no one could have predicated the extraordinary forms assumed by these bones in *Pezophaps*.

§ 5. *Sternum.*

Of this bone the collection contains fragments belonging to at least six individuals, but only one that is at all complete, though some portions of the others furnish information as to certain not unimportant details. The most perfect specimen (Plate XVIII. figs. 71–74) also has received much damage on its sides and posterior extremity, so that their contour can only be inferred—a matter perhaps of the less consequence since the characters afforded by the posterior extremity of the sternum are certainly not so constant in birds generally as those to be deduced from the anterior end*. This same specimen is also one of the smallest (female?), and on that account it is possible that some of the most remarkable characters of *Pezophaps* may not be very strongly developed in it.

* In support of this opinion the following cases may be cited among many others that could be mentioned. In the *Limicolæ* (NITZSCH) (= *Charadriidæ* and *Scolopacidæ*, AUCTT.), an eminently natural group, a very great diversity of conformation of the posterior margin of the sternum exists even among forms which are, both in general habits and outward structure, very closely allied. *Totanus ochropus* differs from all other *Totaninæ* known to us by possessing only one posterior emargination on each side the keel, where they (and even *T. glareola*, so long confounded with *T. ochropus*) have two. *Machetes* has a single emargination. *Charadrius morinellus* is unlike any other Limicoline bird we know, inasmuch as both "ectolateral" and "entolateral" processes extend backward nearly as far as the keel does, and the former are somewhat everted as in *Phalaropus*. But *Œdicnemus* and *Vanellus* have the extremity of the inner emarginations bridged across so as to become fenestræ. *C. pluvialis* and *C. hiaticula*, with many of the smaller Plovers, have two emarginations like most *Totaninæ*. *Scolopax gallinula* resembles these last and the majority of *Tringinæ*, while *S. gallinago*, *S. wilsoni*, and *S. bernieri* have only one large, and *S. major*, like *S. americana* and *S. rusticola*, one small emargination on each side. As great differences are to be found in other *Limicolæ*; but here we have only mentioned cases of very closely-allied species; for in aberrant forms like *Œdicnemus* and *Machetes* some diversity is not much to be wondered at. Examples might also be taken from the *Procellariidæ*; but, to come nearer our present subject, among the *Columbæ* we have *Didunculus* with a single pair of broad lateral processes, and the space between the emarginations without any fenestration (Plate XXIII. fig. 170). In the more normal *Columbæ* the structure is much the same (Plate XXIII. fig. 169), except that the lateral processes are much narrower, but a monstrous example of *C. livia* (Plate XXIII. fig. 172) shows two pairs. In a specimen of *Patagioenas caribbæa* in our possession, where the middle of the hinder end of the sternum is very broad, there is a small circular fenestra on the left side, but none on the right (Plate XXIII. fig. 173). In *Goura* and *Leucosarcia* (Plate XXIII. figs. 176, 171) there are two pairs of lateral processes, and in the former (*Goura*) the outer pair is wide and its front edge distinctly turned forwards at its upper extremity. In the latter (*Leucosarcia*) the anterior pair of processes is tapering. In *Geopelia* (Plate XXIII. fig. 175) and for a long time in *Chamæpelia* there are two pairs of lateral processes, but in the latter (Plate XXIII. fig. 174) the inner pair eventually become connected at their extremities, and thus fenestræ are formed.

A singular modification of the posterior margin of the sternum seems, in some birds, to be connected with diving habits. In this respect we have *Cinclus* differing from nearly all the *Passeres*, and *Pandion* from all the *Falconidæ*; while an extraordinary development of the representative of the ensiform cartilage is observable in the Ducks of the group *Clangula* and the *Merginæ* when compared with other *Anatidæ*, and these birds, as is well known, are among the most excellent of divers.

None of the other fragments (Plate XVIII. fig. 75), though undoubtedly belonging to larger (male?) birds, could have come from individuals equalling in size that whose sternum is now at Paris, and believed to have been hitherto unique. Even making every allowance for the stalagmitic incrustation with which the specimen is encumbered, this bird could not have been less bulky than the largest *Didus* whose sternum has been preserved.

The very near affinity of *Pezophaps* to *Didus* is nowhere better shown than on comparison of the sterna of the two forms; and it is self-evident on the most cursory inspection of them. So far as can be determined from the imperfect state of all the specimens of each, hitherto recovered, there is no distinction of any very great importance to be detected. The general configuration of the two—their extraordinary helmet-like form—is precisely similar and unlike that of any other bird known to us. It may be that the keel in *Pezophaps* is relatively somewhat deeper than that of *Didus*, but *Didus* varies not inconsiderably in this respect, and it would be very unsafe, from what we see of other parts of the skeleton, to assert that *Pezophaps* did not vary also.

Yet a closer examination will reveal several characters more or less diagnostic. First it may be remarked that the coracoidal grooves are shallower and narrower in *Didus* than in *Pezophaps*, and in all the specimens, save one of the last, there is a deeper depression, almost a little pit, at the inner extremity of the groove, in a line and, occasionally, to some degree confluent with it. Above this pit there is in one specimen a second, smaller and subtriangular shallow depression. This same specimen is the only one that exhibits any part of the costal processes, and even here their edges have been broken off. It has the same distal enlargement and concave extremity as *Didus*; but the concavity is semi-divided by a well-defined ridge stretching halfway across the cup. A more remarkable difference is presented by the costal border in this specimen, which shows articular surfaces for four sternal ribs only instead of five, which is the normal number in *Didus**, and, so far as can be determined from the broken state of the remaining specimens, there is nothing to induce the belief that they possessed more than four such surfaces. A third character which seems to prevail in *Pezophaps* (being found in all our six examples, and also remarkably well shown in the specimen of the Museum at Paris) is by no means constant in *Didus*, such indication of it as exists at all occurring there, so far as our experience goes, only in the smaller examples. This is the presence in *Pezophaps* of a very deep median hollow near the anterior end of the internal surface of the sternum—a hollow so deep that in five (including the Paris example) out of seven instances the bone is actually perforated, and in one the perforation (the external orifice of which is situated below the coracoid grooves) is of very considerable size. In most specimens of *Didus* where anything like this depression exists it seems to be connected with the

* Too much importance must not be placed on this character. In one of the sterna of *Didus* in our possession, *five* articular surfaces are exhibited on the left side and only *four* on the right. In another specimen all the five surfaces on the left side are bilobed, while on the right (as seems to be usually the case) this is so with four only, and the anterior surface is simple.

pneumatic foramina of the bone, and in the largest examples it is, with them, altogether obliterated. In *Pezophaps*, however, it has a very distinct existence apart from the pneumaticity of the bone, and (as has just been said) it occurs conspicuously in the Paris specimen, which is equal in size to the largest *Didus*, where no trace of any depression, much less of a deep hollow, is present*.

§ 6. *Scapular Arch.*

Of this portion of the skeleton we have numerous scapulæ and coracoids, but unfortunately not a fragment which we can identify as belonging to the furcula. None of the specimens offer any trace of the remarkable anchylosis, observed not unusually in *Didus*, of these three bones, and from the considerable series in our possession it is perhaps not too much to conclude that such anchylosis never took place in *Pezophaps*.

The scapula (Plate XIX. figs. 84–86, 97–99, XX. figs. 132, 133), of which thirty-six specimens are contained in the collection, differs very greatly in its general form from that of *Didus*. Instead of being sabre-shaped and curved decidedly backward, it is rather straight, or, if anything, inclining generally somewhat forward—a character not known by us to exist in any other bird, and only pointing backward and inward at its extremity when it becomes spatulate in form. Considerable individual variety of shape, however, is found in this part in *Pezophaps*, as is also the case in *Didus*. As a whole the bone is comparatively stouter in *Pezophaps*, especially its proximal part, and the markings of muscular attachments are not so well defined, there being in this respect little difference between its inner and outer surface. The latter does not so often display towards the distal extremity the slight elongate cavity observable in *Didus*; but is, on the contrary, very frequently somewhat convex, while the inner surface is, almost without exception, decidedly concave, though to a variable extent. The process sent off from the lower and outer border, which is a marked and (so far as we can judge) an invariable feature in *Didus*, is represented merely by a slight notch formed by the abrupt termination of the ridge, which eventually becomes the edge of the thinner and more distal extremity of the bone. The articular end of the scapula is relatively thicker than in *Didus*, with the coracoidal surface occupying a proportionately larger space, while the humeral surface is not so extensive, and the acromial process is proportionately smaller, much less curved, and not containing in its hollow the pneumatic foramen which appears to be always present in *Didus*.

* In *Didunculus* there is at this part a good-sized hollow, at the bottom of which are some four or five pneumatic foramina. In *Goura* the median hollow is very deep and elongate, constricted in the middle; behind it lies a series of pneumatic orifices, and on each side a shallow depression. *Leucosarcia* is simpler, having a single deep median hollow and two lateral depressions. The same appearance is presented by *Trocaza* (a Mauritian form), *Columba*, and the more normal Doves. *Geopelia* and *Patagioenas* much resemble *Goura* in this part. In *Chamæpelia* there is little appearance of the deep hollow, and the lateral depressions are nearly obliterated. Of the normal *Gallinæ*, most forms, as *Phasianus*, *Perdix*, and *Coturnix*, have no lateral depressions here; but the genera *Cuvcabis*, *Francolinus*, *Ortyx* and *Lophortyx* possess them, while *Odontophorus* does not. A single median depression, which is often thickly set with orifices, exists in most of the genera *Tetrao*, *Bonasia*, and *Lagopus*.

Of the coracoid (Plates XIX. figs. 76–83, XX. fig. 136) the collection contains twenty-seven more or less perfect specimens, besides the anterior extremity of eleven others, thirty-eight in all. This most significant bone differs very greatly from the corresponding part in *Didus*—a fact not perhaps so surprising when the exceedingly abnormal form which it assumes there is taken into consideration. It is in *Pezophaps* proportionately much more bulky, and generally smoother. At its sternal end it differs from *Didus*, and indeed from all Columbine birds, in the extension and rounding off of the outer border so as entirely to mask the more or less elongate process which in some of the *Columbidæ* (*Columba livia* for instance) is turned distinctly forwards. Corresponding with this the inner border is produced into a sharper angle, though blunt at the tip, than it is in *Didus*, and much more so than in most other Pigeons*. Making allowance for the extreme attenuation of the coracoid in *Didus*, which is its especial characteristic, the surfaces of the coracoid in *Pezophaps* present, on closer study, much the same aspect. The upper surface at the sternal end (Plate XIX. figs. 78, 82) is more convex, and the lower one is less flat, but generally perforated with similar pneumatic foramina†, though these are variable in shape and extent, and sometimes, though seldom, entirely disappear. The muscular ridge and rough surface which mark the back part of this bone, below the middle of the shaft, in *Didus* are not usually so distinct in *Pezophaps*; the former, however, can be traced in all the specimens. At the upper end of the coracoid the articular surfaces (Plate XIX. figs. 77, 81) have much the same aspect as in *Didus*, an allowance being made for its increased relative thickness. The surface for the articulation of the scapula is, however, proportionately larger. It is very variable in form, being in most of the larger (male?) examples roughly quadrate, in the smaller (female?) triangular‡. In these last there commonly springs from the inner corner a somewhat elongate process, directed inward and slightly downward, and then curving backward. The humeral articular surface is not nearly so well defined, and is not oval and smooth as in *Didus*, but generally somewhat semilunar in shape, with two or three depressions, caused (as some of the specimens show) by minute foramina (which in others are entirely obliterated)§. The surface for the articulation of the

* In *Didunculus* the coracoid exhibits certain well-marked characters; whether they are diagnostic we will not say. Chief among them is the increased development of a muscular ridge along the inner edge of the sternal end. Arising at the front part of the sternal articulation it passes obliquely upward and backward till it ceases abruptly on meeting the usual ridge running along the internal side of the bone. At the other end of this bone also the usual internal curved process which serves as the basis of attachment of the upper side of the scapula is more largely developed than seems to be usual with the Doves, being extended downward so as most nearly to meet the furcula.

† The absence of pneumaticity in the coracoids of the *Ardeidæ*, and their consequent difference of colour, is a striking feature when the specimens have not been macerated so much as to remove all the grease from the bones.

‡ An additional proof of the value of the characters furnished almost invariably by this bone in birds, on which I have elsewhere slightly touched (Ibis, 1868, p. 95).—A. N.

§ In one specimen of the coracoid of *Didus* in our possession there exists a good-sized deep circular hollow on the lower side of the bone, immediately opposite to the humeral articular surface. There is no trace of this hollow in any of the other six coracoids we have. This same specimen presents on its upper aspect a singularly

furcula in *Pezophaps* is shorter and broader than in *Didus*. There is no trace in any of the specimens of an actual anchylosis of the coracoid with the furcula any more than with the scapula; but in many of the larger (male ?) ones, there is a considerable bony development on the edges of the articular surfaces, as if resulting from the partial ossification of the integument, which may perhaps be an approximation to that condition of the three bones which seems usually to have existed in *Didus*.

The exact angle formed by the axes of the coracoid and scapula at the point where they meet cannot positively be determined. To the best of our judgment, however, it cannot have been much less than it is in *Didus**.

§ 7. *Bones of the Wing.*

Compared with the wing-bones of *Didus* those of *Pezophaps* present, even on the most cursory inspection, a more massive aspect with a smoother surface. The humerus (Plate XIX. figs. 91, 92), of which the collection contains forty-six specimens, most of them in a sufficiently perfect condition†, has the proximal extremity considerably more rounded, with the oval convexity of the articular head absolutely (even in the smallest specimens) more elongated outwardly, and in front more transversely concave. The pectoral process projects very slightly, and the ulnar tuberosity is less developed. Of the pits at its base, the upper one is narrower and more oblique, the lower one much narrower, and with a very much larger pneumatic foramen. The longitudinal ridge, so strongly developed on the upper portion of the shaft in *Didus*, is here generally almost, and sometimes quite, obsolete. The muscular ridges of the distal end resemble more closely those of *Didus*. The cancellous structure of either end is perhaps rather less developed in *Pezophaps*.

The radius (Plates XIX. figs. 94, 95, XX. figs. 137, 138), of which forty-three specimens are contained in the collection, is a stout nearly straight subcylindrical bone in-

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increased development, into a similar but deeper and elongate hollow, of the depression commonly situated between the scapular and furcular articulating surfaces; a depression, however, very variable in its extent in other examples.

* This very marked characteristic of the sternal apparatus in *Didus* I had the pleasure of first making publicly known at the Meeting of the Zoological Society of London on the 12th of December, 1865, when I exhibited a drawing of the confluent coracoid scapula and furcula of that bird from a specimen sent me by my brother, which specimen arrived a few days after the meeting. On learning that Professor Owen was anxious to have the task of describing the skeleton of the Dodo (from the remains sent him by Mr. George Clark), I suppressed the paper containing the remarks I had made on that occasion (Proc. Zool. Soc. 1865, p. 732). I therein stated that (always excepting the *Struthiones*) I knew of no other bird but *Oeydromus* in which the angle formed by the axes of the coracoid and scapula was greater than a right angle, and I have not in the meantime been able to discover any other form presenting this remarkable feature, since recognized by Professor Huxley in his course of Hunterian Lectures for 1867, and again in his proposed scheme for the classification of birds (Proc. Zool. Soc. 1867, p. 425), as one of the distinguishing characters of the Orders (Subclasses ?) *Carinatæ* and *Ratitæ*.—A. N.

† The shaft of a left humerus, wanting its extremities, was picked up by me on my visit to Rodriguez in 1864, and has been figured (Proc. Zool. Soc. 1865, pl. viii. fig. 3).—E. N.

creasing in thickness from near the proximal end, and widening laterally very considerably towards the distal, where it expands to nearly double the original diameter of the shaft, and in the larger specimens (males?) is clothed on the side furthest from the ulna for nearly half an inch by an irregularly-shaped bony growth, having at first sight much the appearance of a stalagmitic encrustation, to which further reference will be made presently. In the smaller specimens the entire bone, though nearly half an inch shorter, is considerably thicker than in *Didus*, especially towards the distal end, in which part there is little resemblance between the two forms. The proximal articular surface though larger is much as in *Didus*.

The ulna (Plates XIX. figs. 93, 96, XX. figs. 134, 135), of which forty-one specimens are present, is cylindrical, straighter, and thicker than in *Didus*, with the muscular depression, described by Professor Owen, much less deeply marked, smaller, and situated more distally. In some specimens as many as ten impressions of the roots of the secondary wing-quills can be distinctly traced along the anconal surface, besides three elongated and transversely-directed depressions towards the distal end. On the radial side of these depressions there is present in two or three large and well-preserved specimens a small bony tuberosity.

Very many of the largest (male?) specimens of both ulna (Plate XX. fig. 135) and radius bear marks of fracture and healing during the bird's life.

Perhaps the most interesting of the bones in the whole collection is the metacarpal (Plates XIX. figs. 87–90, XX. fig. 131, XXII. fig. 148), of which no specimen belonging to any Didine bird has hitherto been seen. We have here thirty-two. That it would be very short was a safe inference from what we know of it in other flightless birds; but it could hardly have been expected to obtain from it such a singular confirmation of Leguat's statement regarding a remarkable peculiarity in the "Solitaire" as observed by him, nor that it should furnish an explanation of the curious bony growth on the distal end of the ulna and radius already mentioned as presented by the specimens of supposed males. All the perfect specimens of the metacarpal have on the radial side a more or less spherical bony knob or callus-like mass developed immediately beyond the proximal end and the pollex, which last would appear to be thrust away by it to some extent. This knob in the larger specimens, which we suppose to have belonged to males, is out of all proportion to anything of the kind yet known in ornithology. It is subject to a good deal of variation in form and to a less degree in size, sometimes projecting rather downward than laterally, but in all cases more or less pedunculate. In the largest (male?) specimens its longest diameter is about half the length of the entire metacarpal, and its transverse breadth in any direction is not much less. The metacarpal itself is pneumatic and light, the largest specimen weighing, the knob and all, just about a quarter of an ounce; but the knob, which has been broken off from another specimen, weighs as much as the remainder. The appearance of the knob is much that of diseased bone; it has probably been covered by a cartilaginous integument, which may have extended to the somewhat similar though

less developed osseous growths on the ulna and radius just mentioned. But unquestionably it answers most accurately to LEGUAT's words—

L'os de l'aileron grossit à l'extrémité, & forme sous la plume une petite masse ronde comme une balle de mousquet. (1st ed. vol. i. pp. 98, 99.)

The Bone of their Wing grows greater towards the Extremity, and forms a little round Mass under the feathers, as big as a Musket Ball. (Engl. transl. p. 71.)

Mr. JOSEPH GEDGE, of Gonville and Caius College, has been good enough to furnish us with the following account of the microscopic appearance of this singular structure:—

"On removing a portion of the exterior (after reducing it to a sufficient tenuity) it presented under a low power the cancellous appearance of moderately old callus. On examining it with a high power the lacunæ were for the most part irregular in form; some round with jagged edges, others irregularly stellate, connected together by processes from the lacunæ without the intervention of true canaliculi. A few normal lacunæ and a few vascular canals existed; but there was no lamination. The osseous matter was arranged so as to give an unusually fibrillated aspect to the bone. Irregularity in the form of the lacunæ may not unfrequently be observed in the exterior portion of osseous outgrowths (*e. g.* spavin), when the structure of the interior in no way departs from the ordinary characters of normal bone. It is, however, a character which belongs only to bone which has passed through the stage of rudimental fibrous tissue, and which has not been taken up and laid down again. This growth I consider to be of a nature that might have been produced by irritation of the periosteum, and its formation would appear to have been physiologically analogous to the commonest form of callus."

The indicial and median portions of the metacarpal bear to each other much the same relation that they appear to do in other *Columbæ*, such as *Columba* and *Didunculus*. The distal end has two well-marked articular surfaces for as many phalanges. There is no trace of quill-marks to be seen on any of the specimens.

The remaining bones of the wing are wanting.

§ 8. *Bones of the Leg.*

The leg-bones of *Pezophaps* when compared with those of *Didus* show very much more strongly-developed ridges and muscular impressions, being exactly the converse of what is to be remarked of the wing-bones of the two birds. As regards massiveness they are more nearly on a par; but in this respect the leg-bones of *Didus* vary considerably from one another*. Generally, however, the leg-bones of *Pezophaps* are decidedly longer than those of *Didus*, and the excess increases toward their extreme members. Thus while the smaller (female?) femora of *Pezophaps* are in many instances greatly surpassed in length by the larger (male?) ones of *Didus*, the corresponding tibio-tarsals of the two are most generally equal, and of the tarso-metatarsals of *Pezophaps* only some

* This variability is not likely to have been sexual. Of the two longest tibio-tarsals of *Didus* in our possession, that which is only very slightly (if at all) shorter than the other is most perceptibly and out of all proportion more slender.

of the very smallest (female?) examples fall short of those of *Didus*, so that it is likely that even the smallest (female?) *Pezophaps* stood higher on the leg than the largest (male?) *Didus*. Comparing, however, the bones of what we imagine to have been the same sexes (and this is, of course, the only fair mode of comparison) it is clear that the body of *Pezophaps* was mounted upon legs at least some two or three inches longer than that of *Didus*.

The principal bones of the leg having been already very minutely and correctly described by Dr. Melville (*op. cit.* pp. 115–119) and Strickland (Trans. Zool. Soc. iv. pp. 192–196), it seems unnecessary here to say much more about them. But there are some points to which we may perhaps advert with advantage.

The femur, of which the collection contains one hundred and nineteen nearly perfect examples, has a very great general resemblance to that of *Didus*; but it is of more uniform thickness, not tapering so much towards the middle or upper third. There is also in our series a total absence of the pneumatic foramen* on the inner side of the anterior ridge of the great trochanter, which is nearly always present and occasionally of large size in *Didus*. On the other hand there is in most cases a single good-sized and apparently pneumatic orifice on the posterior aspect of the neck of the bone, below the ridge of the articular surface which is adapted to the trochanterian prominence of the pelvis. This single orifice we have not found in *Didus*; but, there, the same part is often perforated by several smaller openings of variable position. Turning to the distal extremity of the femur, the popliteal fossa in *Pezophaps* generally presents at least one large, and often two or three smaller pneumatic orifices, which are perhaps more distinct than in *Didus*. The muscular ridges and impressions are usually very strongly marked.

The tibio-tarsal (Plate XX. figs. 100, 102), of which the collection contains sixty-six specimens in a fair state, besides a vast number in a fragmentary condition, has been already minutely described by the authors above mentioned. It need therefore only be said that the inner anterior ridge at the proximal end (which was broken off in Strickland's specimen) is generally fully as much developed as in *Didus*, and in many examples is certainly thicker, but its precise form is very variable in both birds.

The fibula (Plate XX. figs. 100–103), of which there are fifty-nine specimens in the collection, is very decidedly (it might almost be said absolutely) stouter and more massive than it is in *Didus*; but owing to the extreme variability of this bone it is difficult to seize on any characters that may be considered to be more than those of the individual. Perhaps on the whole the fibula of *Pezophaps* has a smoother surface; but two or three of the examples have their ridges and depressions as strongly marked as any of those of *Didus* which we have seen. As the tibio-tarsal is relatively longer in the former than in the latter, so is it (as might be expected) with the fibula.

The tarso-metatarsus, of which the collection contains one hundred and twenty-three examples in fair preservation, besides a great number of larger and smaller fragments,

* This foramen is said by Dr. Melville to be absent in all Pigeons except *Goura*. The last statement we can confirm of our own observation. It is also wanting in *Didunculus*.

has already been not only elaborately described by Dr. Melville and Strickland (*opp. citt.*), but also compared by them with the corresponding bone of *Didus*. It is accordingly unnecessary here to say more about it than to remark that the large series of specimens in our possession forbids us from coinciding in the generalization propounded by Strickland (Trans. Zool. Soc. iv. p. 196) as to the "inner or longest calcaneal process" in *Pezophaps* being considerably less developed than in *Didus*. This process, now regarded* as the head of the third (anchylosed) metatarsal, does not project posteriorly (it is true) so much as in *Didus*; but this fact appears to us to be in consequence of the heads of the second and fourth (anchylosed) metatarsals being expanded laterally rather than antero-posteriorly. In like manner, and for the same reason, we are unable to confirm Strickland's generalization as to the three trochleæ at the lower extremity being placed more nearly in the same vertical plane in *Didus* than in *Pezophaps*; for in this last the large series of examples shows that the angle formed by joining the centres of the three trochleæ (of course the only mode of accurately testing the observation) is much more obtuse in the smaller (female?) specimens than in the larger (male?). In this case, if any stress is to be laid on Strickland's inference (*ut suprà cit.*), it would follow that the larger (male?) examples of *Pezophaps* did not run so quickly as ordinary examples of *Didus*, while the smaller (female?) attained a greater speed. The orifice of the calcaneal canal is absolutely smaller in *Pezophaps* than in *Didus*, a fact apparently arising from the greater thickening in the former of the "calcaneal processes." For the rest we may safely leave this bone as already accurately and sufficiently described.

The posterior metatarsal (Plate XX. figs. 104, 105), of which fifteen specimens are contained in the collection, is one of the few bones we have not had the opportunity of comparing with the corresponding part of *Didus*. Judging, however, from the figures and a model of the celebrated Oxford specimen, it would appear to be more massive and to present a smoother surface, the furrow for lodging the flexor tendons of the hallux not being so deep, and the whole bone is less crooked. The proximal articular surface is more convex in the middle than it would appear to be in *Didus*; but its lower portion is roughened just as is said to be the case there. The styloid process is much less developed than it would seem to be in *Didus*, and on the surface which is twisted forwards it becomes almost flush with the trochlea, so that its boundaries are not strongly defined; on the other hand the pit on the inner extremity of the trochlea is to all appearance as deep as in *Didus*. Generally this bone in *Pezophaps* would seem to be longer, which is of course in proportion to the increased length of the lower extremities.

Among the phalanges, of which the collection contains in all one hundred and fifty-specimens, there is wanting the second and third of the inner, and the second, third, and fourth of the outer toes. The first, or proximal phalanx of the hind toe (Plate XX. figs. 106, 107) seems to be a good deal stouter relatively than it is in *Didus*,

* *Cf.* Gegenbaur, Arch. für Anat. und Physiol. 1863, pp. 450–472; Untersuchungen zur vergleichenden Anatomie der Wirbelthiere (4to, Leipzig: 1864), pp. 93–108, pl. vi.

and the ungual segment (Plate XX. figs. 108, 109) proportionally longer. In the inner toe the proximal phalanx (Plate XX. figs. 128–130) is the longest in the whole foot, whereas in *Didus* it would appear to be about equal in length to that of the middle toe. This (Plate XX. figs. 116–118), again, is relatively and in large (male?) specimens absolutely longer and especially thicker than that of *Didus*, and the next two phalanges of this toe (Plate XX. figs. 119–124) are in proportion to it, but the ungual segment (Plate XX. figs. 125–127) is longer than the third. Of the outer toe the first (Plate XX. figs. 110–112) is the shortest of the proximal phalanges of the whole foot. The other segments of this toe, except the ungual (Plate XX. figs. 113–115), are missing. This is also, like the rest of the ungual phalanges, larger and especially longer than in the single corresponding example of *Didus*.

§ 9. *Skull.*

Of the cranial portion of the skull (Plates XXI. figs. 139–147, XXII. figs. 149, 150) the collection includes five specimens, more or less imperfect, but all containing the foramen magnum, which, as in *Didus*, but not to the same extent, varies not a little in shape, one specimen (of a male?) having it much wider below than above, but in the others the upper and lower curves are more nearly alike. So also the occipital condyle is to some extent inconstant. In the (male?) specimen just mentioned, and in two at least of the smaller ones, it slopes gradually from its base; but in an example of median size it rises abruptly, and on one side (the left) slightly overhangs its base, so that regarding it from that aspect it looks as if it were pedunculate.

Three of the specimens (Plate XXI. figs. 141, 144, 147) are sufficiently perfect to show the supraoccipital orifice, the existence of which in *Pezophaps* was surmised by Dr. Melville (*op. cit.* p. 114). The other small foramina and protuberances correspond singularly with those in *Didus*, exhibiting the same amount of individual variation.

On the other hand the cranium of *Pezophaps* differs remarkably from that of *Didus* in several important features, some of which have been already pointed out by Dr. Melville (*loc. cit.*), while others will be rendered more apparent by an inspection of the accompanying figures. It is narrower and longer, entirely destitute of the peculiar frontal protuberance, and generally more compressed and less ventricose, while the orbital chambers are larger. In one respect, however, that accurate osteologist has been misled. He attributes the apparent depression of the central tract of the upper surface entirely to the stalagmitic deposit on its anterior and posterior parts in the only specimen then known. In the specimens in the present collection, which are wholly free from that deposit, this apparent depression is equally to be observed. It is due, however, to the elevation of the anterior and particularly of the posterior parts into two bony ridges of the cancellous structure which in *Didus* occupies the whole surface, and indeed is most prominently developed on its central portion (Trans. Zool. Soc. vi. pl. 23. fig. 1), but in *Pezophaps* does not rise in the middle to any considerable degree, and consequently leaves the exterior surface almost perfectly flat, while the posterior ridge, being continued

forward on either side, a somewhat dish-like formation is given to the top of the head*.

The frontals rise abruptly as in *Didus*, but, as will have been gathered from what has just been said, to a much less extent. The cranium also in this place is much compressed, the largest specimen measuring only 1·76 inch at the narrowest part between the orbits, and the interorbital septum is not so thick as in *Didus*, but composed of the same cancellous structure. None of the specimens are sufficiently perfect to show the entire extent of the posterior olfactory chambers, which are, as in *Didus*, partially divided. At the junction of the nasals with the nasal process of the premaxillary, the proximal portion of which is not preserved in any of the specimens, the surface is extremely rugose, indicating the position of the caruncular ridge, which, from Leguat's figure and description, we already knew the bird to have possessed at the base of its bill.

In the upper mandible (Plate XXII. figs. 151–158) we have presented one of the most remarkable differences between *Didus* and *Pezophaps*. Instead of that singular characteristic of the former (which has been duly dwelt on by previous investigators), the vertical enlargement and coalescence, before reaching the core of the dertrum, of the maxillary branches of the premaxillary, we have in the latter a structure much more in accordance with the normal *Columbæ*, and presenting an appearance laterally though not from below much as in *Treron*. The maxillary branches of the premaxillary in *Pezophaps* approach each other gradually, but do not coalesce until about halfway between the commencement and extremity of the core of the dertrum, and thus they do not meet the nasal process of the premaxillary at an acute angle immediately behind the core of the dertrum, but the core of the dertrum intervenes vertically (Plate XXII. figs. 152, 155, 157). In other words, the axes of the nasal process and maxillary branches in *Didus* are convergent, in *Pezophaps* divergent.

There is remarkable variation in the size of the upper mandible in different individuals to the extent of very nearly one-half the linear dimensions between the largest and smallest specimens, of which the collection contains thirteen in all, some of which, however, are merely fragmentary, and the best exceedingly imperfect. No portion of the palatals is preserved.

Of the lower mandible (Plate XXII. figs. 159–162) the collection contains thirty-four fragments, one-half belonging to the distal and the other half to the proximal end. It has not been possible to fit together with certainty any two of them so as to form a perfect ramus. To one of the dentary portions, however, part of the core of the symphysis (Plate XXII. fig. 162) remains attached, and there is a second such core which is perfect though detached from both its rami. Still the exact outline of the lower as of the upper mandible must be left in some degree a matter of conjecture. Enough, however, is present to prove that it was sufficiently different in shape from that of *Didus*,

* It is proper to observe that in the only portrait we possess of the bird, that of Leguat, there is no trace of this formation to be found.

and still more from that of *Didunculus*. In the sudden vertical narrowing of the dentary in front of its junction with the surangular (Plate XXII. fig. 160) *Pezophaps* shows a greater approach to some of the more normal *Columbæ*, *Treron* for example. While in *Didunculus* the lower mandible sweeps on with a continuous downward curve, and in *Didus* it exhibits but a slight projecting angle at its splenial region (in both cases preserving nearly the same vertical height throughout), in *Pezophaps* (the upper edge of the whole mandible being apparently much straighter than in *Didus*) the angular and surangular slope steeply down to meet the dentary, which immediately afterwards rises as steeply, thereby forming on its lower edge a salient angle altogether wanting in *Didunculus* and not very considerable in *Didus*. From this the dentary rapidly tapers till it descends to form the gonys, which is fully as much developed proportionately as in *Didus*, while the angle formed by the two rami at their symphysis is somewhat more acute, and thus, with the less massive proportions of the upper mandible already described, the whole bill of *Pezophaps* must have been very much more slender and less powerful than the formidable, not to say monstrous weapon of *Didus*. The various foramina for the passage of the different nerves or vessels are situated much as in *Didus*. This is especially noticeable in the case of the large dental canal, which opens externally nearer the upper than the lower edge, while in *Didunculus* it occupies a position nearly midway.

The proximal portion of the lower mandible (Plate XXII. figs. 159, 161) offers further differential characters between *Didus* and *Pezophaps*. Notwithstanding the small size of the entire head in the latter, the angular and articular bones are fully as large as in the former, though neither the internal pyramidal process nor the external semilunar plate are so largely developed. The basal facet therefore assumes a very different form. Instead of presenting as in *Didus* the shape of a nearly isosceles triangle, the equal angles of which are subtended by the sides forming the internal process, it is in *Pezophaps* more equilateral; its extreme width only slightly exceeds its height, and its nearly vertical surface is less concave. The walls of the articular surface also are much thicker absolutely, and of course still more so comparatively, in *Pezophaps* than in *Didus*, and that which supports the concave reniform tract on its outer surface thrust out as to form a much more considerable projection in place of the elongated tubercle in *Didus*. As if further to add to the solidity of this part of the jaw, the tract for the insertion of the external pterygoid muscle is perceptibly less deeply concave.

Of the quadrate or "tympanic" (Plate XXII. figs. 163–168), the collection includes nineteen specimens. We have not had any opportunity of comparing this bone in *Pezophaps* with the same in *Didus*, and the latter has not been figured separately, but only represented *in situ*, and consequently with some of its parts foreshortened and concealed. To judge, however, from the figures, the bone in the two birds is of much the same general form, though in *Didus* the angle formed by the mastoid articular segment and the orbital process seems to be more filled up than in *Pezophaps*, thereby making this process more slender, while in *Pezophaps* the mandibular condyle appears to

be more elongated and transverse, and the process to which the squamosal is articulated does not project backward so far as in *Didus*. There is still less resemblance in the structure of this part to the same in *Didunculus*, where the inferior articular surface deviates, as has been already stated by Dr. MELVILLE (*op. cit.* pp. 99, 100), most greatly from the normal Columbine form. In point of bulk, the quadrate of *Pezophaps* would seem to be nearly absolutely as large as that of *Didus*. The intero-posterior cavities present very considerable individual variation both in shape and extent.

The remaining parts of the skull are unfortunately wanting.

§ 10. *Comparison of the Skeleton.*

As the whole of the preceding description has been in a manner comparative, there remains little to be said that can be placed under this heading. Nevertheless, to preserve the uniformity of arrangement with Professor OWEN'S treatise on the Dodo, we here include such general remarks on the subject as we have to make.

The most important distinctions between *Pezophaps* and *Didus*, as may be gathered from what we have already said, are on the whole such as not to remove the former further from the normal *Columbæ* than the latter. In several respects (of which the enormous development of the cancellous boss on the top of the head, the general form of the lower mandible, and, above all, the extreme attenuation of the coracoid in *Didus* may be mentioned as chief) *Pezophaps* holds a middle place between that form and ordinary Pigeons. On the other hand the armature of the wing is a character equally remote from either*. We had expected to have found that *Pezophaps* would prove to be a link between *Didus* and *Didunculus*; but this is not the case: the latter seems to be a perfectly distinct form, and, in our opinion, sufficiently isolated from other *Columbæ* to be considered the type of a separate family, in which light it has before been regarded; but at the same time this family *Didunculidæ* must not be considered to be so aberrant as the *Didide*, which we look upon as the most remotely connected of the Order COLUMBÆ, consisting of three well-marked families of equal rank, (1) DIDIDÆ, (2) COLUMBIDÆ, and (3) DIDUNCULIDÆ, the first and last of which are only related to each other by means of the second, and have no direct affinity, however much superficial resemblance there may be between the beak of *Didunculus* and that of *Didus*.

The grounds on which STRICKLAND ('The Dodo' &c. p. 54) established the genus *Pezophaps*, by separating it from *Didus*, were merely "the greater length of the legs and the less development of the beak." The result of our investigations prove that he was amply justified in coming to this conclusion. Still we would desire to see his genus more firmly established, and accordingly we venture here to propound its characters in set phrase as follows:—

PEZOPHAPS, Strickland, 1848. *Genus* Columbarum *Didinum*.

Char. gener. Rostrum mediocre, curvatum, processu nasali et ramis maxillaribus antice divergentibus. Frons plana, porcâ osseo-cancellatâ circumdata. Ossa coracoidea

* In *Goura*, as Professor HUXLEY has lately most kindly pointed out to us, the wing is armed with a knob, but it is, even comparatively, much smaller than in *Pezophaps*.

robusta. Alæ breves, involatiles. Manus singulis bullis osseo-callosis armatæ. Collum et pedes longiores.

In like manner the allied genus might be defined:—

Didus, Linnæus, 1766.

Char. gener. emend. Rostrum magnum, aduncum, processu nasali et ramis maxillaribus antice convergentibus. Frons tumida in umbonem hypoconicum osseo-cancellatum surgens. Ossa coracoidea attenuata scapulis obtuse attingentia. Alæ breves, involatiles. (Manus inermes?) Collum et pedes breviores.

§ 11. *Concluding Observations.*

It may be convenient here to quote at length the account given by Leguat of this bird as observed by him during his sojourn at Rodriguez from the 1st of May, 1691, till the 21st of May, 1693.

Voyage et Avantures de François Leguat *& de ses Compagnons, en deux isles désertes des Indes Orientales.* Londres: 1708. 2 vols. 12mo.

De tous les oiseaux de l'Isle, l'espèce la plus remarquable est celle à laquelle on a donné le nom de *Solitaires*, parce qu'on les voit rarement en troupes quoi qu'il y en ait beaucoup.

Les mâles ont le plumage ordinairement grisâtre & brun, les pieds de coq d'Inde, & le bec aussi, mais un peu plus crochu. Ils n'ont presque point de queuë, & leur derrière couvert de plumes est arrondi comme une croupe de cheval. Ils sont plus haut montez que les coqs d'Inde, & ont le cou droit, un peu plus long, à proportion, que ne l'a cet oiseau quand il lève la tête. L'œil noir & vif, & la tête sans crête ni houpe. Ils ne volent point, leurs ailes sont trop petites pour soutenir le poids de leurs corps. Ils ne s'en servent que pour se battre, & pour faire le moulinet, quand ils veulent s'appeler l'un l'autre. Ils font avec vitesse vingt ou trente pirouëttes tout de suite, du même côté, pendant l'espace de quatre ou cinq minutes: le mouvement de leurs ailes fait alors un bruit qui approche fort de celui d'un Crécerelle; & on l'entend de plus de deux cens pas. L'os de l'aileron grossit à l'extrémité, & forme sous la plume une petite masse ronde comme une balle de mousquet: cela & le bec, sont la principale défense de cet oiseau. On a bien de la peine à les attraper dans les bois, mais comme on court plus vite qu'eux, dans les lieux dégagez, il n'est pas fort difficile d'en prendre. Quelquefois même on en approche fort aisément. Depuis le mois de Mars jusqu'au mois de Septembre, ils sont extraordinairement gras, & le goût en est excellent, sur tout quand ils sont jeunes. On trouve des mâles qui pèsent jusques à quarante cinq livres.

A New Voyage to the East Indies by Francis Leguat *and His Companions. Containing their Adventures in two Desert Islands.* London: 1708. 1 vol. 8vo.

Of all the Birds in the Island, the most remarkable is that which goes by the Name of the *Solitary*, because 'tis very seldom seen in Company, tho' there are abundance of them. The Feathers of the Males are of a brown, grey Colour: The Feet and Beak are like a Turkeys, but a little more crooked. They have scarce any Tail, but their Hind-part cover'd with Feathers is Roundish, like the Crupper of a Horse, they are taller than Turkeys. Their Neck is straight, and a little longer in proportion than a Turkeys, when it lifts up his Head. Its Eye is black and lively, and its Head without Comb or Cop. They never fly, their Wings are too little to support the weight of the Bodies; they serve only to beat themselves, and flutter when they call one another. They will whirl about for twenty or thirty times together on the same side, during the space of four or five Minutes: The Motions of their Wings makes then a Noise very like that of a Rattle; and one may hear it two hundred Paces off. The Bone of their Wing grows greater towards the Extremity, and forms a little round Mass under the Feathers, as big as a Musket Ball: That and its Beak are the chief Defence of this Bird. 'Tis very hard to catch it in the Woods, but easie in open Places, because we run faster than they, and sometimes we approach them without much Trouble. From *March* to *September* they are extremely fat, and tast admirably well, especially while they are young, some of the Males weigh forty-five pounds.

La femelle est d'une beauté admirable ; il y en a de blondes & de brunes ; j'appelle blond, une couleur de cheveux blonds. Elles ont une espèce de bandeau comme un bandeau de veuves au haut du bec qui est de couleur tanée. Une plume ne passe pas l'autre sur tout leur corps, parce qu'elles ont un grand soin de les ajuster, & de se polir avec le bec. Les plumes qui accompagnent les cuisses sont arrondies par le bout en coquilles ; & comme elles sont fort épaisses en cet endroit-là cela produit un agréable effet. Elles ont deux élévations sur le jabot, d'un plumage plus blanc que le reste, & qui représente merveilleusement un beau sein de femme. Elles marchent avec tant de fierté & de bonne grace tout ensemble, qu'on ne peut s'empêcher de les admirer & de les aimer ; de sorte que souvent leur bonne mine leur a sauvé la vie.

Quoique ces oiseaux s'approchent quelquefois assez familièrement quand on ne court pas après eux, on ne peut jamais les apprivoiser : si tôt qu'on les arrêtez ils jettent des larmes sans crier, & refusent opiniâtrement toute sorte de nourriture, jusqu'à ce qu'ils meurent enfin. On leur trouve toûjours dans le gésier, (aux femelles aussi bien qu'aux mâles) une pierre brune de la grosseur d'un Oeuf de poule ; elle est un peu raboteuse, platte d'un côté & arrondie de l'autre, fort pesante, & fort dure. Nous avons jugé que cette pierre naît avec eux ; parce que quelque jeunes qu'ils soient, ils en ont toûjours, & n'en ont jamais qu'une ; & qu'outre cela, le canal qui va du jabot au gésier, est trop étroit de moitié pour donner passage à une pareille masse. Nous nous en servions préférablement à aucune autre pierre pour aiguiser nos couteaux.

Quand ces oiseaux veulent bâtir leur nids, ils choisissent un lieu net, & ils l'élèvent à un pied & demi de terre sur un tas de feuilles de palmier qu'ils ont ramassées pour ce dessein. Ils ne pont qu'un Oeuf, qui est beaucoup plus gros que celui d'une oye. Le mâle & la femelle le couvent tour à tour, & il n'éclôt qu'après sept semaines. Pendant tout le tems qu'ils couvent, ou qu'ils élèvent leur petit, qui n'est capable de pourvoir seul à ses besoins qu'après plusieurs mois, ils ne soufrent aucun oiseau de leur espèce à plus de deux cens pas à la ronde ; & ce [*lege* ce] qui est assez singulier, c'est que le mâle ne chasse jamais les femelles ; seulement, quand il en aperçoit quelqu'une, il fait en pirouëttant son bruit ordinaire, pour apeller la femelle qui vient donner aussi-tôt la chasse à l'étrangère, & qui ne la quitte que lorsqu'elle l'a conduite hors de ses limites. La femelle en fait de même & laisse chasser

The Femals are wonderfully beautiful, some fair, some brown; I call them fair, because they are of the colour of fair Hair: They have a sort of Peak like a Widow's upon their Breasts [*lege* beaks], which is of a dun Colour. No one Feather is stragling from the other all over their Bodies, they being very careful to adjust themselves, and make them all even with their Beaks. The Feathers on their Thighs are round like Shells at the end, and being there very thick, have an agreeable effect: They have two Risings on their *Craws*, and the Feathers are whiter there than the rest, which livelily represents the fine Neck of a Beautiful Woman. They walk with so much Stateliness and good Grace, that one cannot help admiring and loving them; by which means their fine Mein often saves their lives.

Tho' these Birds will sometimes very familiarly come up near enough to one, when we do not run after them, yet they will never grow Tame: As soon as they are caught they shed Tears without Crying, and refuse all manner of Sustenance till they die.

We find in the Gizards of both Male and Female a brown Stone, of the bigness of a Hens Egg, 'tis somewhat rough, flat on one side, and round on the other, heavy and hard. We believe that this Stone was there when they were hatch'd, for let them never be so young, you meet with it always. They have never but one of 'em, and besides, the Passage from the Craw to the Gizard is so narrow, that a like Mass of half the bigness cou'd not pass. It serv'd to whet our knives, better than any other Stone whatsoever.

When these Birds build their Nests, they choose a clean Place, gather together some Palm-Leaves for that purpose, and heap them up a foot and a half high from the Ground, on which they sit. They never lay but one egg, which is much bigger than that of a Goose. The Male and Female both cover it in their turns, and the young is not hatch'd till at seven Weeks end: All the while they are sitting upon it, or are bringing up their young one, which is not able to provide for its self in several Months, they will not suffer any other Bird of their Species to come within two hundred yards round of the place: But what is very singular, is, The Males will never drive away the Females, only when he perceives one he makes a noise with his Wings to call the Female, and she drives the unwelcome Stranger away, not leaving it till 'tis without her Bounds. The Female do's the same as to the Males, whom she leaves to the

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les mâles par le sien. C'est une particularité que nous avons tant de fois observée, que j'en parle avec certitude.

Ces combats durent quelquefois assez long-tems, parce que l'étranger ne fuit qu'en tournant, sans s'éloigner directement du nid; cependant, les autres ne l'abandonnent jamais qu'ils ne l'ayent chassé. Après que ces oiseaux ont élevé leur petit & l'ont abandonné à lui-même, ils ne se départent pas comme font tous les autres, mais ils demeurent toujours unis & compagnons, quoi qu'ils aillent quelquefois se mêler parmi d'autres de leur espèce. Nous avons souvent remarqué que quelques jours après que le jeune étoit sorti du nid, une compagnie de trente ou quarante en amenoient un autre jeune, & que le nouveau déniché avec ses père & mère, se joignant à la bande, s'en alloient dans un lieu écarté. Comme nous les suivions souvent, nous voyions qu'après cela, les vieux fe [*lege* se] retiroient chacun de leur coté, ou seuls, ou couple à couple, & laissoient les deux jeunes ensemble; & nous appellions cela un mariage.

Il y a dans cette nouvelle circonstance, quelque chose qui semble un peu fabuleux: mais ce sont pourtant des véritez pures, & des choses que j'ai bien souvent remarquées avec soin, & avec plaisir. Je ni pouvois m'empêcher non plus, d'abandonner mon esprit à diverses reflexions.—vol. i. pp. 98–102.

Male, and he drives them away. We have observ'd this several times, and I affirm it to be true.

The Combats between them on this occasion last sometimes pretty long, because the Stranger only turns about, and do's not fly directly from the Nest: However, the others do not forsake it, till they have quite driv'n it out of their Limits. After these Birds have rais'd their young One, and left it to its self, they are always together, which the other Birds are not, and tho' they happen to mingle with other Birds of the same Species, these two Companions never disunite. We have often remark'd, that some days after the young one leaves the Nest, a Company of thirty or forty brings another young one to it; and the new fledg'd Bird with its Father and Mother joyning with the Band, march to some bye Place. We frequently follow'd them, and found that afterwards the old ones went each their way alone, or in Couples, and left the two young ones together, which we call'd a *Marriage*.

This Particularity has something in it which looks a little Fabulous, nevertheless, what I say is sincere Truth, and what I have more than once observ'd with Care and Pleasure; neither cou'd I forbear to entertain my Mind with several Reflections on this Occasion.—pp. 71–74.

The only other original testimony, of which we know, with regard to the Solitaire (if we except the vague notice, already mentioned, of Sir Thomas Herbert) is that of D'Heguerty* in 1754, as quoted in 1849 by Strickland†, who states that D'Heguerty had been Governor of the Island of Bourbon (now Réunion) about the year 1734, and a few years afterwards thus expressed himself of the sister Island of Rodriguez:—

"On y trouve aussi des oiseaux de différentes espèces, que l'on prend souvent à la course, et entre autres des Solitaires, qui n'ont presqu' point de plumes aux ailes; cet oiseau, plus gros qu'un Cygne, a la physionomie triste; apprivoisé on le voit toujours à la même ligne, tant qu'il a d'espace, et retrograder de même sans s'en écarter. Lorsqu'on en fait l'ouverture, on y trouve ordinairement des Bézoards, dont on fait cas, et qui sont utiles dans la médecine."

Such is the meagre information‡ we have respecting the history of this remarkable form, blotted out from existence within the last hundred and fifty years, by means at which we can only guess. It is now no longer a matter of doubt that many species of

* "Mémoires de la Société Royale des Sciences et Belles Lettres de Nancy, vol. i. p. 79." We have not been able to consult this work.

† Ann. and Mag. Nat. Hist. 2nd ser. iii. pp. 138, 139.

‡ For a further notice, which renders it probable that the species lingered so late as 1761, see Postscript.

animals have become extinct through the agency of man; but in nearly all the cases the history of which is sufficiently well known to us, the extirpation has been caused less by his direct than by his indirect agency. The most noticeable exceptions to this rule are perhaps those of the Gare-fowl (*Alca impennis*)* and the Northern Manatee (*Rhytina gigas*). And while in the first of these the submersion of its chief remaining breeding-station, owing to a volcanic eruption (and, we may presume, an accompanying earthquake), unquestionably was not without its influence, it does seem as if man himself actually, though unintentionally, had destroyed these two species. But with the Solitaire of Rodriguez, as with the Dodo and many other animals, it seems to us that there is no good ground for believing that they were pursued to the death of the race by man. It is far more likely that they succumbed to other forces set in motion indeed by him but without a thought of thereby accomplishing their destruction. It does not seem at all unreasonable to suppose that either or both might have survived for many years his direct attacks. They would doubtless have become more shy, and have retired to the most secluded parts of the islands they inhabited. They might even have acquired greater wariness, and been enabled in some manner to baffle his pursuit. But they had not to contend with man alone. He had allies fighting against them. In our own country we see that man has not, after many hundred years of incessant warfare, succeeded in extirpating by direct action a single species of bird. Birds of prey still exist, and in some districts in not inconsiderable numbers, in spite of the war of extermination carried on against them. In those districts where their destruction has been most nearly accomplished, it is because man has fought them with other weapons, more fatal because apparently more peaceful. It is to the plough and the draining-spade, with the changes that have followed their use over large areas, rather than to the gun or the gin that they have succumbed. The Crane and the Wild Goose have been banished from the English fens with the Harriers, and more species than we can here stay to enumerate by the simple act of bringing under cultivation by means of improved drainage the extensive tract of the "Bedford Level." The Bustard has yielded to the driver of the horse-hoe and the maker of plantations†. These have gone from us without an idea that any such effects would follow the causes employed; nay, they have gone from us, some of them, in spite of legal protection, and therefore against the will of man. Now, so far as we are able to judge, the Solitaire of Rodriguez has been subject to no disturbances of this kind. The island seems to bear now much the same appearance as it presented to the eyes of LEGUAT‡, and if the popular tradition be true that forests once existed in it which were destroyed some forty or fifty years ago by fire, it seems to be certain, from GRANT'S silence in 1801§, that there were then no Solitaires left to perish in the flames. Some other cause of extinction must therefore be sought, and it

* Though it may still be premature to speak of this species as extinct, there can be no doubt about its fate should a colony yet be discovered. (*Cf.* Natural History Review, October 1865, pp. 467–488.)

† STEVENSON'S 'Birds of Norfolk,' vol. ii. pp. 9–17. ‡ Ibis, 1865, p. 148.

§ History of Mauritius and the Neighbouring Islands, pp. 100–145.

has been already long since suggested* that Cats, turned out by the Dutch settlers may have destroyed the species "by devouring the young ones as soon as they were hatched." That this may have been the case is quite possible; but it seems quite as, or even more, likely that a feller enemy than the Cat may have been introduced. We mean the Hog, which we cannot but believe was the principal agent in destroying the Dodo in Mauritius†, though, so far as we are aware, the efficacy of this agent of destruction has not been sufficiently appreciated by former writers. It has been always so universal a practice to liberate Pigs in countries newly discovered by Europeans that we cannot suppose any exception to have been made in the case of Rodriguez, and at almost every place where these omnivorous animals have been set free we know them to have speedily increased and multiplied, replenished the land, and in most instances to have subdued it.

In every way save one, perhaps, the account given by Leguat of the Solitaire of Rodriguez seems to be fully borne out by a study of its osteology, so far as that can throw any light on the subject. The rugose surface at the junction of the nasal bones with the nasal process of the premaxillary indicates the position of the caruncular ridge which he likens to a "widow's peak," and in his figure represents at this place. The curved contour-lines of the upper and posterior part of the pelvis is quite in accordance with his statement that the "hind-part" of this bird was "roundish, like the crupper of a horse." We can well imagine that it was a somewhat less heavy bird than the Dodo‡, and, judging from its longer legs, that it was capable of attaining considerable speed. The remarkable and unexpected confirmation of his description of the extraordinary knob on the wing has been already noticed—a fact which alone would place him high in the ranks of truthful observers. The appearance presented by this knob is so exactly that of diseased bone that one is led to believe that it had its origin in injuries received by the birds in such combats as are mentioned by Leguat, and was aggravated by a continuance of fighting-propensities transmitted from generation to generation. Other Pigeons, despite their proverbially peaceful reputation, are as much given to fighting as

* Proc. Zool. Soc. 1833, pp. 31, 32.

† In 'A Voyage to *Arabia Fœlix* through the Eastern Ocean and the *Streights* of the Red-Sea, being the First made by the French in the Years 1708, 1709, and 1710. Translated from the French' (London: Printed for E. Symon, 1732, small 8vo, pp. xvi. and 372), the editor, whose name (as appears for the anonymous translator's preface, p. vii) was La Roque, says that the expedition touched at Mauritius in Sept. 1708, when they found (p. 147) that "on the other side of the Isle beyond the Mountains, there were Droves of wild Boars; that, not long before, a general Hunting had been order'd to root them out; and that, the Inhabitants having assembl'd for that purpose, they slew above fifteen hundred of them in one Day." It is plain that inactive birds could not long be the inhabitants of an island swarming with feral swine. It is perhaps possible to imagine a venerable Dodo capable of assuming sufficient dignity to disconcert an impetuous Hog, but the awe of the latter once dispelled by any mark of resentment the result would be the same as when the Roman Senator provoked the fury of the Gaulish soldier; while the more helpless part of the community would rapidly fall victims, young birds and eggs being inevitably traced to destruction by the unerring power of scent possessed by the riotous new-comers. (*Cf.* Leguat, 1st ed. ii. pp. 70, 71, Engl. transl. p. 170.)

‡ Herbert (*cf.* Strickland, 'The Dodo, &c.', pp. 19, 20) puts the weight of the Dodo at fifty pounds; Leguat (*ut supr. cit*) that of the Solitaire at forty-five.

many other birds, and theirs is literally *pugnacity*, for, the majority not possessing any other weapons of attack or defence, their battles consist of a succession of cuffs and buffetings administered by the wings. LEGUAT speaks of his having witnessed the combats of his Solitaires, and it is quite possible that these engagements were occasionally even more serious than any of which he was an eye-witness, for we find in the collection a fair proportion of broken bones, broken and mended during the bird's life. Among these are examples of the ulna (Plate XX. fig. 135), radius, and coracoid (Plate XX. fig. 136)*.

The particular in which we think LEGUAT may have erred is in the assertion (or perhaps rather inference) as to their monogamous habits, and the cause of the error (if such it be) may, we think, be ascribed, without derogation of his truthfulness or accuracy, to his anxiety to point a moral which has led him to imagine he saw what he wished to see†. We think we are right in saying that most monogamous animals when they fight at all fight without regard to sex. Now in describing the combats of these birds he especially mentions that the opposite sexes would not fight with one another, but that the combatants were invariably of the same sex, and this is just what occurs in polygamous birds. However it is not always easy to decide whether a species is monogamous or the contrary. Respecting a species which has been so much studied, and of which so much has been written as the Bustard (*Otis tarda*), it seems undecided whether it is polygamous or not, though the testimony is on the whole in favour of its occasionally being so‡. It is therefore not to be wondered at if LEGUAT, setting aside even his evident leaning, should have been mistaken in this particular. We venture here to reproduce some remarks on this matter, which one of us has already made.

"So far as I know, none of the existing *Columbæ* represent any remarkable sexual distinctions§ either in bulk or otherwise. As a group, the Pigeons are remarkably, and, relatively to most birds, abnormally uniform in this respect. In the species [*Pezophaps*] to which the bones now exhibited belong, however, the contrary is most conspicuously

* This last is exactly matched by a specimen of *Didus* in our possession.

† A great part of the misfortunes with which LEGUAT was afflicted during his absence from Europe arose from the determination of leaving Rodriguez, at which his companions, overruling his own wish, arrived. This determination was, as he naively tells us (*op. cit.* 1st ed. vol. i. pp. 148–154, Engl. transl. pp. 107–112), finally brought about by the fact that, in founding their proposed colony, they had altogether forgotten the necessity of including the softer sex in their company! He consequently never loses an opportunity of inculcating the prudence of making early marriages, and calls his dear Solitaires to witness to their expediency.

‡ *Cf.* NAUMANN, 'Vögel Deutschlands,' vol. vii. pp. 35–41; STEVENSON, 'Birds of Norfolk,' vol. ii. p. 13.

§ Mr. DARWIN (Variation of Animals and Plants under Domestication, vol. i. p. 162, note) demurs to this assertion and cites the case of *Carpophaga oceanica*, in which he says the excrescence at the base of the beak is "sexual." Whether or not it is so we have no means of judging, but the word "sexual" has been accidentally substituted by him for "seasonal," and therefore his remark does not bear on the question. On the other hand we are ready to admit that the assertion is not so universally true as it was thought to be when made. Our friend Mr. OSBERT SALVIN has shown us two species of *Chamæpelia* in which the plumage of the two sexes exhibits a manifest difference, and Mr. WALLACE (Ibis, 1865, pp. 365–400) cites some similar instances among the Malayan Pigeons.

the case; and one naturally turns to seek other instances in which a species differs from most of its allies in a similar manner. Two, if not more, such instances will at once occur to every ornithologist. They are those of the Capercally (*Tetrao urogallus*) and the Great Bustard (*Otis tarda*). In both these birds, I believe, the disproportion in size between the sexes is not greater than in the remains of the species now before the Society. But then another idea is called up: *Tetrao urogallus* departs from the usual habits of the *Tetraonidæ* by being polygamous; and (though I am aware that the statement has been disputed) the same is said to be the case with *Otis tarda*. Indeed I am inclined to suppose that with all species of birds the practice of polygamy is accompanied with more or less disproportion in the size of the sexes. Whether the converse is true I am not in a position to declare."—Proc. Zool. Soc. 1865, pp. 716, 717.

The strongest objection that seems possible to the suggestion here thrown out is in the fact recorded by LEGUAT, that the male birds of the Solitaire of Rodriguez assisted the females in the work of incubation; but the question of its polygamy or monogamy is not likely to meet with a solution now.

From the amount of variability which every portion of the skeleton of this species displays, it may not unreasonably be supposed that as much was exhibited in those parts of its structure which have perished, even if we had not LEGUAT's direct evidence as to the individual difference in plumage among the female birds. There would accordingly have been abundant room for the operation of any such process as that which has been described under the names of "Natural Selection" and "Survival of the Fittest" to have operated; and it may therefore appear at first sight, there having been known to us only one species of *Pezophaps*, that an argument against the existence of such a process might hereon be founded. A little reflection will, we think, show that such an argument would be unsound. Confined in a space so restricted as one small island, every individual of the species must have lived, moved, and had its being under conditions essentially identical in all cases. Whatever power such a process might have had of operating, there would be neither occasion nor opportunity for it so long as no change took place in the physical character of the island. But, if we venture to indulge our fancy and regard what would have been the inevitable consequences of a gradual upheaval of the land and a corresponding extension of its area until it became vastly increased, and its original low rounded hills were exalted into mountains, it is plain that a great variety of physical conditions would be thereby incurred. One side of the island would be exposed to the full force and direct influence of the trade-winds, the other side would be completely sheltered from them. The climate of these two portions would accordingly differ, and a great difference would be speedily wrought in the character of their vegetation, while that of the lofty central part would undergo a corresponding modification. After some longer or shorter period of time, we can conceive the island itself being broken up into two portions, separated from one another by a strait, such as divides the North and Middle

Islands of New Zealand. This rupture would certainly tend still more to affect the existing fauna and flora, and, at the end of another epoch, there can be little doubt but the animals and plants of each portion, exposed to different influences, would present a decidedly different appearance, and the eastern and western islands (supposing the separation to have taken place longitudinally) might each possess its own special form of Solitaire, as the islands composing New Zealand have their peculiar species of *Apteryx*.

But it is only in such case as that which we have imagined that considerable modifications would be likely to be effected. It seems, therefore, to us not to be an argument against the existence of such a process as that of "Natural Selection," but in reality an argument in its favour to find a small oceanic island tenanted by a single species, restricted in its range, which was subject to very great individual variability. Indeed a believer in this theory would, we think, be inclined to predicate that it would be under just such very circumstances that the greatest amount of variability would be certain to occur. In its original state, attacked by no enemies, its increase would be only dependent on the supply of food, which, one year with another, would most likely not vary much; the form would continue without any predisposing cause to change, and thus no advantage would be taken of the variability of structure presented by its individuals.

On the other hand, instead of speculating on what might have come to pass, it is perhaps more profitable to reflect on what certainly did take place. Of the other terrestrial members of the avifauna of Rodriguez but few now remain. A small Finch (*Foudia flavicans**) belonging to a group almost entirely confined to Madagascar and its satellite-islands, but specifically distinct, and a Warbler (*Drymœca rodericana**) belonging to a genus extending to Africa and Australia, though mainly represented in India and the adjacent territories, but also specifically distinct, are the only two land-birds of its original fauna which, so far as we know, now exist. The Guinea-fowl and the Love-bird have in all probability been introduced from Madagascar*; but the blue and green Parrots†, and the slate-coloured Pigeons‡, of which Leguat speaks, have vanished with the Solitaire. The remains of one of the first, and the description of the last, leave little room to doubt but they also were closely allied to the forms found in the other Mascarene Islands and Madagascar, and thus it is clear that certainly *four* out of the *five* or *six* indigenous species had their natural affines in other species belonging to the same zoological province. It seems to us impossible on any other reasonable supposition than that of a common ancestry to account for this fact. We are compelled to the

* Proc. Zool. Soc. 1865, pp. 46–48, pl. i.; Ibis, 1865, pp. 148–150.

† Leguat, *op. cit.* 1st ed. i. pp. 67, 107, and 132; Engl. transl. pp. 49, 77, and 95. On the fragment of a maxilla of a Parrot sent with the Solitaire's bones, Professor Alphonse Milne-Edwards has founded his *Psittacus rodericanus* (Ann. des Sc. Nat. 5e sér. Zool. viii. pp. 145–156, pls. 7, 8; Comptes Rendus, lxv. (Dec. 30, 1867) pp. 1121–1125.

‡ Leguat, *op. cit.* 1st ed. i. p. 104; Engl. transl. p. 75.

belief that there was once a time when Rodriguez, Mauritius, Bourbon, Madagascar, and probably the Seychelles were connected by dry land, and that that time was sufficiently remote to permit of the descendants of the original inhabitants of this now submerged continent to become modified into the many different representative forms which are now known to the ornithologist. Whether this result can have been effected by the process of "Natural Selection" must be regarded as an open question; but that the Solitaire of Rodriguez and the Dodo of Mauritius, however much they eventually came to differ, sprang from one and the same parent stock, seems to us a deduction from the facts so obvious, that we can conceive no one fully acquainted with them hesitating about its adoption any more than he can doubt the existence of the POWER by whom they were thus formed.

POSTSCRIPT, August 1869.

Since this paper was written a communication made some years ago to 'Notes and Queries' (First Series, vol. vi. p. 83, 24 July, 1852) by Mr. PINKERTON has attracted our attention. That gentleman refers to the account of Rodriguez, given by M. EUGÈNE DE FROBERVILLE in the 'Iles de l'Afrique' of M. D'AVEZAC (part iii. pp. 67–80, Paris: 1848), forming a volume of the series entitled 'L'Univers, ou Histoire et Description de tous les peuples,' &c., wherein the writer quotes freely from a manuscript written by the Abbé PINGRÉ, who passed some months of the year 1761 in Rodriguez, which he visited for the purpose of observing the transit of Venus, as already mentioned by Dr. HAMEL* and STRICKLAND†. As this manuscript was in existence not long prior to 1848, there seemed a good chance of its being found, and accordingly we applied to our good friend Professor ALPHONSE MILNE-EDWARDS, requesting him to make some inquiries respecting it. With his wonted readiness he most kindly undertook the research at once, and after some little time we had the pleasure of hearing from him that the manuscript had been found in the library of Ste. Geneviève at Paris. Subsequently he was so obliging as to transcribe for us that part which relates to the zoology of the island, and from his copy the following is an extract;—

"Relations du voyage de l'abbé Pingré à l'île de Rodrigue.

* * * * * *

"Les *Solitaires* étaient communes à Rodrigue du temps de *François Leguat*: Mr. de Puvigné‡ m'a assuré que la race n'en était pas encore détruite, mais ils se sont retirés dans les endroits de l'isle les plus inaccessibles."

This brief though interesting notice undoubtedly renders it probable that the Solitaire of Rodriguez existed as lately as 1761, but taken alone it is insufficient to afford foundation for the statement of M. DE FROBERVILLE (*ut supr. cit.* p. 71), that "ces tristes restes d'une population dont Leguat admirait la beauté et les mœurs curieuses, étaient sans

* 'Der Dodo, der Einsiedler', &c. 8vo, St. Petersburg: 1848, pp. 16, 17.

† 'The Dodo', &c. pp. 64, 65.

‡ "M. de Puvigné commandant de l'île."

cesse pourchassés sans pitié par les nègres, et ne vivaient que dans une inquiétude qui fut bientôt fatale à leur propagation;" and in the absence of any other evidence we shall continue to entertain the belief that the cause we have suggested in the foregoing pages was that which brought about the extinction of this species. How PINGRÉ and LE MONNIER elevated it to a place among the constellations has been already told by Dr. HAMEL and STRICKLAND—*sic itur ad astra*!

DESCRIPTION OF THE PLATES.

*** Unless the contrary is stated the figures are of the natural size.

PLATE XV.

Figs. 1–5. Atlas and axis (partially anchylosed ?), from the side, front, behind, above and below.

Figs. 6–8. Third cervical vertebra, from the side, above and below.

Figs. 9–12. Fourth cervical vertebra, from the side, behind, above and below.

Figs. 13–16. Fifth cervical vertebra, from the side, front, above and below.

Figs. 17–20. Sixth cervical vertebra, from the side, behind, above and below.

Figs. 21–23. Seventh (?) cervical vertebra, from the side, above and below.

Figs. 24–26. Eighth (?) cervical vertebra, from the side, above and below.

Figs. 27–30. Ninth cervical vertebra, from the side, front, above and below.

Figs. 31–34. Tenth cervical vertebra, from the side, behind, above and below.

Figs. 35–37. Eleventh cervical vertebra, from the side, above and below.

Figs. 38–41. Twelfth (and last ?) cervical vertebra, from the side, front, above and below.

Figs. 42–44. First (?) dorsal vertebra, from the side, above and below.

Figs. 45–47. Second (?) dorsal vertebra, from the side, above and below. Fig. 45 (the original having been damaged on the left side) has been drawn unreversed to preserve the continuous appearance of the series.

Figs. 48–50. Third (?) dorsal vertebra, from the side, above and below.

Figs. 51–55. Three anchylosed dorsal vertebræ (fourth, fifth, and sixth ?) from the side, front, behind, above and below. To fig. 51 the upper portion of the corresponding dorsal ribs are shown attached.

Figs. 56–59. Penultimate (seventh ?) dorsal vertebra from the side, behind, above and below.

PLATE XVI.

Fig. 60. The same three dorsal vertebræ as in figs. 51–55, but from another specimen, and incompletely anchylosed (♂?).

Fig. 61. Left sternal rib (seventh ?), outer side (♂?).

Fig. 62. Left dorsal rib (eighth ?), outer side, showing marks of pressure or friction (♂?).

Fig. 63. Right dorsal rib (sixth ?), outer side, showing similar marks (♂?).
Fig. 64. Sternal extremity of right dorsal rib (eighth ?), coalescent with preceding (seventh ?) sternal rib (♂?).
Fig. 65. Pelvis, from above (♂?).

PLATE XVII.

Fig. 66. Pelvis, from below (♀?).
Fig. 67. The same, from before, showing anterior surface of last (eighth ?) dorsal vertebra.
Fig. 68. The same, from above.
Fig. 69. The same, from behind.
} Slightly reduced.

PLATE XVIII.

Fig. 70. The same pelvis, from the side (♀?). Slightly reduced.
Fig. 71. Sternum, from before (♀?).
Fig. 72. The same, from the side.
Fig. 73. The same, from below.
Fig. 74. The same, from above.
Fig. 75. Posterior portion of sternal keel from below (♂?).

PLATE XIX.

Figs. 76–79. Left coracoid, from below, front, behind and above (♂?).
Figs. 80–83. Left coracoid, from below, front, behind and above (♀?).
Figs. 84–86. Left scapula, outer, lateral and inner surfaces (♂?).
Fig. 87. Left metacarpal, upper view (♂?).
Figs. 88–90. Right metacarpal, front, lower and upper views (♂?).
Figs. 91, 92. Right humerus, lower and upper views (♂?).
Fig. 93. Right ulna, upper view (♂?).
Figs. 94, 95. Right radius, upper and lower views (♂?).
Fig. 96. Right ulna, lower view (♂?). (From same specimen as fig. 93.)
Figs. 97–99. Right scapula, inner, lateral and outer surfaces (♂?).

PLATE XX.

Fig. 100. Left tibia, with fibula attached, from front (♂?).
Fig. 101. The same fibula, from behind.
Fig. 102. Left tibia, with fibula attached, from front (♀?).
Fig. 103. The same fibula, from behind.

Figs. 104, 105. Right posterior metatarsus, from behind and outside (♂?).
Figs. 106, 107. Proximal phalanx of right hallux, from above and side (♂?).
Figs. 108, 109. Ungual phalanx of right hallux, from above and side (♂?).
Figs. 110–112. Proximal phalanx of right outer toe, from above and from each end (♂?).
Figs. 113–115. Ungual phalanx of right outer toe, from above, below, and side (♂?).
Figs. 116–118. Proximal phalanx of right middle toe, from above and from each end (♂?).
Figs. 119–121. Second phalanx of right middle toe, from above and from each end (♂?).
Figs. 122–124. Third phalanx of right middle toe, from above and from each end (♂?).
Figs. 125–127. Ungual phalanx of right middle toe, from above, below, and side (♂?).
Figs. 128–130. Proximal phalanx of right inner toe, from above and from each end (♂?).
Fig. 131. Right metacarpal, lower view (♂?). (From a second specimen.)
Figs. 132, 133. Left scapula, outer and inner surface (♀?).
Fig. 134. Right ulna, upper view, showing impressions of secondary quill-feathers (♀?).
Fig. 135. Left ulna, upper view, showing the same, and also mark of fracture and healing (♂?).
Fig. 136. Right coracoid, side view, showing mark of fracture and healing (♂?).
Figs. 137, 138. Right radius, upper and hinder view (♂?). (From a second specimen.)

PLATE XXI.

Figs. 139–141. Cranium, from above, below, and behind (♀?).
Figs. 142–144. Cranium, from above, below, and behind (♂?).
Fig. 145. Cranium, from below (♀?). (From a second specimen.)
Fig. 146. Cranium, from behind (♀?). (From a third specimen.)
Fig. 147. Cranium, from behind (♂?). (From a second specimen.)

PLATE XXII.

Fig. 148. Left metacarpal, upper view (♀?).
Fig. 149. Cranium from the side (♂?). (From same specimen as figs. 142–144.)
Fig. 150. Cranium from the side (♀?). (From same specimen as figs. 139–141.)
Figs. 151, 152. Maxilla, from below and side (♂?).
Fig. 153. Dertrum, from side (♂?).
Figs. 154, 155. Maxilla, from side and below (♂?). (From a second specimen.)
Figs. 156–158. Maxilla, from above, side, and below (♀?). (Fig. 157 shows a portion of the outside of the left lower branch of the premaxillary drawn unreversed —the right being injured.)
Figs. 159, 160. Proximal and distal portions of mandible (possibly belonging to the same individual), from the side (♀?).
Figs. 161, 162. The same, from above.
Figs. 163–168. Quadrate from outside, inside, above, below, behind, and front (♂?).

PLATE XXIII.

Fig. 169. Sternum of *Columba livia.*
Fig. 170. Sternum of *Didunculus strigirostris* (Mus. Coll. Reg. Chirurg. Lond.).
Fig. 171. Sternum of *Leucosarcia picata.*
Fig. 172. Sternum of *Columba livia*, var. *domestica* (exempl. monstr.).
Fig. 173. Sternum of *Patagiœnas caribbœa.*
Fig. 174. Sternum of *Chamæpelia trochila.*
Fig. 175. Sternum of *Geopelia striata.*
Fig. 176. Sternum of *Goura coronata.*

PLATE XXIV.

Fig. 177. Skeleton of *Pezophaps solitaria*, from front (♀?).
Fig. 178. Skeleton of *Pezophaps solitaria*, from front (♂?).
Fig. 179. The same from the side.

} From photographs of the mounted specimens (p. 332) deposited in the Museum of the University of Cambridge. Greatly reduced.

IX. *On the Solar Variations of Magnetic Declination at Bombay.* *By* CHARLES CHAMBERS, *Esq.*, *Superintendent of the Colaba Observatory.* *Communicated by* BALFOUR STEWART, *F.R.S.*

Received June 30,—Read December 10, 1868.

1. THE observations discussed in the following pages were made at the Government Observatory at Bombay, during the years 1859 to 1865, and constitute a portion of a much larger series of observations of which the remaining part is still awaiting reduction. With the exception of Sundays, and eight or ten complete days in each year, the observations were taken continuously at hourly intervals throughout the period of seven years. The observers were carefully trained Brahmins, who were under the immediate oversight of highly intelligent assistants of the same caste.

2. The instrument used was made by GRUBB of Dublin, and consists of a rectangular bar-magnet suspended horizontally, and carrying a divided scale and a lens, by means of which its position can be determined from time to time by reading the scale with a fixed telescope properly placed. The dimensions of the magnet are 15 inches by 0·86 inch by 0·25 inch, and the broad surface is made to lie horizontal; the suspension-thread was formed of about forty fibres of untwisted silk, and is 35 inches long, being protected (but not concealed from view) by a glass tube 1·3 inch in diameter: the bottom of the glass tube rests upon the top of a cylindrical mahogany box (8·5 inches in height) which surrounds the magnet, and the top supports a horizontal divided circle (the torsion-circle), and a brass cross piece to which the suspension-thread is attached. The tube is secured in a vertical position by a mahogany cross bar which has a circular hole in its centre that fits over the upper end of the tube, and which is fixed at its extremities to two copper pillars whose feet are screwed into the marble basement of the instrument; and the upper aperture of the tube with the attachments is screened from the outer air and dust by an inverted hemispherical glass vessel. A sliding-frame which carries above a finely divided scale etched upon glass is fixed by a binding-screw near the northern extremity of the magnet, and a similar frame carrying a lens, whose focal length is about 12·7 inches, is secured to the magnet at that distance to the southward of the glass scale, so that the latter is approximately in the principal focus of the lens. The vertical lines which form the scale are equidistant and generally of uniform length; but every fifth division is slightly and every tenth division considerably prolonged, and over the tenth divisions numbers are marked in consecutive order, thus allowing a numerical designation to be given with facility to every point of the scale when viewed by the telescope. To permit the scale to be viewed through its lens by a telescope outside

the mahogany box, the latter has two small windows of flat glass suitably inserted in its curved side, and a lamp is placed on a stool to the northward of the masonry pillar which supports the declinometer, and is kept constantly burning. The joints and crevices, and for greater security, the whole outside surface of the cylindrical box was covered with paper to keep out small spiders or other insects.

3. The observations are made by a transit instrument (30 inches long and with pivots 15 inches apart) which is fixed on an isolated pillar of masonry, at a distance of 27 feet to the southward of the declinometer pillar, and which in this position is capable of being used also for observing the transits of stars through an opening (provided with shutters) in the roof of the observatory. This instrument was adjusted in a line with the centres of the scale and lens attached to the magnet, and its basement was then firmly imbedded in masonry, and the telescope was focused so as to give a distinct view of the scale. A vertical spider-line is placed in the common principal focus of the object-glass and eye-piece, and the instrument was adjusted so that an object on the true meridian should be cut by this line. With these arrangements any change in the azimuth of the freely suspended magnet is indicated by a change of corresponding magnitude in the position of the image of the collimator scale, as seen in the field of view of the telescope; and the actual position of the magnet at any moment may be found by noting the scale-division that is cut by the reading-wire of the telescope, if only we know the position corresponding to any one reading of the scale, and the change of position for a change of unity in the reading. The latter constant, the scale-coefficient, is easily determined by substituting an azimuth instrument in the place of the transit instrument, and observing with it the horizontal angle included between any two widely different divisions of the scale when the magnet is at rest. Experiments recorded on pages x and xi of the Introduction to the Bombay Magnetical Observations for 1864 showed that the scale was very correctly placed in the principal focus of its lens, and that the scale-coefficient is 6′·806; but the actual value of the coefficient used in the reduction of the observations was 6′·841, that having been the result of a first determination made in 1865, when the scale and lens were apparently differently adjusted with respect to each other. No correction has, however, been applied to the reduced observations for the present purpose, since the *laws of periodical variations* are exhibited with equal clearness whatever the unit with which they are expressed; and it will only be necessary to bear in mind what is the real unit made use of if we should wish to compare the variations of this element of the terrestrial magnetic force at Bombay with those of either or both of the two others, or with the results of observations made elsewhere, so that all may be reduced to a common unit as a preliminary step*.

4. The direction of the magnetic axis of the bar corresponding to a particular scale-reading was found in the following manner:—

First, the level and collimation errors of the transit instrument were approximately destroyed, and the telescope was then pointed upon Polaris when on the true meridian,

* It has not been deemed requisite to do this for any of the discussions that follow.

and the azimuthal adjusting-screw turned until the star was bisected by the vertical spider-line at the proper moment, as taken from the Nautical Almanac: it appeared, from observations made in May 1866, and described in detail in the Introduction to the Bombay Magnetical Observations for 1864, pages x to xii, that this adjustment was at that time very nearly correct. Secondly, a bundle of about forty threads of untwisted silk having been prepared with care to avoid entangling the different fibres, a brass bar of the same weight as the magnet was suspended and allowed to swing until it took up a position of rest; the arms of the torsion-circle and the attached suspension-pin, to which the upper end of this thread was bound, were then turned in azimuth until the brass bar lay at rest approximately in the true meridian: the magnet was now substituted in place of the brass bar, and a reading of the scale was taken and recorded; next, the magnet being provided with a suspension-hook that allows the bar to be inverted about its own length as an axis, without disturbing any of the attachments, this inversion was made and the scale again read. Now the position of the magnetic axis of the bar in the direct and reverse positions would be the same, but the scale-reading would be increased in the one case and diminished in the other, by an amount corresponding to the deviation of the magnetic axis from true meridian; hence the average of the two readings will give the reading that would be found if the magnet lay with its axis in the true meridian, and half their difference represents the deviation (in scale-divisions) from that position. When the instrument was last adjusted, on the 9th November, 1861, the true meridian reading was 28·2733; and the scale being placed in an inverted position it appeared erect in the telescope, so that increase of reading represents an easterly movement of the north end of the magnet.

5. One other element, taking account of the magnet being restrained from responding fully to the changes of magnetic force by the torsion-force of the suspension-thread, enters into the reduction of the observations:—the torsion-force is assumed, as in Coulumb's Electrometer, to be proportional to the angle of torsion, that is, to the deviation of the magnet from the position of no torsion—in this case the true meridian; the torsion-coefficient, or unity *plus* the ratio between the torsion-force and the directive force of the earth acting upon the particular magnet, is determined by observing the change of scale-reading produced when the arms of the torsion-circle are turned through 90°, dividing the corresponding angle (α) by ($90°-\alpha$), and adding unity to the quotient. In November 1861 its value was found to be 1·0028, and this has been used without alteration over since.

6. The formula used in the reduction of the observations was

$$D=6'\cdot841(f-R).c,$$

where 6′·841 is the adopted value of a unit of the declinometer scale, R the true meridian reading, c the torsion-coefficient, and D is the absolute easterly declination when f is the observed scale-reading. The values of D thus obtained, in which form the

declination records have been printed in the several volumes of 'Bombay Magnetical Observations,' form the fundamental data for the following discussion*.

7. It ought to be mentioned that the sliding tube of the transit instrument with which the declinometer was read was rather loosely fitted in its socket, a fault which might cause sensible change in the collimation-error whenever the focusing adjustment was used; now the instrument was once every month turned upon a divided scale fixed upon the north wall of the magnetometer-room and adjusted for distinct vision, after which the adjustment was restored for the collimator scale of the magnetometer. It is suspected that small but important irregularities in the observations, which will be pointed out later, are due chiefly to this fault.

8. Throughout the periods January 1, 1859 to July 2, 1861, and November 13, 1861 to December 31, 1865, the adjustments of the instrument were undisturbed; but between July 3 and November 12, 1861 the adjustments were several times renewed, and the action of the instrument was at times so unsatisfactory that the observations of July 3 to 8, August 7 to 12, and October 28 to November 12 are considered unfit for incorporation with the general body. The following statement shows the several periods made use of, the observed and adopted values of the true meridian reading, and the values of the torsion-coefficient as experimentally determined:—

Periods.	True meridian reading.		Torsion-coefficient.
	Observed.	Adopted.	
January 1, 1859 to July 2, 1861	28·3064	28·379	1·003009
July 9, 1861 to August 6, 1861	28·5334	31·153	1·00039
August 13, 1861 to October 27, 1861	28·2515	28·2515	1·00088
November 13, 1861 to December 31, 1865	28·2733	28·2733	1·0028

No reason is assigned in the records for the adoption for the first two periods of a different true-meridian reading to that which was observed; but it may be presumed that in the latter case the intention was to produce that accordance, which doubtless existed in fact, between the values of declination in that and the preceding and following periods. Any periodical variations which the observations may, however, exhibit will be unaffected by the meridian reading, so long as none of the observations of any single period involved belong to more than one series, or so long as each series of observations is treated independently of the others; and it is only when combining the different series together for the elimination of secular variation and for the evaluation of absolute declination at a given epoch, that regard must be paid to the actual disconnexion of the several series. In the investigations of solar and lunar diurnal variations which follow or are in progress, no incomplete days, nor any comprising observations from different series have been made use of.

* I should have preferred to operate directly upon the observed quantities (f), but was led to believe, at the commencement of the work, that part of the original registers had been destroyed; I have since found that this was incorrect.

9. *Method of Reduction of the Observations.*—The singularly fruitful nature of the mode of treatment of observations of a like character with those now under discussion, introduced and extensively applied by Lieut.-General SABINE, P.R.S., is such as to leave no doubt as to the propriety of its adoption in the present instance, especially as the whole series of observations, the greater part of which remains still unreduced, extends over nearly a quarter of a century. That mode of treatment is described by General SABINE as follows*:—"The hourly directions of the magnet [in this case expressed in minutes of deviation of the north end of the magnet to the eastward of true north] are entered in monthly tables, having the days of the month in successive horizontal lines, and the hours of the day in vertical columns. The 'means' of the entries in each vertical column indicate the mean direction of the magnet at the different hours of the month to which the table belongs, and have received the name of 'First Normals.' On inspecting any such monthly table, it is at once seen that a considerable portion of the entries in the several columns differ considerably from their respective means or first normals, and must be regarded as 'disturbed observations.' The laws of their relative frequency and amount of disturbance, in different years, months, and hours, are then sought ought, by separating for that purpose a sufficient body of the most disturbed observations, computing the amount of departure in each case from the normal of the same month and hour, and arranging the amounts in annual, monthly, and hourly tables. In making these computations, the first normals require to be themselves corrected, by the omission in each vertical column of the entries noted as disturbed, and by taking fresh means, representing the normals of each month and hour after this omission, and therefore uninfluenced by the larger disturbances. These new means have received the name of 'Final Normals,' and may be defined as being the mean directions of the magnet in every month and every hour, after the omission from the record of every entry which differed from the mean a certain amount either in excess or in defect.

"In this process there is nothing indefinite; and nothing arbitrary save the assignment of the particular amount of difference from the normal which shall be held to constitute the measure of a large disturbance, and which, for distinction sake, we may call 'the separating value.' It must be an amount which will separate a sufficient body of disturbed observations to permit their laws to be satisfactorily ascertained; but in other respects its precise value is of minor significancy; and the limits within which a selection may be made, without materially affecting the results, are usually by no means narrow; for it has been found experimentally on several occasions that the ratios by which the periodical variations of disturbance in different years, months, and hours are characterized and expressed do not undergo any material change by even considerable differences in the amount of the separating value. The separating value must necessarily be larger at some stations than at others, because the absolute magnitude of the disturbance-variation itself is very different in different parts of the globe, as well as its comparative magnitude in relation to the more regular solar-diurnal variation; but it

* Proceedings of the Royal Society, vol. x. pp. 624–626.

must be a constant quantity throughout at one and the same station, or it will not truly show the relative proportion of disturbance in different years and different months."

In two instances only were days of a certain month included, in consequence of interruption of the observations, in the monthly Tables of a different month, viz.:—the 1st and 2nd of July with the month of June 1861, and from the 1st to the 6th of August with the month of July 1861; and the only incomplete months were those of August, October, and November 1861. The first step to be taken was to fix upon a suitable *separating value*, and in doing this regard was paid to the desirableness of having the results derived for Bombay comparable with those for some one of the stations where similar observations had been made and similarly treated. Now the ordinary reductions of the Bombay observations already made showed that the station amongst those which it most nearly resembled was St. Helena; for whilst both are equatorial stations in the sense of the ordinary solar-diurnal variation being at certain seasons overridden by the semiannual inequality of that variation, Bombay has very nearly the same preponderance of the character of a north-latitude station that St. Helena has as a south-latitude station. The separating value adopted by General SABINE in the reduction of the St. Helena observations was 1′·78, or a disturbing force equal to ·0029 English units; a force of this magnitude acting at Bombay would produce a deviation of the declination-magnet of 1′·25. The value finally fixed upon was 1′·4, which gives the ratio of the number (3516) of disturbed observations to the total number (50736) of observations in the seven years as 1 to 14·4, the similar ratio for St. Helena having been 1 to 13·7.

10. Trials were also made with separating values 1′·0 for the years 1859 and 1860, and 1′·7 for 1860, the results of which (being further confirmatory of the position taken above that "the *ratios* by which the periodical variations of disturbance in different hours are characterized and expressed, do not undergo any material change by even considerable differences in the amount of the separating value") are shown in the subjoined Tables, and by Curves Nos. 1 to 4 of Plate XXV.

TABLE I.—Showing, for each hour of the day, the aggregates of Easterly disturbances of Declination exceeding 1′·0, 1′·4, and 1′·7 respectively, in the years 1859 and 1860, and the corresponding ratios of the aggregate disturbance at each hour to the average aggregate hourly disturbance for the twenty-four hours.

Bombay Astronomical Hours.	Aggregates of Easterly Disturbances.					Ratios.				
	1859.		1860.			1859.		1860.		
	Separating value.					Separating value.				
	1′·0.	1′·4.	1′·0.	1′·4.	1′·7.	1′·0.	1′·4.	1′·0.	1′·4.	1′·7.
h m										
4 12	51·757	35·466	46·246	29·107	18·025	0·90	0·88	0·90	0·83	0·70
5 12	51·593	29·690	42·672	19·311	13·192	0·90	0·74	0·83	0·56	0·52
6 12	43·782	28·741	31·687	22·608	19·348	0·76	0·71	0·61	0·65	0·76
7 12	48·294	34·288	35·280	35·102	30·890	0·84	0·85	0·68	1·01	1·21
8 12	61·750	43·332	37·917	29·834	21·345	1·08	1·07	0·73	0·86	0·83
9 12	50·127	31·550	39·530	26·678	23·296	0·88	0·78	0·76	0·77	0·91
10 12	59·087	41·861	40·473	23·614	15·830	1·03	1·04	0·78	0·68	0·62
11 12	47·550	40·412	42·193	21·482	12·942	0·83	1·00	0·82	0·62	0·51
12 12	44·935	36·934	25·639	22·429	8·458	0·78	0·91	0·50	0·65	0·33
13 12	41·108	31·866	36·812	18·459	14·628	0·72	0·79	0·71	0·53	0·58
14 12	36·100	29·017	28·555	16·674	11·715	0·63	0·72	0·55	0·48	0·46
15 12	39·732	25·540	36·595	25·299	17·069	0·57	0·63	0·71	0·73	0·67
16 12	32·534	22·446	40·806	17·891	13·079	0·57	0·56	0·79	0·52	0·51
17 12	37·221	22·906	51·522	31·804	28·145	0·65	0·57	1·00	0·92	1·10
18 12	53·392	37·587	77·986	60·232	51·330	0·93	0·93	1·51	1·74	2·01
19 12	68·025	59·644	83·650	61·639	47·653	1·19	1·48	1·62	1·78	1·86
20 12	84·294	43·181	61·085	40·580	31·984	1·47	1·07	1·18	1·17	1·25
21 12	82·185	49·738	71·760	31·803	19·012	1·44	1·23	1·39	0·92	0·74
22 12	116·645	71·755	63·296	49·933	38·281	2·04	1·78	1·23	1·44	1·50
23 12	101·169	90·538	82·663	56·567	29·285	1·77	2·24	1·60	1·63	1·14
0 12	74·977	57·241	65·006	51·857	41·839	1·31	1·42	1·26	1·50	1·64
1 12	55·486	45·681	69·136	57·587	41·671	0·97	1·13	1·34	1·66	1·63
2 12	42·159	30·646	78·016	40·832	38·878	0·74	0·76	1·51	1·18	1·52
3 12	56·927	29·154	49·730	40·975	25·997	0·99	0·72	0·96	1·18	1·02

TABLE II.—Showing, for each hour of the day, the aggregates of Westerly disturbances of Declination exceeding 1′·0, 1′·4, and 1′·7 respectively, in the years 1859 and 1860, and the corresponding ratios of the aggregate disturbance at each hour to the average aggregate hourly disturbance for the twenty-four hours.

Bombay Astronomical Hours.	Aggregates of Westerly Disturbances.					Ratios.				
	1859.		1860.			1859.		1860.		
	Separating value.					Separating value.				
	1′·0.	1′·4.	1′·0.	1′·4.	1′·7.	1′·0.	1′·4.	1′·0.	1′·4.	1′·7.
h m										
4 12	24·787	15·013	50·816	32·064	26·760	0·71	0·72	1·27	1·27	1·49
5 12	10·189	7·905	26·883	19·412	16·416	0·29	0·38	0·67	0·77	0·91
6 12	10·201	4·717	17·461	8·013	8·013	0·29	0·23	0·44	0·32	0·45
7 12	6·486	3·263	10·855	10·143	0·000	0·19	0·16	0·27	0·40	0·00
8 12	1·011	0·000	9·468	1·609	0·000	0·03	0·00	0·24	0·06	0·00
9 12	3·310	0·000	7·202	3·117	0·000	0·07	0·00	0·18	0·12	0·00
10 12	1·395	0·000	7·959	6·438	0·000	0·04	0·00	0·20	0·25	0·00
11 12	3·237	2·024	8·552	1·569	0·000	0·09	0·10	0·21	0·06	0·00
12 12	1·887	1·896	14·075	0·000	0·000	0·05	0·09	0·35	0·00	0·00
13 12	0·000	0·000	7·433	3·114	0·000	0·00	0·00	0·19	0·12	0·00
14 12	2·198	0·000	10·457	5·461	2·159	0·06	0·00	0·26	0·22	0·12
15 12	3·535	0·000	11·472	6·754	2·025	0·10	0·00	0·29	0·27	0·11
16 12	4·749	0·000	10·787	2·011	2·156	0·14	0·00	0·27	0·08	0·12
17 12	7·771	2·881	21·923	11·698	4·078	0·22	0·14	0·55	0·46	0·23
18 12	44·972	25·868	33·496	7·822	9·742	1·30	1·24	0·84	0·31	0·54
19 12	48·941	13·587	51·874	25·168	16·693	1·41	0·65	1·29	1·00	0·93
20 12	41·231	23·262	64·509	40·914	18·695	1·19	1·11	1·61	1·62	1·04
21 12	73·392	51·357	79·574	69·518	61·456	2·12	2·46	1·96	2·75	3·42
22 12	68·559	53·587	119·854	78·239	65·378	1·89	2·57	2·99	3·09	3·64
23 12	125·160	71·600	97·152	68·135	65·226	3·61	3·43	2·40	2·70	3·63
0 12	109·060	83·193	102·258	69·272	59·325	3·15	3·99	2·55	2·74	2·91
1 12	113·927	75·811	84·736	60·539	33·296	3·29	3·63	2·11	2·39	1·85
2 12	90·291	47·939	58·058	46·296	22·661	2·60	2·30	1·45	1·83	1·26
3 12	39·807	16·962	55·962	29·424	24·285	1·15	0·81	1·40	1·16	1·35

The number of observations reckoned as disturbed with the different separating values was as follows:—

TABLE III.

	Number of disturbed observations.			Number of observations out of which on an average one is disturbed.		
Separating value...	1″·0	1″·4	1″·7	1″·0	1″·4	1″·7
1859	1259	672	.....	5·8	10·9	
1860	1318	685	434	5·6	10·7	16·9

11. *Disturbance-variations, Diurnal, Annual, and Decennial.*—The following Tables exhibit the general results as to diurnal, annual, and decennial variations of disturbance when the separating value made use of is 1″·4.

TABLE IV.—Showing the aggregates of Easterly disturbances exceeding 1″·4 in amount for each hour of the day in each of the years 1859 to 1865, and in the whole period of seven years; also, for the whole period, the ratios of aggregate disturbance at the several hours to the average aggregate hourly disturbance in the twenty-four hours.

Bombay Astronomical Hours.	Aggregates of Easterly Disturbance.								Ratios.
	1859.	1860.	1861.	1862.	1863.	1864.	1865.	Seven years, 1859 to 1865.	
h m									
4 12	35·466	29·107	17·153	33·364	22·841	10·058	22·777	170·766	0·94
5 12	29·690	19·311	7·649	48·925	16·214	17·529	15·789	155·107	0·86
6 12	28·741	22·608	6·637	26·097	23·345	18·817	25·042	151·287	0·84
7 12	34·288	35·102	13·825	20·864	18·871	5·743	13·132	141·825	0·78
8 12	43·332	29·834	20·293	30·670	19·230	8·054	16·396	167·809	0·93
9 12	31·550	26·678	14·459	19·417	7·742	10·225	19·112	129·183	0·71
10 12	41·861	23·614	16·697	14·826	3·987	4·817	11·363	117·165	0·65
11 12	40·412	21·482	21·490	23·522	6·162	3·819	5·607	122·494	0·68
12 12	36·934	22·429	26·900	21·564	0·000	5·079	9·773	122·679	0·68
13 12	31·866	18·459	25·131	14·455	8·746	7·880	13·594	120·131	0·66
14 12	29·017	16·674	13·612	21·417	7·993	9·229	14·274	112·216	0·62
15 12	25·540	25·299	22·647	28·693	12·297	16·307	20·598	151·381	0·84
16 12	22·446	17·891	20·821	13·856	10·591	14·974	16·375	116·954	0·65
17 12	22·906	31·804	13·247	15·917	6·514	15·691	8·254	114·333	0·63
18 12	37·587	60·232	18·583	19·855	3·238	9·840	14·236	163·571	0·90
19 12	59·644	61·639	27·029	26·175	14·956	13·030	13·250	215·723	1·19
20 12	43·181	40·580	27·107	27·248	9·460	12·929	5·265	165·770	0·92
21 12	49·738	31·803	32·626	45·258	12·444	13·964	19·449	205·282	1·14
22 12	71·755	49·933	32·302	71·258	18·259	30·885	30·277	304·669	1·69
23 12	90·538	56·567	38·052	68·850	33·287	24·832	29·167	341·293	1·89
0 12	57·241	51·857	27·281	63·044	63·427	32·828	29·614	325·292	1·80
1 12	45·681	57·582	26·904	58·423	45·280	30·713	25·231	289·159	1·60
2 12	30·646	40·832	22·609	44·571	34·410	36·667	34·197	243·932	1·35
3 12	29·154	40·975	21·219	32·459	17·198	20·459	29·100	190·564	1·05

The hourly ratios are also represented, in the usual manner, by curve No. 5, Plate XXV.

TABLE V.—Showing the aggregates of Westerly disturbances exceeding 1′·4 in amount for each hour of the day in each of the years 1859 to 1865, and in the whole period of seven years; also, for the whole period, the ratios of aggregate disturbance at the several hours to the average aggregate hourly disturbance in the twenty-four hours.

Bombay Astronomical Hours.	Aggregates of Westerly Disturbance.								Ratios.
	1859.	1860.	1861.	1862.	1863.	1864.	1865.	Seven years, 1859 to 1865.	
h m									
4 12	15·013	32·064	14·454	32·204	6·893	11·174	9·100	120·902	1·08
5 12	7·905	19·412	9·513	12·966	1·418	9·040	8·438	68·692	0·61
6 12	4·717	8·013	6·502	11·999	1·404	2·512	9·008	44·155	0·39
7 12	3·263	10·143	5·509	17·113	0·000	3·768	7·906	47·702	0·43
8 12	0·000	1·609	1·733	8·432	3·140	0·000	5·486	20·400	0·18
9 12	0·000	3·117	0·000	7·209	1·504	0·000	2·908	14·738	0·13
10 12	0·000	6·438	0·000	6·841	1·472	0·000	3·568	18·319	0·16
11 12	2·024	1·569	0·000	1·655	1·518	0·000	1·561	8·327	0·07
12 12	1·896	0·000	0·000	3·661	0·000	0·000	0·000	5·557	0·05
13 12	0·000	3·114	1·550	6·526	0·000	0·000	4·854	16·044	0·14
14 12	0·000	5·461	2·137	6·513	1·655	0·000	0·000	15·766	0·14
15 12	0·000	6·754	0·000	6·413	0·000	0·000	1·710	14·877	0·13
16 12	0·000	2·011	1·464	4·880	3·653	0·000	6·662	18·670	0·17
17 12	2·881	11·698	0·000	6·891	5·446	1·567	13·378	41·861	0·37
18 12	25·868	7·822	6·827	23·963	3·607	4·765	13·781	86·635	0·77
19 12	13·587	25·168	21·578	18·685	9·431	8·544	16·417	113·410	1·01
20 12	23·262	40·914	23·882	33·729	31·408	15·059	17·349	185·603	1·66
21 12	51·357	69·518	25·055	43·036	32·460	28·909	15·863	266·198	2·37
22 12	53·587	78·239	48·552	41·718	37·928	27·151	26·749	313·924	2·80
23 12	71·600	68·135	37·509	37·271	41·551	28·587	25·384	310·037	2·76
0 12	83·193	69·272	49·884	42·012	20·564	17·279	26·744	308·939	2·75
1 12	75·811	60·539	46·012	41·489	26·947	21·303	11·828	283·929	2·53
2 12	47·939	46·296	22·632	33·045	34·339	12·880	15·951	213·082	1·90
3 12	16·962	29·424	23·511	37·265	15·289	21·635	9·483	153·569	1·37

The hourly ratios are also represented, in the usual manner, by curve No. 6, Plate XXV.

TABLE VI.—Showing, for the period 1859 to 1865, the algebraical sum of all disturbances, those to the Eastward being reckoned as positive, those to the Westward as negative, exceeding 1′·4 in amount for each hour of the day.

	Bombay Astronomical Hours.											
	h m 4 12	h m 5 12	h m 6 12	h m 7 12	h m 8 12	h m 9 12	h m 10 12	h m 11 12	h m 12 12	h m 13 12	h m 14 12	h m 15 12
Excess of Easterly Disturbances.	+49·864	+86·415	+107·132	+ 94·123	+147·409	+114·445	+98·846	+114·167	+117·122	+104·087	+96·450	+136·504
	Bombay Astronomical Hours.											
	h m 16 12	h m 17 12	h m 18 12	h m 19 12	h m 20 12	h m 21 12	h m 22 12	h m 23 12	h m 0 12	h m 1 12	h m 2 12	h m 3 12
Excess of Easterly Disturbances.	+98·284	+72·472	+ 76·936	+102·313	− 19·833	− 60·916	− 9·255	+ 31·256	+ 16·353	+ 5·230	+30·850	+ 36·995

The curve which represents the disturbance diurnal variation, constructed from these numbers, is marked fig. 7 on Plate XXV.

TABLE VII.—Showing, for the period 1859 to 1865, the aggregates of Easterly and of Westerly disturbance in each month of the year, the corresponding ratios to the average aggregate monthly disturbance, and the numbers of observations separated as disturbed in each month.

Months.	Easterly Disturbances.			Westerly Disturbances.			Easterly and Westerly Disturbances.		
	Aggregates.	Ratios.	Numbers of disturbed observations.	Aggregates.	Ratios.	Numbers of disturbed observations.	Aggregates.	Ratios.	Numbers of disturbed observations.
January	304·347	0·84	155	198·089	0·88	105	502·436	0·86	260
February ...	323·306	0·89	165	150·901	0·67	79	474·207	0·81	244
March	465·344	1·29	225	209·485	0·93	114	674·829	1·15	339
April	376·052	1·04	185	289·836	1·29	153	665·888	1·14	338
May	207·107	0·57	110	224·211	1·00	117	431·318	0·74	227
June	264·433	0·73	136	183·258	0·82	96	447·691	0·76	232
July	428·079	1·18	211	409·885	1·83	204	837·964	1·43	415
August	592·799	1·64	270	288·258	1·29	155	881·057	1·50	425
September ...	434·502	1·20	195	234·283	1·04	127	668·785	1·14	322
October	551·198	1·52	245	293·979	1·31	151	845·177	1·44	396
November ...	144·255	0·40	76	91·034	0·41	51	235·289	0·40	127
December ...	247·163	0·68	126	118·117	0·53	65	365·280	0·62	191

TABLE VIII.—Showing the aggregates of Easterly and of Westerly disturbance in each of the years 1859 to 1865, and in the whole seven years, and the ratios of the aggregates for the individual years to the average annual aggregate; also the numbers of observations separated as disturbed in each year and in the whole period.

Years.	Easterly Disturbances.			Westerly Disturbances.			Easterly and Westerly Disturbances.		
	Aggregates.	Ratios.	Numbers of disturbed observations.	Aggregates.	Ratios.	Numbers of disturbed observations.	Aggregates.	Ratios.	Numbers of disturbed observations.
1859.	969·214	1·56	417	500·865	1·30	255	1470·079	1·46	672
1860.	532·297	1·34	385	606·730	1·58	300	1439·027	1·43	685
1861*.	513·573	0·83	262	348·304	0·91	189	861·877	0·86	451
1862.	790·728	1·28	388	485·518	1·26	262	1276·246	1·27	650
1863.	416·532	0·67	217	281·627	0·73	157	698·159	0·70	374
1864.	374·369	0·60	198	214·164	0·56	120	588·533	0·59	318
1865.	441·872	0·71	232	254·128	0·66	134	696·000	0·69	366
Seven years.	4338·585	.....	2099	2691·336	.....	1417	7029·921	.....	3516

12. *Remarks on the Disturbance-variations.*—At nearly all stations whose disturbance-diurnal variation curves have been found by the process described above, one or other of these curves, either the easterly or the westerly, closely corresponds in shape to a general type. The westerly disturbance- diurnal variation curve for Bombay conforms to this type, of which the characteristics are, that during about half the day the ordinates are very small, whilst there is a gradual and regular rise on either side of this quiescent period towards a maximum, which occurs near the middle of the remaining

* The deficiency in the ratios and numbers of observations for the year 1861 is partly accounted for by the fact that during an aggregate of thirty-one days in the months of July, August, October, and November the instrument was acting irregularly or was under adjustment, and the observations had to be rejected.

part of the day; such curves are the easterly for Kew, St. Helena, Toronto, and the Cape of Good Hope, and the westerly for Hobarton, St. Helena, Pekin, and Nertschinsk. The correspondence as to shape does not, however, hold as to time, either absolute or local, for the hours of maxima are not confined to any particular portion of the day. The Bombay astronomical hours of little or no westerly disturbance are from 6 to 17, the considerable and regular variation from hour to hour occurring during the remaining hours. The principal features of the easterly disturbance- diurnal variation curve for Bombay are like those of the westerly curve, with this distinction, that there is a considerable amount of easterly disturbance at all hours of the day, the least hourly ratio being 0·62; the easterly curve is also less remarkably regular than the westerly. There is great similarity in the declination-disturbance diurnal variations of St. Helena and Bombay, and the principal inflections of the curves occur at the same local hours; but the night hours at St. Helena are, unlike the same at Bombay, almost entirely free from easterly disturbance. In the disturbance- annual variations there is, as at Toronto, a tendency, though not very decided, towards maxima at the equinoxes and minima at the solstices. Having regard to the note at foot of Table VIII., it is seen that the decennial variation of aggregate disturbance is distinctly indicated, as far as seven years are capable of showing it, both in the easterly, westerly, and combined disturbances, the minimum occurring in 1864 and the maximum about the end of 1859. The aggregate of disturbances in the latter year exceeds that in the former in the proportion of 2·5 to 1. The disturbance- diurnal variation curve shows a preponderance of easterly disturbance at all hours except 20^h and 21^h; from 5^h to 19^h the preponderance is great, but is less considerable during the remaining hours: on the whole the proportion of aggregate easterly disturbance to westerly is as 1·6 to 1.

13. *The regular Solar-diurnal Variation.*—After the separation of disturbed observations differing from the final normals by 1′·4, the averages were taken of the monthly final normals of each hour—in the first place for every month, and in the second place for each of the fourteen half years; and by means of these averages were calculated the diurnal variations* for each month (on an average of seven years), for the whole year, and for each individual half year from 1859 to 1865, the one half year comprising the months April to September, and the other the months January to March and October to December. The diurnal variations are shown in the following Tables.

* By diurnal or annual variations is always to be understood the series of hourly or monthly values expressed by the excess of each above the mean value for the whole day or for the whole year respectively.

TABLE IX.—Showing, for the period 1859 to 1865, the mean diurnal variation of Declination for each month of the year and for the whole year*.

Bombay Astronomical Hours.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Year.
h m													
0 12	−0·554	−0·460	−1·217	−2·226	−2·536	−2·424	−1·969	−2·849	−2·937	−1·382	−0·256	−0·213	−1·585
1 12	−0·183	−0·421	−1·389	−2·182	−2·099	−2·428	−2·172	−2·574	−2·588	−1·090	+0·244	+0·225	−1·388
2 12	+0·003	−0·183	−0·639	−1·610	−1·599	−2·067	−1·860	−1·881	−1·678	−0·366	+0·427	+0·342	−0·896
3 12	+0·136	−0·072	−0·153	−0·704	−0·751	−1·279	−1·097	−0·847	−0·183	+0·311	+0·280	+0·321	−0·337
4 12	+0·210	+0·066	+0·146	−0·065	−0·088	−0·395	−0·372	+0·021	+0·508	+0·412	+0·136	+0·280	+0·072
5 12	−0·019	−0·130	−0·261	−0·024	+0·184	−0·017	+0·004	+0·417	+0·266	−0·157	−0·125	+0·040	+0·015
6 12	+0·005	−0·289	−0·508	−0·267	+0·065	−0·038	−0·030	+0·225	−0·110	−0·268	−0·026	+0·069	−0·098
7 12	+0·169	+0·012	−0·301	−0·323	−0·333	−0·385	−0·407	−0·200	−0·092	−0·073	+0·163	+0·269	−0·125
8 12	+0·168	+0·098	−0·201	−0·253	−0·402	−0·395	−0·594	−0·210	−0·064	−0·057	+0·212	+0·220	−0·123
9 12	+0·155	+0·067	−0·082	−0·052	−0·223	−0·298	−0·401	−0·163	−0·033	−0·064	+0·173	+0·211	−0·059
10 12	+0·131	+0·056	−0·081	+0·084	−0·077	−0·173	−0·240	−0·098	+0·020	+0·074	+0·238	+0·209	+0·012
11 12	+0·164	+0·113	+0·090	+0·203	+0·034	+0·028	−0·082	+0·055	+0·099	+0·144	+0·334	+0·239	+0·118
12 12	+0·107	+0·175	+0·149	+0·302	+0·151	+0·236	+0·073	+0·147	+0·190	+0·183	+0·282	+0·223	+0·185
13 12	+0·051	+0·176	+0·166	+0·414	+0·276	+0·366	+0·256	+0·249	+0·291	+0·218	+0·218	+0·161	+0·237
14 12	−0·115	+0·130	+0·109	+0·453	+0·341	+0·460	+0·346	+0·307	+0·302	+0·213	+0·111	+0·026	+0·215
15 12	−0·224	−0·014	−0·037	+0·267	+0·293	+0·477	+0·368	+0·358	+0·294	+0·019	−0·033	+0·092	+0·155
16 12	−0·400	−0·230	−0·136	+0·116	+0·326	+0·478	+0·435	+0·420	+0·312	−0·080	−0·251	−0·173	+0·068
17 12	−0·443	−0·330	−0·155	+0·162	+0·630	+0·870	+0·850	+0·882	+0·533	−0·150	−0·502	−0·465	+0·157
18 12	−0·551	−0·367	+0·007	+0·974	+1·873	+2·080	+2·064	+2·068	+1·582	+0·218	−0·608	−0·687	+0·721
19 12	−0·648	−0·249	+0·693	+1·962	+2·641	+2·708	+2·544	+2·937	+2·494	+0·898	−0·705	−0·971	+1·192
20 12	+0·066	+0·409	+1·486	+2·242	+2·426	+2·507	+2·344	+2·586	+2·310	+1·338	−0·178	−0·445	+1·424
21 12	+0·978	+0·840	+1·582	+1·532	+1·136	+1·455	+1·272	+0·996	+1·009	+0·904	+0·199	+0·172	+1·006
22 12	+1·055	+0·660	+1·012	+0·188	−0·501	−0·216	−0·020	−0·742	−0·713	−0·081	+0·048	+0·119	+0·067
23 12	−0·264	−0·053	−0·280	−1·194	−1·778	−1·537	−1·326	−2·104	−2·080	−1·176	−0·376	−0·277	−1·037

The variation for the year is also represented by curve No. 8, Plate XXV.

TABLE X.—Showing, for the period 1859 to 1865, the excess of the mean diurnal variation of Declination for each month over the mean diurnal variation for the year.

Bombay Astronomical Hours.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.
h m												
0 12	+1·031	+1·125	+0·368	−0·641	−0·951	−0·839	−0·384	−1·264	−1·352	+0·203	+1·329	+1·372
1 12	+1·205	+0·967	−0·001	−0·794	−0·711	−1·040	−0·784	−1·186	−1·200	+0·298	+1·632	+1·613
2 11	+0·899	+0·713	+0·257	−0·714	−0·703	−1·171	−0·964	−0·985	−0·520	+0·630	+1·323	+1·238
3 12	+0·473	+0·265	+0·184	−0·367	−0·414	−0·942	−0·760	−0·510	+0·154	+0·648	+0·617	+0·658
4 12	+0·138	−0·006	+0·074	−0·137	−0·160	−0·467	−0·444	−0·051	+0·436	+0·340	+0·064	+0·298
5 12	−0·034	−0·145	−0·276	−0·039	+0·169	−0·032	−0·011	+0·402	+0·251	−0·172	−0·140	+0·025
6 12	+0·103	−0·191	−0·410	−0·169	+0·163	+0·060	+0·068	+0·323	−0·012	−0·170	+0·072	+0·167
7 12	+0·294	+0·137	−0·176	−0·198	−0·208	−0·260	−0·282	−0·075	+0·033	+0·052	+0·288	+0·394
8 12	+0·291	+0·221	−0·078	−0·130	−0·279	−0·272	−0·471	−0·087	+0·059	+0·066	+0·335	+0·343
9 12	+0·214	+0·126	−0·023	+0·007	−0·164	−0·139	−0·342	−0·104	+0·026	−0·005	+0·232	+0·270
10 12	+0·119	+0·044	−0·093	+0·072	−0·089	−0·185	−0·252	−0·110	+0·008	+0·062	+0·226	+0·197
11 12	+0·046	−0·005	−0·028	+0·085	−0·084	−0·090	−0·200	−0·063	−0·019	+0·026	+0·216	+0·121
12 12	−0·078	−0·010	−0·036	+0·117	−0·034	+0·051	−0·112	−0·038	+0·005	−0·002	+0·097	+0·038
13 12	−0·186	−0·061	−0·071	+0·177	+0·039	+0·129	+0·019	+0·012	+0·054	−0·019	−0·019	−0·076
14 12	−0·330	−0·085	−0·106	+0·238	+0·126	+0·245	+0·131	+0·092	+0·087	−0·102	−0·104	−0·189
15 12	−0·379	−0·169	−0·192	+0·112	+0·128	+0·322	+0·213	+0·203	+0·139	−0·136	−0·122	−0·063
16 12	−0·468	−0·298	−0·204	+0·048	+0·258	+0·410	+0·367	+0·352	+0·244	−0·148	−0·319	−0·341
17 12	−0·600	−0·487	−0·312	+0·005	+0·473	+0·713	+0·693	+0·725	+0·376	−0·307	−0·659	−0·622
18 12	−1·272	−1·088	−0·714	+0·253	+1·152	+1·359	+1·343	+1·347	+0·861	−0·503	−1·329	−1·408
19 12	−1·840	−1·441	−0·499	+0·770	+1·449	+1·516	+1·352	+1·745	+1·302	−0·294	−1·897	−2·163
20 12	−1·358	−1·015	+0·062	+0·818	+1·002	+1·083	+0·920	+1·162	+0·886	−0·086	−1·602	−1·869
21 12	−0·028	−0·166	+0·576	+0·526	+0·130	+0·449	+0·266	−0·010	+0·003	−0·102	−0·807	−0·834
22 12	+0·988	+0·593	+0·945	+0·121	−0·568	−0·283	−0·087	−0·809	−0·780	−0·148	−0·019	+0·052
23 12	+0·773	+0·984	+0·757	−0·157	−0·741	−0·500	−0·289	−1·067	−1·043	−0·139	+0·661	+0·760

The curves corresponding to these excess variations are numbered 9 to 20, Plate XXVI.

* In this and all the Tables that follow, the positive sign indicates a movement of the north end of the declination-magnet to the eastward of its normal direction, and the deviations from the normal are expressed in minutes of arc.

TABLE XI.—Showing, for the half years including the months from April to September in the several years 1859 to 1865, and in the whole period of seven years, the mean diurnal variation of Declination.

Bombay Astronomical Hours.	1859.	1860.	1861.	1862.	1863.	1864.	1865.	Seven Years, 1859 to 1865.
h m								
0 12	−2·715	−2·867	−2·590	−2·354	−2·461	−2·335	−2·118	−2·490
1 12	−2·625	−2·623	−2·436	−2·245	−2·304	−2·128	−2·021	−2·349
2 12	−2·038	−1·992	−1·748	−1·508	−1·657	−1·689	−1·537	−1·738
3 12	−1·108	−0·997	−0·777	−0·605	−0·785	−0·717	−0·682	−0·810
4 12	−0·138	−0·122	+0·022	+0·149	−0·086	−0·115	−0·165	−0·065
5 12	+0·065	+0·223	+0·262	+0·135	+0·119	+0·102	+0·063	+0·138
6 12	−0·010	+0·164	+0·028	−0·111	−0·013	−0·109	−0·130	−0·026
7 12	−0·320	−0·206	−0·302	−0·326	−0·274	−0·313	−0·288	−0·290
8 12	−0·297	−0·281	−0·312	−0·383	−0·353	−0·360	−0·250	−0·319
9 12	−0·153	−0·208	−0·218	−0·223	−0·181	−0·209	−0·162	−0·193
10 12	−0·095	−0·064	−0·101	−0·102	−0·078	−0·079	−0·046	−0·081
11 12	+0·011	−0·001	+0·078	−0·008	+0·084	+0·073	+0·160	+0·057
12 12	+0·180	+0·109	+0·166	+0·157	+0·247	+0·182	+0·243	+0·183
13 12	+0·294	+0·259	+0·295	+0·308	+0·336	+0·336	+0·332	+0·309
14 12	+0·277	+0·282	+0·384	+0·350	+0·419	+0·417	+0·449	+0·368
15 12	+0·282	+0·222	+0·325	+0·351	+0·434	+0·407	+0·385	+0·344
16 12	+0·200	+0·301	+0·326	+0·300	+0·403	+0·409	+0·395	+0·348
17 12	+0·590	+0·758	+0·704	+0·626	+0·683	+0·612	+0·609	+0·655
18 12	+1·969	+2·008	+1·925	+1·794	+1·643	+1·613	+1·464	+1·774
19 12	+2·797	+2·969	+2·775	+2·497	+2·348	+2·287	+2·162	+2·548
20 12	+2·854	+2·891	+2·426	+2·235	+2·156	+2·243	+2·013	+2·403
21 12	+1·676	+1·556	+1·145	+1·107	+1·007	+1·242	+0·901	+1·233
22 12	−0·141	−0·348	−0·459	−0·472	−0·229	−0·314	−0·372	−0·334
23 12	−1·651	−2·025	−1·917	−1·674	−1·459	−1·555	−1·406	−1·670

Curves representing these variations are figured 23 to 30 on Plate XXVII.

TABLE XII.—Showing, for the half years including the months from January to March and October to December in the several years 1859 to 1865, and in the whole period of seven years, the mean diurnal variation of Declination.

Bombay Astronomical Hours.	1859.	1860.	1861.	1862.	1863.	1864.	1865.	Seven Years, 1859 to 1865.
h m								
0 12	−0·493	−0·556	−0·749	−0·754	−0·664	−0·664	−0·881	−0·680
1 12	−0·419	−0·458	−0·591	−0·446	−0·333	−0·332	−0·479	−0·436
2 12	−0·135	−0·156	−0·287	+0·270	+0·029	−0·087	+0·001	−0·052
3 12	−0·006	−0·019	+0·076	+0·285	+0·134	+0·158	+0·333	+0·137
4 12	−0·075	+0·126	+0·225	+0·305	+0·226	+0·192	+0·462	+0·209
5 12	−0·412	−0·236	−0·096	−0·215	−0·011	+0·024	+0·186	−0·109
6 12	−0·540	−0·468	−0·046	−0·187	−0·071	−0·037	+0·164	−0·169
7 12	−0·077	−0·077	+0·018	+0·013	+0·032	+0·079	+0·294	+0·040
8 12	+0·011	−0·034	+0·089	−0·011	+0·106	+0·089	+0·264	+0·073
9 12	−0·016	−0·039	+0·062	+0·050	+0·124	+0·106	+0·254	+0·077
10 12	+0·073	−0·021	+0·045	+0·065	+0·105	+0·187	+0·280	+0·105
11 12	+0·181	+0·141	+0·132	+0·098	+0·192	+0·213	+0·308	+0·181
12 12	+0·158	+0·140	+0·158	+0·120	+0·193	+0·285	+0·254	+0·187
13 12	+0·160	+0·154	+0·106	+0·138	+0·162	+0·229	+0·209	+0·165
14 12	+0·078	+0·073	+0·057	+0·055	+0·066	+0·124	−0·014	+0·063
15 12	−0·056	−0·058	−0·088	+0·168	−0·079	−0·041	−0·071	−0·032
16 12	−0·284	−0·242	−0·210	−0·158	−0·274	−0·140	−0·171	−0·211
17 12	−0·230	−0·396	−0·366	−0·313	−0·390	−0·361	−0·311	−0·338
18 12	−0·365	−0·431	−0·105	−0·267	−0·426	−0·393	−0·328	−0·331
19 12	−0·178	−0·058	−0·053	−0·139	−0·329	−0·260	−0·127	−0·163
20 12	+0·756	+0·688	+0·608	+0·431	+0·257	+0·216	+0·169	+0·446
21 12	+1·236	+1·207	+0·865	+0·783	+0·643	+0·524	+0·214	+0·782
22 12	+0·764	+0·864	+0·515	+0·311	+0·771	+0·246	−0·189	+0·469
23 12	−0·139	−0·144	−0·357	−0·561	−0·454	−0·350	−0·824	−0·404

Curves representing these variations are figured (in broken lines) on Plate XXVII. Nos. 23 to 30. Viewed together with the continuous curves from April to September,

they show very distinctly the consistent character of the semiannual inequality in the diurnal variation of declination in different years.

The half-excess for each hour of the numbers in the last column of Table XI. over those in the last column of Table XII., or the semiannual inequality of the mean diurnal variation of declination, is as follows, the excesses being written in the order of the hours 0 to 23:—

−0·905 −0·952 −0·843 −0·473 −0·137 +0·123 +0·071 −0·165 −0·175 −0·135 −0·093 −0·062
−0·002 +0·072 +0·152 +0·188 +0·279 +0·496 +1·052 +1·355 +0·978 +0·225 −0·399 −0·633

Fig. 21, Plate XXVI. is constructed from these numbers.

TABLE XIII.—Showing the mean diurnal variation of Declination for each of the years 1859 to 1865.

Bombay Astronomical Hours.	1859.	1860.	1861.	1862.	1863.	1864.	1865.
h m							
0 12	−1·604	−1·711	−1·669	−1·554	−1·562	−1·494	−1·499
1 12	−1·517	−1·540	−1·513	−1·345	−1·318	−1·230	−1·250
2 12	−1·086	−1·074	−0·917	−0·619	−0·814	−0·888	−0·768
3 12	−0·557	−0·508	−0·350	−0·160	−0·325	−0·279	−0·174
4 12	−0·106	+0·002	+0·123	+0·227	+0·070	+0·038	+0·148
5 12	−0·173	−0·006	+0·085	−0·040	+0·054	+0·063	+0·134
6 12	−0·375	−0·152	−0·009	−0·149	−0·042	−0·073	+0·017
7 12	−0·198	−0·141	−0·142	−0·156	−0·121	−0·117	+0·003
8 12	−0·143	−0·157	−0·111	−0·197	−0·123	−0·135	+0·007
9 12	−0·084	−0·123	−0·078	−0·086	−0·028	−0·051	+0·046
10 12	−0·011	−0·042	−0·028	−0·018	+0·013	+0·054	+0·117
11 12	+0·096	+0·070	+0·105	+0·045	+0·138	+0·143	+0·234
12 12	+0·169	+0·124	+0·162	+0·138	+0·220	+0·233	+0·248
13 12	+0·227	+0·206	+0·200	+0·223	+0·249	+0·282	+0·271
14 12	+0·177	+0·177	+0·220	+0·202	+0·242	+0·270	+0·217
15 12	+0·113	+0·082	+0·118	+0·258	+0·177	+0·183	+0·157
16 12	+0·008	+0·029	+0·058	+0·071	+0·064	+0·134	+0·112
17 12	+0·180	+0·181	+0·169	+0·156	+0·146	+0·125	+0·149
18 12	+0·802	+0·788	+0·910	+0·763	+0·608	+0·610	+0·568
19 12	+1·309	+1·455	+1·361	+1·179	+1·009	+1·013	+1·017
20 12	+1·805	+1·789	+1·517	+1·333	+1·206	+1·229	+1·091
21 12	+1·456	+1·381	+1·005	+0·945	+0·825	+0·883	+0·557
22 12	+0·311	+0·258	+0·028	−0·080	+0·271	−0·034	−0·280
23 12	−0·895	−1·084	−1·137	−1·117	−0·956	−0·952	−1·115

14. *Remarks on the regular Solar-diurnal Variations.*—Although the semiannual inequality in the diurnal variation of declination has been established by the researches of General SABINE as beyond a doubt a general feature in that variation, possessing the same characteristics in all parts of the globe, yet the regular progression from month to month in the diurnal variation has not, that I am aware, been much dwelt upon. It is shown, however, so distinctly in the Bombay Observations as to lead, on a first inspection of the curves, figs. 9 to 20, Plate XXVI., to the supposition that the law of variation is identical throughout the year, the extent only (including a reversal of direction) varying from month to month. But in this respect a different exposition of the character of the variation in different months shows that the first thought would be inaccurate. The deviations from the normal position of the magnet (δ_h) at each hour (h) may be expressed by the series

$$\delta_h = A_1 \cos n + B_1 \sin n + A_2 \cos 2n + B_2 \sin 2n + A_3 \cos 3n + B_3 \sin 3n + \&c. \ldots (y),$$

where $n = h \times 15°$; and it is easy from each complete set of hourly values to deduce the

values of the coefficients A_1, B_1, A_2, B_2, A_3, B_3, &c. This has been done for the mean solar-diurnal variation for each month of the year, and also for each half year in the period 1859 to 1865, with results as shown in the following Tables:—

TABLE XIV.—Showing the calculated values of the coefficients A_1, B_1, A_2, B_2, A_3 and B_3 in the series (y) which expresses the mean solar-diurnal variation for each month of the year, for the whole year, and for different half years*.

Months.	Values of the coefficients of the first three pairs of terms of the series (y).					
	A_1.	B_1.	A_2.	B_2.	A_3.	B_3.
January	+0·147	+0·028	−0·247	+0·025	+0·183	+0·209
February	−0·060	−0·003	−0·221	+0·034	+0·187	+0·184
March	−0·407	−0·175	−0·267	+0·367	+0·535	+0·220
April	−0·908	+0·048	−0·097	+0·828	+0·799	+0·153
May	−1·038	+0·044	+0·197	+1·057	+0·839	−0·051
June	−1·271	−0·011	+0·090	+1·119	+0·777	+0·075
July	−1·151	−0·130	+0·102	+1·018	+0·700	+0·059
August	−1·123	+0·160	+0·296	+1·262	+0·898	−0·059
September	−0·910	+0·230	+0·300	+1·090	+0·943	−0·096
October	−0·305	+0·063	+0·032	+0·379	+0·573	−0·040
November	+0·183	+0·200	−0·091	−0·187	+0·076	+0·019
December	+0·250	+0·225	−0·046	−0·196	+0·013	+0·082
Year	−0·549	+0·057	+0·004	+0·566	+0·544	+0·062
Half Years. April to September	−1·067	+0·057	+0·148	+1·062	+0·826	+0·013
Half Years. October to March	−0·032	+0·056	−0·140	+0·070	+0·261	+0·111
Half Years. January to June	−0·589	−0·011	−0·091	+0·572	+0·553	+0·130
Half Years. July to December	−0·509	+0·125	+0·099	+0·561	+0·534	−0·006

TABLE XV.—Showing the calculated values of the coefficients A_1, B_1, A_2, B_2, A_3 and B_3 in the series (y), which expresses the mean solar-diurnal variation for each half year, April to September and October to March, in the period 1859 to 1865.

Years.		Values of the coefficients of the first three pairs of terms of the series (y).					
		A_1.	B_1.	A_2.	B_2.	A_3.	B_3.
April to September.	1859	−1·183	−0·007	+0·037	+1·230	+0·916	+0·073
	1860	−1·168	+0·046	+0·138	+1·308	+0·954	+0·020
	1861	−1·084	+0·085	+0·217	+1·125	+0·882	−0·028
	1862	−0·996	+0·042	+0·216	+0·989	+0·823	−0·043
	1863	−1·040	+0·088	+0·158	+0·973	+0·760	+0·041
	1864	−1·029	+0·048	+0·139	+0·952	+0·766	+0·030
	1865	−0·967	+0·096	+0·133	+0·857	+0·681	+0·001
October to March.	1859	−0·141	−0·103	−0·300	+0·043	+0·246	+0·141
	1860	−0·102	−0·107	−0·266	+0·058	+0·288	+0·188
	1861	−0·073	+0·028	−0·151	+0·165	+0·294	+0·143
	1862	−0·033	+0·042	−0·045	+0·048	+0·268	+0·053
	1863	+0·037	+0·076	−0·147	+0·081	+0·237	+0·141
	1864	+0·006	+0·147	−0·096	+0·022	+0·223	+0·108
	1865	+0·079	+0·309	+0·031	+0·127	+0·272	+0·002

The curves (Nos. 31 to 33, Plate XXVIII.) are constructed to exhibit the variations, in magnitude and direction, of the resultants of A_1 and B_1, A_2 and B_2, A_3 and B_3 respectively; O being the centre of coordinates, the A coefficients are set off in a vertical direction, upward if positive, and the B coefficients in a horizontal direction, to the right if positive, so that the distances from O to the points marked 1, 2, 3, &c. to 12 and $\odot r$

* Göttingen time was inadvertently used instead of Bombay time in the calculation of the coefficients A_1, B_1, &c.

indicate the maximum deviations expressed by each pair of terms in the respective months January, February, March, &c. to December, and for the whole year, and the angles A O1, A O2, A O3, &c., reckoned in a right-handed direction from A O, when converted into time, show the hours at which the several maxima occur. The curves having reference to Göttingen astronomical hours, it will be necessary, in order to adapt them to Bombay time, to turn backward (*i. e.* in a left-handed direction) the zero direction line A O by an equivalent in each figure to $4^h\,12^m$. It will be seen that this method of exhibiting variations brings to light in the regular solar-diurnal variations niceties of change in character which would fail to be noticed on a mere inspection of the respective ordinary representative curves. For instance, if the supposition mentioned above, that the change from month to month in the character of the solar-diurnal variation was one of extent only, not affecting the relative deviation at different hours, the variation at different parts of the year might be expressed in the form

$$\delta_h = C\{(A)_1 \cos n + (B)_1 \sin n + (A)_2 \cos 2n + (B)_2 \sin 2n + (A)_3 \cos 3n + (B)_3 \sin 3n + \&c.\},$$

where $(A)_1$, $(B)_1$, $(A)_2$, B_2, &c. would be constants, and C a function of the day of the year, or of the concurrent variable, the declination or right ascension of the sun. In this case a change in the value of any one of the coefficients A_1, B_1, A_2, B_2, &c. (equal respectively to $C\times(A)_1$, $C\times(B)_1$, $C\times(A)_2$, $C\times(B)_2$, &c.) would be attended by a proportional change in all the others, and the *direction* of the resultants of A_1 and B_1, A_2 and B_2, A_3 and B_3, &c. respectively, would be invariable throughout the year. Thus in the curves Nos. 31 to 33, Plate XXVIII., the points 1, 2, 3, &c. to 12 and $\odot r$ should in each case lie upon a straight line directed through the centre of coordinates. To such a requirement their positions do roughly conform; but there is sufficient evidence of system in their departures from the rule to render the latter worthy of independent consideration. The most striking feature about all the three curves is undoubtedly the existence of the semiannual inequality in the values of the several maximum deviations, the greater values occurring about the northern solstice, the lesser about the southern solstice: this is in accord with the now well-known semiannual inequality in the diurnal variation, which inequality is shown for Bombay by the curve No. 21, Plate XXVI., and by the numbers at the foot of Table XII., and possesses the character commonly attributed to it as general throughout the globe. The points *n*, *s*, *a*, *d* in figures 31 to 33, Plate XXVIII., mark the mean values of the coefficients respectively for the half years, April to September, October to March, January to June, and July to December. Let us now confine our attention to the monthly excess or defect of the coefficients A_1, B_1, A_2, B_2, A_3, B_3, &c. over the annual mean values; or, what is to the same effect, let us imagine the centre of coordinates in the figures 31 to 33, Plate XXVIII. removed to the points $\odot r$, and let vertical lines be drawn upward from these points to A′; then will the lines $r1$, $r2$, $r3$, &c. to $r12$ represent in magnitude and direction the monthly maximum deflections that must be geometrically superimposed upon the annual mean maximum deflection in order to produce the actual monthly maximum deflections; and the angles A′$r1$, A′$r2$, A′$r3$, &c., converted into time, will show the hours at which the monthly superimposed variation gives a maximum of deflection for each pair of terms of the

series (y) which expresses it. Now, on a single glance at the three curves, it is perceived that (speaking generally) the points 1, 2, 3, &c. to 12 circulate in each case in a left-handed direction about the points $\odot r$, the equinoctial months (3 and 4) and (9 and 10) lying on opposite sides of the closed curves. Moreover, the half-yearly points n and s and a and d lie in the same order about $\odot r$; and the interval between the latter two, though smaller, is not inconsiderable in comparison with the interval between the two former points; and the relation of the four points to each other is much the same in each of the three figures, the straight line joining a and d approaching a direction at right angles to the line of junction of n and s. This would seem to imply that the summer form of the diurnal variation merges into the winter form, not solely by the gradual fading away (and inversion at other places) of the former, in which case the diurnal variation at the opposite equinoxes would be the same, but also partly by the superposition of a distinct variation, whose turning-points are considerably removed from those of the ordinary diurnal variation, and the extent of which is much more limited, and which moreover has an inverse character at the opposite equinoxes, at which times its influence on the ordinary mean diurnal variation is most sensible. It does not imply a retrograde rotation of the hours of maxima and minima accompanied by a gradual fading away of the variation, the variation still preserving a constant character in all respects except extent and turning-points, because in that case, whilst a single rotation occurs in the arrangement of the points 1, 2, 3, &c. to 12 in figure 31, there should be two and three rotations respectively in figures 32 and 33, and, in fact, there is only a single rotation in each of the three curves. Now, on referring to plate XXV. of the Philosophical Transactions of the Royal Society for 1863, where appear similar figures representing (in part) the diurnal variations of declination at Toronto and St. Helena, it is seen that these figures possess the characteristics just described of those for Bombay, with the exception that the rotation in the St. Helena curve is in a right-handed instead of a left-handed direction. To examine now more directly whether there be a semiannual inequality in the diurnal variation of declination having opposite characteristics at the opposite equinoxes, and at the same time to eliminate that other semiannual inequality whose times of opposition are the solstices, the mean diurnal variation was calculated for the half year January to June, and compared with the same for the half year July to December; half the excess, for each hour, of the former half year over the latter was taken to represent the inequality sought, which proved to be of so definite a character, and so similar for Bombay and Toronto, as to tend to the extension of the examination to all other stations for which reduced observations were accessible. Distinct similarity was then found to exist in the curves representing the inequality at all the north-latitude stations, Toronto, Kew, Greenwich, Pekin, and Bombay, whilst curves of form approaching to an inversion, with some modifications of the above, was found for the south-latitude stations, St. Helena and the Cape of Good Hope. Table XVI. and curves Nos. 37 to 43, Plate XXX., show the character of this inequality for each station.

TABLE XVI.—Showing, for different stations, the mean diurnal variation of Declination for the half year including the months January to June, and for the half year including the months July to December, and also the half excess, at each hour, of the former over the latter.

Local Astronomical Hours (nearest).	Toronto.			Kew.			Greenwich.			Pekin.			Bombay.			Five North-Latitude Stations.	St. Helena.			Cape of Good Hope.			Two South-Latitude Stations.
	January to June.	July to December.	Half excess.	January to June.	July to December.	Half excess.	January to June.	July to December.	Half excess.	January to June.	July to December.	Half excess.	January to June.	July to December.	Half excess.	Mean of Half excess.	January to June.	July to December.	Half excess.	January to June.	July to December.	Half excess.	Mean of Half excess.
h																							
0	−3·86	−4·19	+0·16	−5·11	−5·15	+0·02	−3·82	−4·13	+0·15	−1·38	−1·72	+0·17	−1·57	−1·60	+0·02	+0·06	+0·81	+0·70	+0·05	+0·13	−0·16	+0·14	+0·09
1	−5·05	−5·08	+0·01	−6·28	−6·11	−0·08	−5·40	−5·30	−0·05	−2·37	−2·15	−0·11	−1·45	−1·33	−0·06	−0·12	+0·79	+0·60	+0·09	+1·12	+0·67	+0·22	+0·15
2	−4·99	−4·75	−0·12	−6·02	−5·59	−0·21	−5·88	−5·20	−0·34	−2·68	−1·95	−0·36	−1·02	−0·78	−0·12	−0·25	+0·49	+0·31	+0·09	+1·54	+1·11	+0·21	+0·15
3	−4·14	−3·51	−0·31	−4·54	−4·03	−0·25	−4·93	−4·03	−0·45	−2·35	−1·35	−0·50	−0·47	−0·20	−0·13	−0·34	+0·16	−0·08	+0·12	+1·28	+1·05	+0·11	+0·11
4	−2·82	−2·14	−0·34	−2·83	−2·37	−0·23	−3·38	−2·40	−0·49	−1·63	−0·79	−0·42	−0·02	+0·16	−0·09	−0·31	−0·24	−0·48	+0·12	+0·74	+0·52	+0·11	+0·11
5	−1·51	−1·07	−0·22	−1·47	−1·05	−0·21	−1·73	−0·83	−0·45	−0·96	−0·33	−0·31	−0·04	+0·07	−0·06	−0·21	−0·48	−0·63	+0·07	+0·32	+0·08	+0·12	+0·09
6	−0·69	−0·26	−0·21	−0·48	−0·29	−0·09	−0·38	+0·15	−0·26	−0·39	−0·19	−0·10	−0·17	−0·02	−0·07	−0·12	−0·33	−0·44	+0·05	+0·21	+0·13	+0·04	+0·04
7	−0·28	−0·04	−0·12	+0·18	+0·25	−0·03	+0·58	+0·88	−0·15	−0·19	−0·22	+0·01	−0·19	−0·06	−0·07	−0·06	−0·02	−0·13	+0·05	+0·37	+0·35	+0·01	+0·03
8	−0·12	−0·24	+0·06	+0·57	+0·79	−0·11	+1·20	+1·38	−0·09	−0·14	−0·24	+0·05	−0·16	−0·08	−0·04	−0·03	+0·09	+0·06	+0·01	+0·44	+0·46	−0·01	0·00
9	+0·42	+0·62	−0·10	+0·88	+1·09	−0·10	+1·67	+1·88	−0·10	−0·07	−0·13	+0·03	−0·07	−0·05	−0·01	−0·05	+0·18	+0·19	0·00	+0·46	+0·51	−0·02	−0·01
10	+0·68	+0·72	−0·02	+1·12	+1·35	−0·11	+1·90	+2·07	−0·08	+0·08	−0·08	+0·08	−0·01	+0·03	−0·02	−0·02	+0·26	+0·29	−0·01	+0·46	+0·50	−0·02	−0·01
11	+0·76	+0·66	+0·05	+1·27	+1·48	−0·10	+1·98	+2·02	−0·02	+0·25	+0·06	+0·09	+0·11	+0·13	−0·01	+0·01	+0·29	+0·28	0·00	+0·41	+0·54	−0·06	−0·03
12	+0·61	+0·64	−0·01	+1·24	+1·62	−0·19	+1·83	+1·82	0·00	+0·43	+0·16	+0·13	+0·19	+0·18	0·00	0·00	+0·29	+0·18	+0·05	+0·40	+0·56	−0·08	−0·01
13	+0·60	+0·52	+0·04	+1·18	+1·39	−0·10	+1·67	+1·48	+0·09	+0·57	+0·33	+0·12	+0·24	+0·23	0·00	+0·02	+0·20	+0·07	+0·06	+0·46	+0·56	−0·05	0·00
14	+0·55	+0·49	+0·03	+1·29	+1·49	−0·10	+1·33	+1·22	+0·05	+0·59	+0·40	+0·09	+0·23	+0·20	+0·01	+0·02	+0·17	0·00	+0·08	+0·42	+0·48	−0·03	+0·02
15	+0·75	+0·68	+0·03	+1·46	+1·57	−0·05	+1·17	+1·03	+0·07	+0·38	+0·46	−0·04	+0·13	+0·19	−0·03	+0·01	+0·17	−0·05	+0·11	+0·42	+0·40	+0·01	+0·06
16	+1·25	+1·00	+0·12	+1·86	+1·89	−0·01	+1·25	+1·00	+0·12	+0·55	+0·50	+0·02	+0·03	+0·11	−0·04	+0·04	+0·18	−0·11	+0·14	+0·37	+0·30	+0·03	+0·08
17	+2·00	+1·86	+0·07	+2·62	+2·40	+0·11	+1·40	+1·15	+0·12	+0·62	+0·52	+0·05	+0·12	+0·19	−0·03	+0·08	+0·27	−0·02	+0·14	+0·30	+0·24	+0·03	+0·08
18	+3·08	+2·89	+0·09	+3·24	+2·89	+0·17	+1·73	+1·35	+0·19	+0·93	+0·94	0·00	+0·67	+0·77	−0·06	+0·08	+0·50	+0·38	+0·06	+0·19	−0·09	+0·14	+0·10
19	+3·97	+4·02	−0·02	+3·86	+3·30	+0·28	+2·07	+1·62	+0·22	+1·47	+1·55	−0·04	+1·18	+1·20	−0·01	+0·11	−0·10	−0·07	−0·01	−0·50	−0·63	+0·06	+0·02
20	+4·55	+4·33	+0·11	+4·18	+3·41	+0·38	+2·47	+1·82	+0·32	+2·13	+1·99	+0·07	+1·52	+1·33	+0·10	+0·21	−1·14	−0·52	−0·31	−2·08	−1·67	−0·20	−0·25
21	+3·85	+3·41	+0·22	+3·64	+2·58	+0·53	+2·40	+1·62	+0·39	+2·40	+1·80	+0·30	+1·25	+0·76	+0·25	+0·35	−1·62	−0·65	−0·48	−3·16	−2·29	−0·43	−0·45
22	+1·52	+1·14	+0·19	+0·88	+0·04	+0·42	+1·28	+0·35	+0·46	+1·55	+0·93	+0·31	+0·37	−0·23	+0·30	+0·32	−1·15	−0·36	−0·39	−2·84	−2·60	−0·12	−0·25
23	−1·45	−1·94	+0·24	−2·43	−2·94	+0·25	−1·03	−1·78	+0·37	+0·03	−0·43	+0·23	−0·85	−1·22	+0·19	+0·21	+0·04	+0·32	−0·14	−1·48	−1·34	−0·07	−0·10

Now an inspection of plate xiv. in the Philosophical Transactions for 1863 suffices to convince that the mean diurnal variation of declination, whilst subject at different places to differences of character of a minor order, is mainly of the same general type throughout the globe: the range may vary or even the inflections of the representative curve may be inverted, but the relation as to the kind of flexure of any one principal feature of the curve to the remaining principal features is the same everywhere: in the typical variation a maximum or minimum occurs between twenty and twenty-one hours, and a minimum or maximum between one and two hours; and the great amount of change takes place between the hours seventeen and five, whilst deviations from the normal position of the magnets of comparatively only small extent occur during the remaining hours. To the same type belongs that semiannual inequality which has opposite features at the opposite solstices, and which may be called the solstitial semiannual inequality to distinguish it from the equinoctial semiannual inequality, as that might be named which has opposite characteristics at the opposite equinoxes,—with this difference, that the turning-point, which in the typical mean variation occurs at twenty or twenty-one hours, takes place in the inequality a full hour earlier. Thus the mean diurnal variation for each of the half years, April to September and October to March, being the result of the combination of two variations of the same kind, will also be of the typical character, but with a small difference in the time of the occurrence of the twenty to twenty-one hours turning-point. Let us now examine the character of the equinoctial semiannual inequality as exhibited in the curves 37 to 43, Plate XXX., and compare it with the type curve of the mean diurnal variation. In the first place, it is seen that, as in the type, there is scarcely any change during the night hours, and that the main variation occurs during half the day, in this case between eighteen hours and six hours; secondly, the range of variation differs from about half a minute to nearly one minute of arc; thirdly, the hour of noon is that at which the deviations due to this variation pass through zero, and on each side of which the inflections of the curve are inversely, but in respect to north-latitude stations symmetrically disposed; fourthly, the turning-points are twenty-one hours and three hours, the former being a maximum and the latter a minimum for north-latitude stations from January to June, and for south-latitude stations from July to December, and *vice versâ*, for north-latitude stations from July to December, and for south-latitude stations from January to June. The concurrent testimony of the five widely distributed north-latitude stations leaves, I think, very little room for doubt that this inequality is an expression of real phenomena common to the whole of the northern magnetic hemisphere. From the general agreement, qualified by the fact that the afternoon inflections are comparatively much subdued, of the curves of two stations of such comparative proximity as St. Helena and the Cape of Good Hope, it would be less safe to generalize as to the southern magnetic hemisphere; but there is abundant evidence of the existence of a definite, and in some respects different law to that which prevails in the northern hemisphere to encourage further inquiry in this direction*. So far as

* An examination of the Hobarton observations (including all disturbances) resulted in a curve corresponding closely with those of the north-latitude stations, but of small range.

I am aware, this is the first time that any diurnal variation of magnetical phenomena has been pointed out in which the sun's position relatively to the meridian of the place appears so directly related to the expressed phenomena. To exhibit the general character of this variation more distinctly the curves Nos. 44 and 45, Plate XXX., are drawn on a larger scale to represent the average variation for the north- and south-latitude stations respectively. For all the stations, except St. Helena and Greenwich, the above are the results derived from the observations after the separation of the larger disturbances by the method above described; for St. Helena the effect of disturbances on the diurnal variation is very slight, and all observations are included; for Greenwich complete days of large disturbance are thrown aside, and the remainder used for the calculation of the diurnal variation.

15. *Solar-diurnal Inequality.*—The following Table shows the mean diurnal inequality when all observations, disturbed and undistubed, are included in the calculations as compared with the mean diurnal variation after the exclusion of disturbances exceeding 1′·4 in amount; and it is seen that at Bombay the disturbances have a very slight effect on the diurnal variation, the largest difference at any hour not exceeding 0′·061.

TABLE XVII.

	Bombay Astronomical Hours.											
	h m 0 12	h m 1 12	h m 2 12	h m 3 12	h m 4 12	h m 5 12	h m 6 12	h m 7 12	h m 8 12	h m 9 12	h m 10 12	h m 11 12
Mean diurnal inequality.	−1·590	−1·424	−0·936	−0·342	+0·064	+0·023	−0·088	−0·108	−0·084	−0·032	+0·025	+0·144
Mean diurnal variation.	−1·585	−1·388	−0·896	−0·337	+0·072	+0·015	−0·098	−0·125	−0·123	−0·059	+0·012	+0·118
Excess of inequality over variation.	−0·005	−0·036	−0·040	−0·005	−0·008	+0·008	+0·010	+0·017	+0·039	+0·027	+0·013	+0·026
	Bombay Astronomical Hours.											
	h m 12 12	h m 13 12	h m 14 12	h m 15 12	h m 16 12	h m 17 12	h m 18 12	h m 19 12	h m 20 12	h m 21 12	h m 22 12	h m 23 12
Mean diurnal inequality.	+0·212	+0·258	+0·238	+0·174	+0·084	+0·159	+0·719	+1·216	+1·388	+0·951	+0·006	−1·059
Mean diurnal variation.	+0·185	+0·237	+0·215	+0·155	+0·068	+0·157	+0·721	+1·192	+1·424	+1·006	+0·067	−1·037
Excess of inequality over variation.	+0·027	+0·021	+0·023	+0·019	+0·016	+0·002	−0·002	+0·024	−1·036	−0·055	−0·061	−0·015

16. *Variation from year to year in the range of the mean diurnal variation of Declination.*

TABLE XVIII.—Showing the range of the mean diurnal variation of Declination for the several years 1859 to 1865, and for the component half years (April to September, and January to March and October to December).

Years.	Range.		
	Year.	Half year, April to September.	Half year, October to March.
1859.	2·913	5·569	1·776
1860.	3·500	5·636	1·763
1861.	3·186	5·365	1·614
1862.	2·887	4·851	1·537
1863.	2·768	4·809	1·435
1864.	2·723	4·612	1·188
1865.	2·590	4·280	1·343

The regular progression in these numbers, the equivalent of which is shown also by the varying range of the curves Nos. 23 to 29, Plate XXVII., is very marked, and tends towards maxima in 1859–60 and minima in 1864–65, in near correspondence with the maxima and minima of the decennial variation of aggregate annual disturbance. In order to show the periodicity, the ranges have also been calculated of the mean diurnal inequality (including disturbances) for the larger number of years 1848 to 1864; and the same has been done for the Greenwich observations (excluding certain complete disturbed days), with the results subjoined.

TABLE XIX.—Showing the range of the mean diurnal inequality of Declination at Bombay in the several years 1848 to 1864, and at Greenwich in the several years 1848 to 1857, with the corresponding ratios of the range of each year to the average range.

Years.....		1848.	1849.	1850.	1851.	1852.	1853.	1854.	1855.	1856.	1857.	1858.	1859.	1860.	1861.	1862.	1863.	1864.
Bombay....	Range.	3·491	3·248	3·294	2·965	2·611	2·728	2·416	2·576	2·453	2·459	2·720	3·117	3·551	3·282	2·790	2·568	2·659
	Ratios.	1·24	1·16	1·17	1·06	0·93	0·97	0·86	0·92	0·87	0·88	0·97	1·11	1·26	1·17	0·99	0·91	0·94
Greenwich	Range.	11·4	10·3	10·0	8·4	8·0	6·9	7·2	7·0	5·9	5·6							
	Ratios.	1·42	1·29	1·25	1·05	1·00	0·86	0·90	0·87	0·74	0·70							

In calculating the average range (2′·808) for Bombay, double value was given to the ranges from 1853 to 1859; for Greenwich it was taken of such a value (8′·0) as would make the sum of the ratios equal to the sum of the ratios at Bombay for the same years. Two periods of maximum range are now seen to exist not far from the years of maximum disturbance with a minimum intervening, and an approach to a second minimum near the last year (1864). The diminishing progression of the ranges for Greenwich being (with one slight exception) continuous, shows no signs of periodicity when viewed by itself; but the succession of ratios so much resembles that which appears for the same years (1848 to 1857) at Bombay, that it is reasonable to suppose that on the extension of the results for Greenwich beyond 1857 an increasing progression would begin to

show itself, as is the case at Bombay. The proportion, however, of the maximum to the minimum range at Greenwich is about 2, whilst at Bombay it is only about 1·5.

The variation in the range of the mean diurnal variation of declination is also indicated by the corresponding variation in the magnitude of the maximum deflections for the half years April to September, for each of the first three pairs of terms of the series (y) as delineated on Plate XXIX. figs. 34 to 36; but there is also in these figures a curious likeness, though not equivalent as to time, in the arrangement of the points for the successive years about the centre of coordinates indicative of systematic change from year to year in the law as well as in the extent of variation.

17. *Annual Variation and Secular Change.*—The observations having been repeatedly interrupted in the year 1861, it is better perhaps to reject that year and use only the two continuous series of observations, 1859 and 1860, and 1862 to 1855, in the deduction of secular and annual variations. The following Tables will, however, be filled up for that year, although no use is made of the numbers in the calculations applied.

TABLE XX.—Showing the mean monthly and annual values of Easterly Declination as deduced from the readings of GRUBB's Declinometer in the several years 1859 to 1865.

Years.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Year.
1859.	20·592	20·835	21·130	20·868	21·227	21·258	21·366	21·190	21·569	22·092	22·375	22·913	21·451
1860.	23·021	23·037	23·498	23·460	23·562	24·045	24·209	24·506	24·296	24·294	24·932	25·150	23·995
1861.	25·414	25·685	26·158	26·136	26·023	26·100	25·902	28·861	29·041	29·185	28·632	28·897	27·169
1862.	29·042	29·014	29·178	29·703	29·367	30·416	30·992	31·049	31·303	31·677	31·820	32·464	30·502
1863.	32·696	32·755	33·948	34·334	33·633	33·424	33·802	33·783	34·407	34·742	34·529	35·471	33·960
1864.	35·262	35·032	35·379	35·331	35·710	36·016	36·459	36·716	37·126	38·043	38·167	38·198	36·453
1865.	38·568	38·618	38·680	37·816	39·004	39·499	40·143	41·251	41·489	41·717	41·241	42·311	40·028
Mean of six years omitting 1861	29·863	29·882	30·302	30·252	30·417	30·776	31·162	31·416	31·687	32·094	32·177	32·751	

The numbers in the last column of Table XX. give values of the annual secular increase of easterly declination as follows:—

From 1859 to 1860 the increase		=2·544
1862 to 1863	„	=3·458
1863 to 1864	„	=2·493
1864 to 1865	„	=3·575
Mean . . .		=3·017*.

This gives a secular increase at the rate of 0′·2515 per month; and applying corrections

* It will be seen by a reference to Table III. on page 4 of the Tables of Results of the 'Bombay Magnetical Observations for 1864,' that the annual secular increase of Declination, which from 1846 to 1865 had an average value of only 1′·24, has since the year 1859 been considerably greater, as shown above.

at this rate to reduce the mean numbers at the foot of Table XX. to the common epoch, the 1st January of the mean year, we have

TABLE XXI.

Months.....	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Year.
Mean for mean year	29·863	29·882	30·302	30·252	30·417	30·776	31·162	31·416	31·687	32·094	32·177	32·751	
Correction	·000	−·251	−·503	−·754	−1·006	−1·257	−1·509	−1·760	−2·012	−2·263	−2·515	−2·766	
Means corrected.	29·863	29·631	29·799	29·498	27·411	29·519	29·653	29·656	29·675	29·831	29·662	29·985	29·682
Annual variation	+·181	−·051	+·117	−·184	−·271	− ·163	− ·029	− ·026	− ·007	+ ·149	− ·020	+ ·303	

The last line of numbers is far from presenting the decidedly systematic and regular appearance that is found by General SABINE (see Philosophical Transactions, 1863, pages 291 to 295) from the similar treatment of the observations of Kew and Hobarton; let us therefore examine, as the similar results for those places and for St. Helena and the Cape of Good Hope would suggest, whether (failing a regular periodical progression from month to month) there be any semiannual inequality of Declination at Bombay. Combining together the months April to September and the months January to March, and October to December, so as to eliminate secular change, we get the following results.

TABLE XXII.—Showing the mean values of Easterly Declination for April to September, and for January to March, and October to December in each of the years 1859, 1860, and 1862 to 1865; and the excess of the mean for the former half year over the mean for the latter in each year.

Years.....	1859.	1860.	1861.	1862.	1863.	1864.	1865.
April to September	21·246	24·001	.....	30·472	33·897	36·226	39·867
January to March, and October to December	21·656	23·989	.....	30·532	34·023	36·680	40·189
Excess of April to September over October to March	−·410	+·012	.....	−·060	−·126	−·454	−·322
Mean excess	−0′·227						

The signs of the differences in five of the six years are negative, whilst the difference for the sixth year has a slight positive value. The inference to be drawn thence is that there is some probability that the north end of the declination-magnet points at Bombay 0′·227 * more to the westward in the months April to September than in the months January to March and October to December; and the same result exactly is arrived at if all the disturbed observations are included in the calculation of the mean values of declination. I cannot regard the evidence as conclusive, however, for two reasons,—first, because the observations at the four stations Kew, Hobarton, St. Helena, and the Cape of Good Hope agree in showing that at those places the north end of the declination-

* The fact that this is exactly the magnitude of the semiannual inequality at St. Helena renders the correctness of the inference more probable.

magnet points, when the sun is north of the equator, to the eastward of its mean annual position, which is contrary to what is found for Bombay; and, secondly, because the annual variations at Bombay, calculated, as in Table XXI., for each individual year, are irregular and inconsistent with each other to an extent much exceeding the whole range of the semiannual inequality. These irregularities are attributable, I believe, almost entirely to the defect of the reading-transit instrument of the declinometer alluded to in paragraph 7. The annual variations found for the different years are as follows:—

TABLE XXIII.—Showing the annual variation of Declination calculated, after the elimination of secular change at the rate of 0′·2515 per month, for the several years 1859, 1860, and 1862 to 1865.

Years.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.
1859.	+·524	+·516	+·559	+ ·046	+·153	−·067	−·211	−·638	−·511	− ·239	−·208	+·079
1860.	+·409	+·174	+·383	+ ·094	−·056	+·176	+·088	+·134	−·398	− ·581	−·195	−·298
1861.												
1862.	−0·77	−·356	−·444	− ·170	−·758	+·040	+·364	+·170	+·172	+ ·295	+·186	+·579
1863.	+·119	−·073	+·868	+1·003	+·050	−·410	−·284	−·554	−·182	− ·098	−·563	+·128
1864.	+·192	−·289	−·194	− ·493	−·366	−·311	−·120	−·114	+·042	+1·410	+·582	+·362
1865.	−·077	−·278	−·468	−1·583	−·647	−·403	−·011	+·846	+·832	+ ·809	+·081	+·900

X. *Observations on the Development of the Semilunar Valves of the Aorta and Pulmonary Artery of the Heart of the Chick.* *By* MORRIS TONGE, *M.A., M.D. Communicated by Dr.* BEALE, *F.R.S.*

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IN the accounts of the development of the heart of vertebrate animals given by various embryological writers, we find an apparently clear description of the mode in which the permanent aorta and pulmonary artery are formed by the longitudinal division of a single large vessel, the truncus arteriosus, into two vessels. The truncus arteriosus is, as is well known, the large arterial trunk that commences in the originally single ventricle of the heart, and terminates by splitting up into the branchial arteries. It conveys the whole of the blood from the ventricle to the system of the embryo, and is also known by the name of *bulbus aortæ* (see Plate XXXI. fig. 1). In studying the descriptions given by different authors of its division into two vessels, it appeared to me very strange that with such a clear description of the mode of division nothing at all, or only very little, should be said about the mode of development of the semilunar valves attached to the commencement of these vessels. KÖLLIKER is the only author I have been able to find who makes any mention of their mode of development, and his account, which I shall presently quote, is very brief and unsatisfactory. Nowhere have I found any drawings of these parts in their rudimentary state. I was therefore obliged to conclude that very little was known about this point, probably in consequence of the difficulty of accurately examining such minute parts at an early period of development, and I was hence led to attempt the observations recorded in the present paper. They were made during the years 1865, 1866, and 1867, on the embryo of the common domestic fowl, artificially incubated. Though not nearly as complete as I could have wished them to be, they nevertheless demonstrate certain new and interesting facts connected with the development of the semilunar valves, and the formation of the aorta and pulmonary artery in the bird's heart. These appear to me to be valuable, as possibly throwing light on some of the congenital malformations of this part of the heart. In working at the development of the semilunar valves, I was also obliged to examine very closely into the mode of division of the truncus arteriosus into two vessels, and found that the manner in which it becomes divided differs from that usually described to occur in some very important particulars. The development of the semilunar valves is so closely connected with the process of division of the truncus arteriosus that I have found it best to unite the description of each stage of the one with that of the corresponding stage of the other.

As the satisfactory examination of these minute parts is difficult, and requires a pecu-

liar mode of preparation of the embryo, I will first give a brief account of the method I followed successfully. It was obvious that these tissues being from the first highly elastic, and also contractile, would collapse immediately the circulation stopped, and in this collapsed condition it would be impossible to examine the anatomy of the interior of the vessel with any degree of certainty, the difficulty being enhanced by the softness of the tissues in their natural state. It seemed to me that this was the point where former investigators must have failed. The desideratum was to obtain the large vessels in a distended and hardened state, and at the same time transparent and fit for examination by transmitted light. To effect this the embryo immediately on its removal from the egg was immersed in strong spirit of wine, the sac of the amnion being first slit open, where this had closed up, in order that the spirit might come freely into contact with every part of the surface of the embryo. As the spirit permeates the tissues of the embryo, which it does rapidly, the capillaries and smaller vessels are soonest contracted by it, and the blood being thus prevented from escaping from the larger arteries before the heart's action is stopped, they become greatly distended. When this distension with blood reaches its maximum in the arteries, the blood accumulates in the cavities of the heart and in the large veins, the impediment caused by the closure of the systemic capillaries and small arteries acting backwards throughout the circulation. By this treatment it was found that the cavities of the heart, and the large vessels nearest to the heart, became fully distended with blood, and becoming coagulated and hardened by the spirit retained the distended condition. The embryos were allowed to remain in the alcohol for a few hours or days, according to their size, in order that they might become completely hardened. By careful management of the position of the embryo in the corked glass tube, I was frequently able to obtain a colourless clot in the large vessels, owing to the sinking of the corpuscles before the blood had coagulated or the spirit had penetrated the walls of the artery. The best position for obtaining this result was to keep the head of the embryo downwards, and the body a little inclined towards its dorsal surface. The embryos after being hardened in spirit were of course perfectly opaque, and could only be examined by reflected light till they had been rendered transparent. This was effected by soaking them in strong PRICE'S glycerine for several days. After this the portions required for examination were removed, and dissected and examined in glycerine by strong transmitted light under a compound microscope fitted with an erector and a low power (1 inch). The hardness of the tissues permitted sections to be made readily in various planes, and their transparency under strong light enabled the course of the vessels distended with coagulated blood to be readily followed. Where it was required to examine the structure of the interior of the cavities of the heart and great vessels the coagula, if opaque, were first carefully picked out by needles: if a colourless coagulum had been obtained this troublesome process was often unnecessary. After dissection in strong glycerine the specimens were mounted for drawing and description. They were preserved in glycerine jelly, this medium retaining the specimen in any position in which it was required to be drawn or described, and allowing of its

being readily altered to a fresh position where it was necessary that the same object should be seen from different points of view. The observations were made on more than fifty embryos in various stages of development, and the most satisfactory dissections were converted into permanent preparations. The preliminary experiments on artificial incubation, as well as on the best method of preparing the embryos, consumed much time. Much time and pains were also spent in conducting the incubating-process, and before I had gained experience the incubation of a number of eggs for several days often led to no result.

The account of my own observations will be best prefaced by quoting the statements of RATHKE and KÖLLIKER with reference to the development of this part of the heart; KÖLLIKER's observations are the latest I have been able to discover.

RATHKE* says, "The anterior chamber of the heart remains single in fishes, and in them developes itself into the so-called bulbus aortæ (Herz-Zwiebel). In the Batrachia it lengthens itself, and forms the trunk for all the arteries of the body. This, however, does not remain so smooth and simple internally as on the exterior; but the wall of the anterior chamber of the heart while it lengthens sends inwards two longitudinal ridges situated opposite to one another, so that the cavity of the arterial trunk which arises from it becomes very incompletely divided into two lateral halves. The same developmental process is also remarked in Birds and Mammalia; in them, however, the two ridges become higher, then grow together throughout their whole length, and completely divide the hollow of the chamber into two lateral halves. When this has taken place, a division results by a resorption through the middle of the wall and newly formed septum of the anterior chamber of the heart; and this now becomes divided into two canals, of which one represents the commencement of the aorta, the other the commencement of the pulmonary artery. In the Snake, and probably also in other reptiles, three partitions arise on the inside of the anterior chamber of the heart, the free borders of which ultimately grow together, and so the chamber gets divided into three canals lying beside one another, of which one represents the commencement of the pulmonary artery, the two others the commencement of the two roots of the aorta." RATHKE's description therefore relates solely to the division of the truncus arteriosus. With reference to the development of the semilunar valves he is silent.

KÖLLIKER's† account of the division of the truncus arteriosus is as follows:—"Simultaneously with the formation of the septum of the ventricles proceeds also the division of the truncus arteriosus into the *pulmonary artery* and *permanent aorta*, which, though apparently only a continuation of the process which takes place in the separation of the ventricles, is yet to be distinguished from it. Thus while in the ventricles it is the muscular structure of the heart which grows out and ultimately developes itself into a complete septum, in the primitive aorta it is the vascular wall, having more the structure of connective tissue, that effects the separation. Therefore the division of the truncus arte-

* Entwickelung der Wirbelthiere, S. 82, pp. 187–188 (1861).

† Entwickelungsgeschichte des Menschen, pp. 404, 405 (1861).

riosus cannot be described as if it were formed by an inward growth of the ventricular septum, as also comes out in the clearest manner from the fact that in certain creatures the aorta divides at a time when the ventricle is still single. This is the case in the Viper, according to RATHKE, in which, at the time when the truncus arteriosus divides into three vessels, the ventricle still possesses no trace of a septum. Therefore, as RATHKE correctly observes, the cause of the division of the primitive aorta into two canals cannot with Von BAER be sought in certain peculiarities of the circulation, in a particular direction of the current of blood, but it depends solely and alone on particular developmental phenomena of the arterial wall. Now, concerning the peculiarities in man, I have in the fourth week found the truncus arteriosus still completely single, and with round *lumen**. Transverse sections examined under the microscope already show clearly three tunics,—a thin tough *adventitia*, a strong clear *media*, and an inner cellular layer as *intima*. In the fifth week likewise the artery was still single, but its *lumen* was now flattened out transversely and slit-shaped. In the seventh and eighth week I found the vessel already completely doubled, and I was not here able to make out the intermediate stages, and to follow the gradual formation of the septum. I was more fortunate with calf embryos of 7‴–8‴ long; and I here first found an aorta with a *lumen* like a figure of 8, or, in other words, with two slight ridges in the interior, which resulted from growths of the tunica media, and, secondly, others which contained two *lumina* within a common *adventitia*, each of which possessed its own separate *tunica intima*, but connected *tunicæ mediæ*. In consequence of this it cannot well be doubted that the division of the truncus arteriosus essentially comes to pass by a growth of its middle coat, followed at a later period by that of the *adventitia*, which is, however, in Man effected very early, since even at the eighth week both the large arteries possess all their proper coats."

With respect to the development of the semilunar valves, he says, "Simultaneously with the division the semilunar valves also become developed, and I saw them already present in both arteries in an embryo of the seventh week. They are, however, at first nothing but horizontally projecting crescentic growths of the middle and of the epithelial coats by which the *lumen* at this spot receives the form of a three-rayed star. At what time they first become visible as distinct pockets I have not yet investigated."

I now pass on to my own observations. They were made only on the embryo of the fowl, as I have had no opportunity of investigating that of Man or any other mammalian animal. But from the great likeness between the hearts of Birds, Mammalia, and Man at different stages of their development, it seems pretty certain that the arterial semilunar valves in Man and Mammalia generally must pass through the same stages of devolopment as those of the Bird, which in the fully developed state quite resemble them.

As is well known, the heart of Birds and Mammalia in its earliest condition is a simple

* I have left this word untranslated because no single English word exactly expresses its meaning. It is obviously the bright area of the interior of a transverse section of the vessel held up to the light. This bright area being bounded by the inner edge of the transverse section, shows the form of the vascular canal at this point.

tube, into the hinder or caudal end of which the omphalo-mesenteric veins open, while the branchial arteries emerge from the anterior or cephalic end. This tube soon increases in length and capacity, and becomes doubly bent upon itself, so that its general direction is at first from the entrance of the veins towards the head and ventral surface of the embryo, then bending backwards towards the caudal end, and from left to right, and then again bending towards the head and ending in the branchial arteries. As the contractile muscular tissue augments about the bends, the cephalic or left bend gradually becomes developed into a rudimentary auricle, the caudal or right bend, and the portion of tube between it and the cephalic bend, into the ventricle, while the considerable portion of the tube situated between the caudal bend and the branchial vessels remains free, or almost entirely so, from muscular tissue, and forms the bulbus aortæ or truncus arteriosus (Plate XXXI. fig. 1). About the third day of incubation this large arterial trunk, into which the still single ventricle pours its blood, does not appear to be separated by any valvular apparatus whatever from the cavity of the ventricle. The vessel is twisted somewhat spirally after its exit from the ventricle, and the aperture of its transverse section is not everywhere circular, being oval or squarish at the bends. It is quite single and undivided throughout, and no ridges can be seen on its interior. It becomes gradually narrower as it approaches the point of division into the branchial arteries, in which it terminates abruptly. There is no valvular apparatus of any kind at the point of division into the branchial arteries, which are three on each side at this period. The first and second pair of branchial arteries have now disappeared; the third, fourth, and fifth pair remain, and unite posteriorly to form the descending aorta. The deficiency of valves seems to be supplied by a considerable development of the elastic wall of the truncus arteriosus next the ventricle. This does not take place equally round the opening into the ventricle, but occurs principally on two opposite sides of the interior of the vessel, so that the circular aperture of communication between the ventricle and truncus arteriosus gradually becomes slit-shaped.

The period at which the division of the truncus arteriosus into two vessels commences is, in the common fowl, about the 106th hour of incubation (four days and ten hours), or rather more than one-fifth ($4\frac{2}{5}$) of the whole period of incubation, which is twenty-one days. There are three branchial arteries on each side, but they do not now come off abruptly from the end of the truncus arteriosus. Looked at sideways, the two anterior or upper pair of arches are seen to be separated from the posterior or fifth pair by a partition just extending a little way into the branchial end of the truncus arteriosus (Plate XXXI. fig. 2). This partition does not pass quite horizontally across the vessel, but is slightly inclined upwards from right to left, and extends a little further along the right side of the truncus arteriosus than it does along the left side; it is in fact a prolongation into the truncus arteriosus of the arterial wall between the fourth and fifth pair of vascular arches, and separates the rudimentary aorta from the rudimentary pulmonary artery, the little channel leading to the third and fourth pair being the first trace of a separate aorta, and that leading to the fifth pair the first trace of a separate

pulmonary artery. On looking from front to back (Plate XXXI. fig. 3) or from back to front through the septum between them, its cardiac edge is seen to be forked, so that it extends further along the sides than the centre of the vessel. Thus, as is well seen in the preparation from which the drawing was taken, the rudimentary aortic and pulmonary vessels are incompletely separated below, where they first leave the truncus arteriosus. That there now exist two channels distinctly separated from one another just above the forked septum is shown by a transverse section of the vessel at this point. Plate XXXI. fig. 5 is a drawing of such a section, looking from the distal side towards the heart. The section made a little lower through the forked part of the septum (Plate XXXI. fig. 6) gives the idea of two prominent ridges forming on the interior of the vessel, and being about to adhere. They are not, however, continued throughout the truncus arteriosus, a section through its central portion being nearly quadrangular and free from ridges (Plate XXXI. fig. 7), but at this period almost directly subside into the wall of the vessel (Plate XXXI. fig. 8). It is very remarkable that even at this early period, when the rudimentary aorta and pulmonary artery have only just appeared, the rudiments of some of the semilunar valves are already to be seen a little below the forked septum and at a considerable distance from the heart. The situation at which they first appear, and their position with reference to the rudimentary vessels, is roughly indicated in Plate XXXII. fig. 37. They are pretty accurately represented in the drawing (Plate XXXI. fig. 9) of this part of the interior of the truncus arteriosus. On looking into the lower end of the section, which has been made just below the rudimentary valves, they appear as somewhat flattened pyramidal projections, passing obliquely upwards on the wall of the vessel and separated by a triangular groove. These are the anterior aortic and pulmonary semilunar valve-rudiments, which are the first to appear. On examination by transmitted light their situation is indicated by a slight opacity of the vessel, which is constricted at this part. Their upper edges are situated just below the termination of the left leg of the fork of the septum, which can be seen above them (Plate XXXI. figs. 8 & 9), and which, as the septum passes down the vessel, gradually grows between these valve-rudiments, passing from the left side rather towards the front of the truncus arteriosus, and separating the anterior aortic from the anterior pulmonary semilunar valve. By the time that the partition has descended as far as these valves, they have become larger and more developed, and the rudiments of the inner semilunar valves have begun to appear. About the time that the anterior valve-rudiments appear, the right side and posterior surface of the interior of the vessel nearly opposite to them becomes thickened and rather prominent in the centre. This is the commencement of a thick pyramidal and somewhat rounded ridge that forms gradually along the posterior surface of the vessel. As the septum grows down the truncus arteriosus, the right leg of the fork becomes continuous with the central portion of this ridge, and follows its line obliquely down the posterior surface of the vessel from right to left, the left leg of the fork, as we have seen, passing down on the front of the vessel obliquely from left to right. The rudiments of the inner semi-

lunar valves of the aorta and pulmonary artery grow out on each side of this pyramidal ridge. Its position and relations to the anterior valve-rudiments are shown in Plate XXXI. figs. 9 & 10, and also a little later, at the 115th hour of incubation in Plate XXXI. fig. 12. In consequence of these formations in the interior of the truncus arteriosus, its canal at the level of the upper margin of the rudimentary valves becomes irregularly slit-shaped, as in Plate XXXI. fig. 12, the long axis of the slit being inclined from the right downwards and to the left across the vessel. As the development of this portion of the truncus arteriosus proceeds, the part situated between it and the heart, and which is at this time free from ridges on the interior, seems to become somewhat shorter and wider.

About the same time, at the middle of the fifth day, or 108th hour of incubation, the opening into the ventricle is a somewhat rectangular slit having a direction nearly horizontally backwards (Plate XXXI. fig. 11) and a little from left to right. The left-hand lip of the slit is more prominent than the right-hand lip, the latter rather nearer the ventricle. A small triangular projection fills up each end of the slit. The lips of the slit are not quite parallel to each other, as the left-hand lip has an upward slope from before backwards. Thus the aortic and pulmonary streams of blood are already in process of separation.

At the next stage of development that I have been able to observe, at the 117th hour of incubation (four days twenty-one hours), the ventricle exhibits a very slight division by the still quite rudimentary septum of the ventricles. The ventricular end of the truncus arteriosus is still slit-shaped, the left lip of the slit running obliquely upwards and backwards, while the right is more horizontally situated and less prominent. Above this the canal of the truncus arteriosus is rather oval, the anterior surface of its interior being smooth and concave, without any appearance of ridges, while its posterior surface is already becoming convex by the formation of the lower part of the rounded ridge, which terminates by passing obliquely into the left lip of the arterio-ventricular slit, and which forms in the lower part of the vessel when scarcely any trace of it is to be seen in the central portion. The extension of the partition between the fourth and fifth pair of branchial arches has advanced still further down the vessel, so that the trunks of the rudimentary aorta and pulmonary artery, when filled with clot, are now distinctly seen by transmitted light (Plate XXXI. fig. 13), though there is not at present any indication of the division on the external surface of the vessel; owing to the advance of the outer semilunar valves of the rudimentary trunks, and the still slight development of the rudiments of the anterior and inner valves, the blood can still pass freely backwards from the aorta and pulmonary artery into the still undivided part of the truncus arteriosus, so that, as is seen in the drawing (Plate XXXI. fig. 13), the clot in the latter vessel is continuous with those in the former. As soon as the outer (lateral) valve-rudiments appear, by which time the anterior and inner rudiments have become still more developed, the clots in the aorta and pulmonary artery become cut off from the clot in the lower part of the truncus arteriosus, which clot is then small, though above the valves

the vessels are quite distended. This is shown in Plate XXXI. figs. 14 & 15; the outer valve-rudiments are there just beginning to appear.

At the 130th hour of incubation (five days ten hours) we find that the pyramidal ridge on the posterior surface of the truncus arteriosus has become more developed at every part of it, being very strongly marked near the valves, only slightly so in the middle portion of the vessel, but again becoming prominent below, where it terminates by suddenly narrowing, and being slightly twisted on itself slants into the edge of the left lip of the opening into the ventricle, passing downwards and forwards. The channel for the pulmonary blood is represented by the deep groove that passes down the left side of the ridge, and terminates in the wide and sloping surface of the left lip of the slit. The groove on the right side of the ridge forms the channel for the aortic blood, and passes almost directly downwards to the posterior end of the slit. Thus the aortic groove passing directly downwards, the pulmonary groove, which is at first situated to its left, gradually twists round the lower end of the ridge separating the two, and expanding out below opens obliquely into it. The figures 16, 17 & 18 are drawings of a section of the truncus arteriosus at this period, which includes the valves and the descending septum, and shows pretty clearly the mode in which the vessel is dividing and the stage of development attained by the valves. In Plate XXXI. figs. 17 & 18, the shape of the rounded pyramidal ridge is well seen. A prominent convex ridge separates the two free convex and sloping sides of the pyramid. Below it is almost lost on the wall of the vessel, whose canal is almost circular a short distance below the valves. Above it projects forwards into the vessel for about two-thirds of the diameter of the vessel at the apex of the fork of the descending septum, and it represents what was originally the right leg of the fork. The other leg of the forked septum is represented by a slighter ridge, which bounds the rudiment of the anterior aortic valve on the left, and is very soon lost on the wall of the vessel. The fork of the septum grows down the vessel along the course of the prominent posterior ridge, and on the anterior aspect of the vessel along the slighter ridge to the left of the anterior aortic valve and the adjacent part of the groove separating the anterior aortic from the anterior pulmonary valve. This anterior portion of the fork is, as we have seen, very slightly marked, and is very soon lost on the wall of the vessel. The posterior ridge is, on the contrary, already distinct along the whole of the truncus arteriosus, though most so above and below. At no time during the division of the vessel does the anterior portion of the fork of the septum extend for more than a very short distance along the anterior surface of the artery; so that the division cannot be correctly described as occurring by the formation of *two* longitudinal ridges situated opposite to one another, and their subsequent growth together throughout their whole length, as RATHKE states to be the case, but must be described as taking place by the growth down the vessel of a forked septum which extends into the vessel nearly horizontally from between the fourth and fifth pair of branchial arteries, and which has already separated the rudimentary aorta and pulmonary artery before any *ridges*, properly so called, have appeared in the vessel, and which

subsequently follows the course of a single prominent pyramidal ridge which forms gradually along the whole of the posterior surface of the vessel, being at first a little on its right side, and afterwards a little on its left. The septum, therefore, grows down the truncus arteriosus in a *spiral* twisting from left to right.

The position and shape of the cardiac surfaces of the rudimentary semilunar valves at the 130th hour is also shown in Plate XXXI. figs. 16, 17, 18. The anterior valves are the most developed, and the outer and inner valves are formed on the same type. They are best described as solid pyramidal growths of the vascular tissue, and have very much the shape of a short crystal of triple phosphate, , the flat base being attached to the wall of the artery, while the sloping ends look upwards and downwards, and the edge projects into the centre of the vessel. This edge is sometimes a little flattened or notched. The upper ends slope upwards and are not at first horizontal, though sometimes slightly concave. The inner semilunar valve of each artery is seen to have a similar form to that of the anterior valve, and to grow out from the corresponding convex side of the posterior pyramidal ridge, into which their sloping lower ends are continued. The main purpose of this very prominent portion of the posterior ridge seems to be to afford a sufficiently large surface on each side for their attachment and continuous development, as well as a firm basis of support for them against the pressure of the blood-column in the rudimentary trunks. The anterior valves have a sufficiently large and firm basis of attachment in the anterior wall of the vessel, and the presence of a very prominent ridge between them would interfere with their development and narrow the artery unnecessarily. We find, therefore, that they are separated only by a very slight edge and a groove. The lateral or outer valves of the aorta and pulmonary artery have just appeared. That of the aorta is well seen in Plate XXXI. fig. 18. It originates nearly opposite to the other valves at the outer portion of the vessel. It has the same pyramidal shape, but is more notched on the inner edge; and if this notch deepens much, the single valve may develope into two valves. The outer valve of the pulmonary artery appears about the same time as the outer aortic valve; it is similar to it in shape, and similarly situated with respect to its fellow valves.

From the foregoing observations it will be seen that KÖLLIKER'S statement, that the semilunar valves are at first nothing but horizontally projecting *crescentic* growths, cannot be accepted as quite applicable to them in this early period of their development.

Ten hours later, at the 140th hour of incubation, we find that the division between the aorta and pulmonary artery has extended much nearer the heart, and that the rudiments of the outer valve in each artery having become more developed, the reflux of blood into the still undivided portion of the truncus arteriosus is very inconsiderable. Plate XXXI. fig. 19 shows the appearance of this part viewed from behind by transmitted light. The clots above the semilunar valves have been removed. The channel followed by the pulmonary blood is indicated by the direction of the clot in the undivided portion of the vessel, and passes obliquely upwards across the anterior part of the truncus arteriosus. The aortic blood passes directly upwards from the ventricular open-

ing towards the aorta, at the posterior part of the vessel. The upper surfaces of the valves are shown in Plate XXXI. figs. 20 & 23, and the upper surface of the anterior valve in each artery is also seen in Plate XXXI. fig. 19, in which figure the profile of the inner and outer valves is also seen. The valves are still quite solid. Their upper surfaces slope upwards from the centre towards the periphery of the vessel, and are slightly concave, especially in the case of the anterior valves. The inner valve of the pulmonary artery and the outer valves of both vessels are seen to be distinctly notched; they do not yet accurately close the canal of the vessel when it is fully distended, so that an irregular aperture is seen in the centre. It does not receive exactly the form of a three-rayed star till some hours later.

The present form and position of the lower forked margin of the descending septum is seen in Plate XXXI. fig. 21. The plane of the septum passes across the vessel from before directly backwards, and the lower forked margin is now very distinctly seen to be formed by the union of the prominent posterior pyramidal ridge with the slighter anterior ridge bounding the anterior aortic valve on the left, and being almost immediately lost on the anterior wall of the vessel. On either side of the fork are seen the apertures of the aorta and pulmonary artery, and the pyramidal prominences of the anterior and inner valves. The under surfaces of the valves are a little more rounded off; the outer valves are still not very conspicuous from below. In Plate XXXI. fig. 22 the shape of the opening into the ventricle is shown. It is no longer a slit, as its two lips have been gradually undergoing the changes by which its anterior portion ultimately becomes the upper part of the infundibulum leading into the pulmonary artery, while the posterior portion becomes the upper part of the aortic infundibulum. This is effected by a widening of the fissure, principally at its ends, by an absorption or arrest of development of the ends of each lip of the fissure, while the development of the central portion proceeds uninterruptedly, and by the hollowing out of the upper surface of the front of the posterior lip. A prominence is thus left about its centre, in which the posterior pyramidal ridge terminates below. The anterior portion of the forked septum ultimately grows into the remains of the central part of the right-hand lip of the fissure. During the time that these changes have been going on the septum of the ventricles has been gradually growing up between the left and right ventricle, and has now attained a considerable development. An oval aperture exists in its upper part, and leads from the left into the right ventricle. This aperture has a right border opening into the right ventricle, and a left border opening from the left ventricle. The anterior part of the right border becomes continuous with the central part of the posterior or left-hand lip of the arterio-ventricular opening, and therefore with the lower end of the pyramidal ridge on the posterior surface of the vessel. At a later period the posterior part of the lower half of the right border of the septal aperture bends round so as to become continuous with the central portion of the anterior or right-hand lip of the arterio-ventricular opening. The bearing this has on the union of the septum between the aorta and pulmonary artery with the upper part of the ventricular septum will be presently

described. Where this union fails to occur, a permanent aperture of communication remains between the right and left ventricle.

Later still, at the 144th hour of incubation, or the end of the sixth day, the semilunar valves are sufficiently developed to prevent the reflux of blood through them, though they are still quite solid. Accordingly we find the aorta and pulmonary artery distended by large clots, while the portion of the truncus arteriosus below them is empty and collapsed, or only has a small clot extending into it from the ventricle. The trunks, however, still exhibit no separation externally, though, as is seen in Plate XXXI. fig. 24, their right and left branches are separating. This drawing (the auricles and sides of the ventricles having been removed) shows remarkably well the considerable distance still separating the arterial aspect of the semilunar valves from the bases of the ventricles, owing to the great distance from the heart at which their rudiments first appear. A small channel between the edges of the valves is seen leading downwards from either vessel, that from the pulmonary artery taking a sweep forwards towards the anterior part of the base of the right ventricle, while that from the aorta passes directly downwards towards the oval opening in the interventricular septum. The appearance of the semilunar valves, as viewed from above, is seen in Plate XXXI. fig. 25. They are three in each artery, are still solid; their upper surfaces, very slightly concave, are now nearly horizontal, and the *lumen* of the vessel at this point, owing to their close approximation, now possesses the form of a three-rayed star. This seems to me to be the stage of their development described by Kölliker, though the valves can still scarcely be called crescentic. The edge of each pyramid that projects into the central canal of the vessel is prolonged for some way down the wall of the undivided portion of the truncus arteriosus towards the ventricles. On looking upwards through the lower end of the section, therefore, as shown in Plate XXXI. fig. 26, we see little angular projections of the vascular wall into the collapsed canals of the aorta and pulmonary artery, which are the lower portions of the rudimentary valves cut across. One of these (inner valve) is seen on each side of the section of the posterior ridge. The enormous development of this ridge, compared with that of the very slightly marked edge forming the left border of the anterior aortic valve, is well seen. The aortic and pulmonary channels are seen to communicate below the valves in front of the posterior ridge.

At the earliest period of the development of the heart the two lips of the opening between the ventricle and truncus arteriosus appear to exercise a valvular function, preventing in a measure the reflux of blood from the truncus arteriosus into the ventricle. As the slit widens, the valvular function gradually becomes less and less, and at the period of development just described is completely abolished.

The next stage in the development which it is important to notice is soon after the end of the sixth day, at the 147th hour of incubation. The development of the heart is now advancing with great rapidity; its appearance at this time, as seen by transmitted light, is shown in Plate XXXI. fig. 27, and Plate XXXII. fig. 28. The breadth of the aorta and pulmonary artery and of the valves has increased very much, and the division

between the vessels has descended so very close to the base of the heart that the undivided portion of the truncus arteriosus has disappeared externally, the bases of the semilunar valves being now on a level with the bases of the ventricles. The root of the pulmonary artery now has a position in front and a little to the left of the root of the aorta, which is becoming hidden behind it. There is still, however, no division externally between the large trunks. The semilunar valves, both aortic and pulmonary, are still more developed in the direction of their depth than transversely; they are still solid. Their upper surfaces are horizontal or slightly concave, and hollowed near the arterial wall, as is well seen from the shape of the base of the large coloured clot which fills the pulmonary artery: this shows that the hollowing-out process, by which they become membranous, is just commencing. The development of the valves proceeds by the gradual hollowing out of the solid pyramidal mass of tissue above and next the arterial wall, while it grows at the sides and below and towards the middle of the artery. The oval aperture of communication between the ventricles is still open; but its channel is becoming more and more directed towards the aorta, and less towards the right ventricle (Plate XXXI. fig. 27). The septum of the ventricles has become much thicker. The margin of the left opening of the channel through the septum is becoming rounded off, so that the canal is funnel-shaped towards the left ventricle. The lower part of the orifice into the right ventricle afterwards closes up, while the upper part leading to the aorta, as well as the funnel-shaped canal leading from the left ventricle, remain permanently open, and become the infundibulum or sinus of the aorta. At the sixth day, and earlier, the orifice in the interventricular septum is seen to be formed of a connective tissue-like structure, similar to that of the adjacent portion of the truncus arteriosus, and quite distinct from the muscular substance of the lower portion of the septum.

At the 165th hour of incubation the union of the upper part of the ventricular septum with the forked lower border of the division between the aorta and pulmonary artery, which has now quite reached the base of the ventricles, is approaching completion: in other words, the separation of the infundibulum or conus arteriosus of the aorta from that of the pulmonary artery is now nearly effected. The drawings (Plate XXXII. figs. 29, 30, 31) from hearts that had been incubated for this space of time (six days twenty-one hours) show the appearance of the right ventricular border of the orifice in the interventricular septum, and of the under aspect of the semilunar valves, and of the division between the vessels. Figs. 30 & 31 were taken from a heart whose development was further advanced than in fig. 29, and show the mode in which the separation between the aortic and pulmonary blood becomes completed.

The lower border of the partition between the aorta and pulmonary artery now appears as a thick round ridge on the left side of the termination of the truncus arteriosus, arching over above and becoming lost on the interior of the right side of the junction of the truncus arteriosus with the right ventricle; its vertical plane is nearly at right angles to that of the septum of the ventricles. The pyramidal inner semilunar valves are seen projecting from either side of this rounded ridge into the orifices of the aorta

and pulmonary artery respectively. The other semilunar valves are seen growing from the wall of the vessel. Viewed from below, they still have a pyramidal shape. The left ventricular border of the channel through the septum of the ventricles has become more sloped off. The right ventricular border is seen in Plate XXXII. fig. 29. It forms an oval orifice whose upper half is sloped and rounded off towards the aortic orifice just above it, to which the channel through the septum is directed. Its lower half is sharp and well defined, and the anterior part of this, near its junction with the upper half, passes off obliquely into the rounded ridge on the left side, which forms the greater portion of the division between the aorta and pulmonary artery; this is well seen in Plate XXXII. figs. 29, 30, 31. As the completion of the partition proceeds, the posterior part of the sharp and well-defined lower margin of the orifice grows off at its junction with the upper half sharply upwards and outwards to the right, bending round the upper part of the inside of the base of the right ventricle, and then passing forwards to join the right side of the division between the vessels, where this loses itself on the wall of the pulmonary infundibulum a little below the remains of the central part of the right leg of the arterio-ventricular fissure. This process is just commencing in Plate XXXII. fig. 29. In Plate XXXII. figs. 30 & 31, it has proceeded to a considerable extent, and the aortic and pulmonary infundibula now communicate by an hourglass-shaped aperture whose plane is twisted on itself, so that the portion on one side of the constriction of the hourglass is nearly at right angles to that on the other. The closure of this aperture by the growth of its margin all round completes the separation between the aortic and pulmonary infundibula. If we consider it closed by an imaginary plane surface, the course of the plane will be at first vertically downwards, then gradually *twisting* from left to right, and at the same time *bending* towards the left. Plate XXXII. fig. 38 further illustrates the mode of completion of the division between the two infundibula, which are already beginning to assume their ultimate positions with respect to each other.

The anterior and inner semilunar valves of the aorta and pulmonary artery are just beginning to form pockets, the part corresponding to the corpus arantii, and the margin of the valve generally, being very thick and solid. The outer valves as yet do not form pockets, and their development is less advanced than that of the other valves, as it has been throughout. In the preparation from which figs. 30 & 31 were taken, there are four valves in the pulmonary artery, the outer valve-rudiment having split into two valves. The valves do not appear to descend any more towards the ventricles in their further development, but remain nearly stationary in position, the further descent of the partition merely completing the arterial infundibula below them.

The trunks of the aorta and pulmonary artery are now beginning to separate externally, and about the middle of the eighth day, at the 180th hour of incubation, the large vascular trunks have become quite separate down to the semilunar valves, and have attained almost their final relative positions. The aperture of communication between the arterial infundibula has closed up till it is now only a semilunar slit, and the thick

rounded ridge on the left side of the termination of the truncus arteriosus forms the principal part of the septum between them. The semilunar valves of each artery are larger and more developed, and the sinuses of Valsalva are beginning to appear; but there is still no pocketing of the outer valves. They are shown in Plate XXXII. fig. 32.

About the end of the eighth day, at the 189th hour of incubation, the aperture in the infundibular septum is a very minute slit, and very soon afterwards is quite closed up. The anterior and inner valves of each vessel are more membranous, with a solid pyramidal corpus arantii; the lateral valves are just beginning to pocket. The valves have gradually become semilunar in shape when viewed from below.

The latest period at which I have been able to observe the development of the valves is the thirteenth day of incubation. They are then miniatures of those of the heart at full term, though rather thicker in comparison with their area. This is still more the case a few days earlier. Hence the principal changes in them after the closure of the infundibular aperture seems to be an increase in area and gradual diminution in thickness, so that, concurrently with the increase in size and strength of the other parts of the heart, they gradually assume the membranous form characteristic of the perfect valves. The great vessels are now quite separated down to their roots, and the heart itself presents externally very much the appearance of the adult heart.

Recapitulation.

I will now recapitulate briefly the principal steps in the formation of the aorta and pulmonary artery.

At first, just before the division commences, the truncus arteriosus, round and smooth on the inside, ends abruptly in the three pairs of branchial arteries, third, fourth, and fifth, which then exist. It is slightly thickened round its ventricular aperture.

Then, at the 106th hour of incubation, the division between the fourth and fifth pair of branchial arteries extends nearly horizontally into the vessel, as a plane septum with forked margin. At the same time the canal of the vessel just below this becomes constricted by the formation almost simultaneously,

1°, on its anterior and left surface, of two flattened pyramidal promimences separated by a groove—the rudiments of the anterior semilunar valve of each artery;

2°, on its posterior and right surface of a flattened ridge, afterwards becoming prominent and pyramidal, and extending gradually down the posterior surface of the vessel.

As these growths enlarge, the forked septum grows downwards into the artery, twisting from left to right, its left leg passing between and separating the anterior prominences, and its right leg passing along the central portion of the posterior ridge.

By the time the forked septum has reached the part where the anterior valve-rudiments first appeared, these are more distinctly developed, and the rudiments of the inner semilunar valves of each artery have begun to grow out from the posterior ridge on each side of the septum, an unoccupied space being left on the wall of the vessel on each side for the development of the outer valve in each artery.

During these changes the aperture into the ventricle has become a slit with two lips, a left-hand and a right-hand lip, the left-hand lip sloping from before backwards and upwards into the artery. The central portion of the vessel remains smooth and free from ridges on its inside, but along the posterior surface, above and near the ventricle, the pyramidal ridge forms, which ultimately joins the left-hand lip of the ventricular slit.

As the forked septum grows down the vessel, the semilunar valves gradually become more developed, and the outer semilunar valve in each artery appears. The leg of the fork which proceeds along the posterior ridge is always prominently developed, while that on the anterior aspect of the vessel is but slightly marked.

By the time the division has reached the ventricular aperture, the original right-hand leg of the fork has wound round to the centre of the left-hand lip of the slit, the left leg to the centre of the right-hand lip, so that the aortic channel has passed from front to back, the pulmonary channel from back to front. The way the septum twists down the vessel is shown in the diagram of imaginary cross sections of the vessel just above the lower margin of the division at different periods (Plate XXXII. fig 39).

During the division of the vessel the ends of each lip of its ventricular orifice have gradually disappeared so as to leave the central portions prominent, and a channel in front and to the left, and behind and to the right. These channels become the roots of the aorta and pulmonary artery, by the growth of the forked septum into the prominent central portions that remain.

The arterial infundibula are finally separated from each other by the union of the lower border of the forked arterial septum with the portion of the right ventricular border of the orifice in the septum of the ventricles, viz. the lower half of this border. Its anterior portion is continued upwards and forwards into the termination of the original right leg of the fork, in the central part of the left-hand lip of the ventricular slit, while its posterior portion passes off slantingly upwards into the termination of the original left leg of the fork in the central part of the right-hand lip of the slit. Thus a twisted hourglass-shaped aperture connects the two arterial infundibula, which become separated by its gradual closure.

The left ventricular opening of the orifice in the septum, with the upper part of the right ventricular opening, and the channel between them, remain permanently open, and develope into the aortic infundibulum.

Conclusions.

The following are the most important new facts at which I have arrived:—

1. That in the heart of the common fowl the division of the truncus arteriosus into the aorta and pulmonary artery does not take place by the formation of *two* longitudinal oppositely situated ridges along the whole length of the vessel and their subsequent growth together, as RATHKE states to occur in Reptiles, Birds, and Mammalia.

2. That this process of division of the truncus arteriosus is accurately described as occurring by the gradual extension into it of the septum between the fourth and fifth

pair of branchial arteries, from the branchial arteries towards the heart, the septum being at first situated nearly horizontally, and then twisting from left to right as it grows down the vessel, and following the line of a single thick rounded and somewhat pyramidal ridge that forms gradually along the posterior surface of the vessel.

3. The opening from the ventricle into the truncus arteriosus is at first circular, but soon becomes a slit with two lips, from a greater development of the vascular tissue in this situation. These lips at first exercise a valvular function, in preventing the reflux of blood from the truncus arteriosus into the ventricle. As the semilunar valves become developed, this valvular function gradually becomes abolished, that of the semilunar valves being substituted for it.

4. The valvular function of the lips of the slit becomes abolished by the widening of its ends and the greater development of the central portion of each lip, especially of the left-hand one. The widened ends of the slit ultimately form the roots of the aorta and pulmonary artery, which have then become separated by the growth of the septum down the vessel into the central portion of each lip.

5. It is a remarkable fact that the rudiments of the semilunar valves first appear on the interior of the truncus arteriosus *at a considerable distance from the heart*, near the termination of the truncus arteriosus in the branchial arteries, and not near the heart, as one might have been led to expect.

6. It is also very remarkable that the rudiments of the anterior and inner semilunar valves of each artery make their appearance before the partition, which has already begun to separate the aorta from the pulmonary artery, has quite descended to that part of the truncus arteriosus in which these valves originate.

7. The rudiments of the anterior semilunar valves of the aorta and pulmonary artery are the first to appear, those of the inner valve of each artery the next, and those of the outer valves the last. The development of the last valve to appear remains behind that of the others throughout.

8. The anterior valve-rudiments appear close together, rather on the right side of the anterior surface of the inside of the truncus arteriosus, about the 106th hour of incubation, simultaneously with the commencement of the division and a short distance below it, and opposite the commencement of the ridge which forms on the posterior surface of the vessel and which appears about the same time.

9. The rudiment of the inner valve of each artery grows from the corresponding side of the ridge which forms gradually on the posterior surface of the vessel a little later than the anterior valves.

10. The rudiment of the outer valve in each artery arises from that part of the inside of the wall of the truncus arteriosus left vacant, between the outer margins of the rudiments of the anterior and inner valves, soon after the 117th hour of incubation. It arises level with the other valves, when the aorta and pulmonary artery are already separated from each other for some little distance, and therefore a little nearer to the heart than the other valves, though still at a considerable distance from it.

11. The anterior valve-rudiments commence as transverse thickenings of the interior of the vessel, sloping off above and below into the general surface of the vessel, and are separated by a slight groove.

12. The inner and outer valves first appear as simple pyramidal thickenings of the vascular wall.

13. All the semilunar valves are solid at first.

14. The anterior and inner valves consist of one single segment for each valve.

15. The outer valve is at first a single pyramidal eminence. It may remain single, or become deeply notched and develope into two valves or more.

16. By the time the third valve in each vessel has appeared, the form of the valves has become more defined. They then have the shape of a short crystal of triple phosphate , its flat surface being attached, its edge projecting into the vessel, and its ends sloping off upwards and outwards above and below. The valves are more developed in the direction of their length than transversely, and their course down the wall of the vessel is parallel to that which the axis of its canal afterwards assumes.

17. About the 144th hour of incubation they are, though still solid, sufficiently developed to close the canal of the vessel pretty completely, and to prevent much reflux of blood into its undivided portion.

18. By this time the valvular function of the two lips of the opening into the ventricle has become quite abolished.

19. The valves are further developed by the hollowing out of the solid pyramid above and near the wall of the vessel, while they grow in other directions.

20. The pocketing of each valve commences in each in the order of its appearance, and begins in the anterior and inner valves of each artery about the time that their bases have descended to the level of the bases of the ventricles, *i. e.* at the 147th hour of incubation, and is distinct in these valves at the 165th hour. The pocketing of the outer valve is not distinct till much later. About the time that it commences, the valves have assumed nearly their final positions with respect to the base of the heart, and the aperture of communication between the arterial infundibula is nearly closed up.

21. After the complete separation of the aortic and pulmonary infundibula from each other, the further changes in the semilunar valves consist principally in increase in size and diminution in thickness, so that they become more and more membranous, *pari passu*, with the growth of the other parts of the heart.

22. The aperture in the septum of the ventricles does not close up entirely, in the manner commonly supposed, but finally developes into the aortic infundibulum. After its closure has proceeded to a certain extent, the lower half of the right ventricular border of the aperture unites with the lower border of the septum between the aorta and pulmonary artery in a peculiar manner, described more at length above. This separates the pulmonary infundibulum and the root of the pulmonary artery from the root of the aorta and the canal of the aperture in the septum, which then forms the aortic infundibulum.

23. The complete closure of the infundibular aperture takes place about the end of the eighth day of incubation.

24. The fifth vascular arch on each side gives off the branch to the lung of that side, and becomes ultimately the corresponding branch of the pulmonary artery, according to the view long ago propounded by Von Baer.

I cannot conclude without thanking several kind friends for assistance received from them during the preparation of this paper, which I here beg leave to acknowledge. In particular Dr. Beale, who suggested the best mode of heating the incubating-apparatus, and has given me much valuable advice; the Rev. George Kempson and my cousin Mr. Charles Paddison, who sent me abundant supplies of fresh eggs; and Dr. Cayley, who has kindly revised the translations from the German authors referred to.

Explanation of the Plates.

PLATE XXXI.

The drawings (with the exception of figs. 37, 38, 39) are all copies of preparations. They were made with the aid of the camera lucida, the preparations being all viewed by *transmitted* light, and variously magnified from 24 to 59 diameters.

Fig. 1. Heart and branchial arches *in situ*, from the Chick at the middle of the fourth day of incubation, showing the convoluted tubular form then possessed by the heart.

a. Auricle. *b*. Ventricle. *c*. Truncus arteriosus. *d*. Branchial arches. *e*. Pericardium.

Fig. 2. Heart and a portion of the branchial arches of Chick at the 106th hour of incubation. The left side of the heart and truncus arteriosus is seen in the drawing.

a. Truncus arteriosus. *b*. Ventricle. *c*. Rudimentary aorta. *d*. Rudimentary pulmonary artery. *e*. Septum forming between rudimentary aorta and pulmonary artery. *f*. Opening into ventricle. 3, 4, 5. Third, fourth, and fifth branchial arteries respectively.

Fig. 3. The same preparation viewed on its anterior surface.

g. Forked lower margin of arterial septum. *h*. Edges of groove between anterior valve-rudiments.

Other letters the same as in fig. 2.

Fig. 4. Diagram of branchial end of truncus arteriosus at the 106th hour of incubation, to show the part at which the sections drawn in figs. 5, 6, and 7 are made.

Fig. 5. Cross section of truncus arteriosus of Chick at 106th hour of incubation, made in the line 1, 1, fig. 4, showing the shape of the channels of the rudimentary aorta and pulmonary artery, separated by the nearly horizontal septum.

a. Aorta. *b*. Pulmonary artery.

Fig. 6 Cross section of same truncus arteriosus made in line 2, 2, fig. 4, through lower forked margin of the septum between the rudimentary aorta and pulmonary artery.

a. Aortic channel. *b*. Pulmonary channel.

Fig. 7. Section of the same truncus arteriosus near its central portion. The slight convexity of the posterior surface indicates the position of the prominent pyramidal ridge afterwards formed here.

Fig. 8. Portion of truncus arteriosus at 106th hour of incubation. The section has been made between the line 2, 2 in fig. 4, and one a little above the centre of the vessel. Figs. 9 and 10 are from the same preparation viewed in different positions.

v. Ventricular end of truncus arteriosus. *e*. Aorta. *f*. Pulmonary artery. *g*. Lower ends of forked margin of septum.

Fig. 9. The lower end of the preparation is looked into obliquely so as to show the left side and corresponding part of the anterior and posterior surfaces of the inside of the vessel. At the upper end the cut end of the left leg of the forked septum is seen terminating on the wall of the vessel, and just below and in a line with it the rudiment of the anterior aortic valve is seen as a flattened ridge, separated from the anterior pulmonary valve-rudiment by a groove expanding out below. The under surface of the rudiment of the anterior pulmonary valve is seen to the left of this in the preparation. The left leg of the forked septum afterwards grows down along the left edge of the rudiment of the anterior aortic valve and the adjacent part of the groove between the anterior valve-rudiments.

a. Anterior aortic valve-rudiment. *b*. Anterior pulmonary valve-rudiment. *c*. Edge of anterior aortic valve along which the septum grows. *d*. Left lower end of forked arterial septum. *e*. Cut edge of cardiac end of section.

Fig. 10. Same preparation more highly magnified and turned into a different position. The lower end is looked into obliquely, so that the right side and posterior part of the inside of the vessel are seen. At the upper aperture of the vessel the termination of the right leg of the forked septum is seen. Below this is the commencement of the ridge on the posterior surface. It is at this time merely a thickened ridge passing obliquely across and down the vessel, from the right side towards the posterior surface. Its ends are the rudiments of the inner semilunar valves of each artery, while its central portion afterwards becomes prominent and pyramidal. The lower face of the ridge slopes off rapidly into the wall of the vessel. Between the right end and the rudiment of the anterior aortic valve (its corner just seen) a light space is seen in the drawing, where the outer aortic valve afterwards becomes developed.

a. Part of anterior aortic valve. *b*. Rudimentary ridge on posterior surface. *c*. Space where lateral valve subsequently developes. *d*. Right lower end of

forked arterial septum losing itself on wall of artery. *e*. Aorta. *f*. Pulmonary artery.

Fig. 11. From Chick at the 108th hour of incubation, to show the form of the opening from the ventricle into the truncus arteriosus. The ventricle has been cut across just below the opening and the truncus arteriosus near the centre of the vessel.

a. Slit-shaped opening into truncus arteriosus. *b*. Thickened lips of slit. *c*. Base of ventricular cavity just below arterial opening. *d*. Cut edge of ventricle.

Fig. 12. Portion of truncus arteriosus from Chick at the 115th hour of incubation; the section made through forked margin of septum above and central part of vessel below.

a. Anterior aortic valve-rudiment. *b*. Anterior pulmonary valve-rudiment. *c*. Commencement of ridge on posterior surface. *d*. Aortic channel. *e*. Pulmonary channel. *f*. Line (left edge of anterior aortic valve) followed anteriorly by arterial septum.

Fig. 13. Left side of heart, branchial arches, and part of descending aorta of Chick at the 117th hour of incubation. The auricles have been removed. Although there is no trace of any division externally, the clots in the vessels show that the division between the rudimentary aorta and pulmonary artery has already advanced some distance. The absence of the rudiment of the outer semilunar valve of the pulmonary artery is shown by the continuity of the clot in that vessel with the clot in the undivided portion of the truncus arteriosus.

a. Ventricle. *b*. Truncus arteriosus. *c*. Carotid. *d*. Aorta. *e*. Pulmonary artery. *f*. Descending aorta. 3, 4, 5. Third, fourth, and fifth branchial arteries respectively.

Fig. 14. Heart, branchial arches, and part of descending aorta of Chick at the 117th hour of incubation. The auricles removed. The division in the clot marks the situation of the upper margins of the semilunar valves at this time. The rudimentary trunks are separate up to this point.

g. Rudimentary semilunar valves.

Other letters the same as in fig. 13.

Fig. 15. Same preparation turned round so as to show the aorta. The clot in the rudimentary aorta is seen, like that in the pulmonary artery, and for the same reason, to be cut off from that in the undivided part of the truncus arteriosus.

Lettering the same as in fig. 14.

Fig. 16. Portion of truncus arteriosus of Chick at the 130th hour of incubation, included between sections made just above the valves and a short distance below them, showing the interior of the vessel below the valves.

a. Anterior aortic valve. *b*. Anterior pulmonary valve. *c*. Pyramidal ridge on posterior aspect of vessel. *d*. Left edge of anterior aortic valve-rudi-

ment, along which the division proceeds anteriorly. *e*. Pulmonary artery. *f*. Aorta. *g*. Cut edge of vessel.

Fig. 17. The same preparation turned, so as to give a better view of the aortic orifice, and to show the pyramidal shape of the rudimentary semilunar valves.

h. Lateral aortic valve. *i*. Inner aortic valve.

Other letters the same as in fig. 16.

Fig. 18. The same preparation turned, so as to give a better view of the outer aortic valve. The rounded form of the ridge on the posterior surface and the pyramidal shape of the anterior aortic valve are well seen.

Letters the same as in fig. 17.

Fig. 19. Portion of truncus arteriosus of Chick at the 140th hour of incubation, included between a section just above the valves and one at the base of the ventricles.

a. Aorta. *b*. Pulmonary artery. *c*, *c*. Anterior semilunar valves. *d*, *d*. Other semilunar valves. *e*. Septum between aorta and pulmonary artery. *f*. Opening from ventricle into lower part of truncus arteriosus. *g*. Clot in truncus arteriosus below valves.

Fig. 20. View of upper surfaces of the semilunar valves of the aorta and pulmonary artery at the 140th hour of incubation. The inner valve of each artery is seen to grow entirely from the side of the thick septum corresponding to the rounded pyramidal ridge on the posterior surface; the outer and anterior valves grow from the corresponding parts of the wall of the artery. The outer valve in the aorta is seen to be deeply notched. The anterior pulmonary valve is also slightly notched, and the outer valve very much so, the notches in the last fitting corresponding projections of the rudimentary inner valve.

a. Aorta. *b*. Pulmonary artery. *c*, *c*. Inner valves. *d*, *d*. Anterior valves. *e*, *e*. Lateral valves.

Fig. 21. The same preparation turned over to show the under aspect of the descending septum and valves.

a. Aorta. *b*. Pulmonary artery. *c*. Anterior aortic valve. *d*. Anterior pulmonary valve. *e*. Inner aortic valve. *f*. Ridge on posterior aspect of vessel.

Fig. 22. Portion of truncus arteriosus remaining between the last section and the ventricle, showing the arterial aspect of the opening into the ventricle. The channel for the pulmonary blood is seen to groove the anterior part of the left-hand lip. The slight prominences on the front of the vessel show the future position of the bases of the anterior and outer pulmonary valves. These are rather exaggerated in the drawing.

a. Left-hand lip of ventricular opening; its prominent central portion becomes continuous with the ridge on the posterior part of the truncus arte-

riosus. *b*. Future base of outer pulmonary valve. *c*. Future base of anterior pulmonary valve. *d*. Future base of anterior aortic valve. *e*. Future base of outer aortic valve.

Fig. 23 shows the upper surfaces of the valves of the aorta and pulmonary artery at the 140th hour of incubation. The outer valve of the pulmonary artery is here developing into two valves.

a. Aorta. *b*. Pulmonary artery. *c*, *c*. Anterior valves. *d*, *d*. Lateral valves. *e*, *e*. Inner valves.

Fig. 24. Heart and truncus arteriosus of Chick at the 144th hour of incubation. The auricles and part of the ventricles removed. Right side of the preparation. A small coloured clot is seen in the aorta. The upper surfaces of the aortic valves are seen to be nearly horizontal, and still at a considerable distance from the bases of the ventricles. The aortic portion of the channel of the undivided part of the truncus arteriosus is seen leading down to the opening in the ventricular septum, between the edges of the anterior and inner aortic valves.

a. Aortic valves. *b*. Channel between edges of aortic valves. *c*. Clot in aorta. *d*. Cavity of right ventricle. *e*. Muscular substance of right ventricle.

Fig. 25. Portion of truncus arteriosus of Chick at the 144th hour of incubation included between sections just above the semilunar valves and about the same distance from the base of the ventricles, turned so as to show the upper surfaces of the valves. These are seen to be three in each artery, and close the canal of each vessel pretty accurately.

a. Aorta. *b*. Pulmonary artery.

Fig. 26. The same preparation turned so as to show its under surface.

a. Aorta. *b*. Pulmonary artery. *c*. Rounded ridge on posterior aspect. *d*. Edge along which arterial septum grows anteriorly.

Fig. 27. Heart and great vessels of Chick at the 147th hour of incubation. The auricles have been removed, as well as the posterior part of each ventricle. The valves completely close the canal of the vessel, a thread-like channel remaining between their edges.

a. Aorta. *b*. Pulmonary artery. *c*. Right branch of pulmonary artery. *d*. Left branch of pulmonary artery. *e*. Aortic valves. *f*. Pulmonary valves. *g*. Septum of ventricles. *h*. Right ventricle. *i*. Left ventricle. *j*. Aperture in septum of ventricle leading from left to right ventricle.

PLATE XXXII.

Fig. 28. The same preparation turned so as to show the anterior surface of the heart instead of the posterior. The branches of the aorta are seen passing in front

of the pulmonary artery. The anterior and outer pulmonary valves are seen extending down the wall of the vessel towards the heart.

k. Branches from arch of aorta. The other letters the same as in fig. 27.

Fig. 29. From the heart of the Chick at the 165th hour of incubation. To show the mode of union of the lower border of the forked division between the aorta and pulmonary artery with the lower portion of the right ventricular border of the aperture in the septum of the ventricles. A horizontal section has been made through both ventricles close to the base of the heart, and just below the orifice of communication between the ventricles. The truncus arteriosus has been cut across just above the rudimentary semilunar valves. The eye looks through the base of the right ventricle into the aortic portion of the truncus arteriosus and the aperture of the aorta, into which the semilunar valves are seen to project.

a. Orifice of aorta; the triangular projections are the aortic valves. *b*. Opening of pulmonary artery. *c*. Rounded ridge with which anterior part of lower margin of septal orifice (*d*) becomes continuous. *h–e*. Muscular substance of ventricle. *f*. Entrance of orifice in septum leading from left to right ventricle. *g*. Base of cavity of left ventricle. *i*. Cut edge of septum ventriculorum. *j*. Right ventricular orifice of channel in septum ventriculorum, leading into truncus arteriosus.

Fig. 30. From the heart of the Chick at the 165th hour, much magnified. Its development was further advanced than in the preceding specimen. Preparation and position similar to that of fig. 29.

The growth of the inner valves from the thick rounded ridge is well shown in this and the following figure.

a. Under surface of anterior pulmonary semilunar valve. *b*. Lower border of fork of arterial septum. *c*. Entrance of pulmonary artery. *d*. Rounded ridge with which anterior part of lower margin of septal opening (*e*) becomes continuous. *f*. Muscular substance of ventricle. *g*. Cavity of left ventricle. *h*. Entrance of canal in septum leading from left to right ventricle. *i*. Cut edge of septum between ventricles. *j*. Aperture of canal in septum opening into right ventricle. *k*. Aortic entrance and valves. *l*. Base of cavity of right ventricle. *m*. Upper part of margin of septal opening continued into right side of forked arterial septum.

Fig. 31. The same preparation turned over a little so as to show all the pulmonary semilunar valves.

a. Inner pulmonary valve. *l*. Inner aortic valve.

The other letters the same as in fig. 30.

Fig. 32. From the heart of the Chick at the middle of the 8th day of incubation, showing the upper surfaces of the semilunar valves. The aorta and pulmonary

artery have been cut across just above the semilunar valves, and the ventricles cut off just below them.

A. Aorta. P. Pulmonary artery. *a, a.* Lateral valves. *b, b.* Inner valves. *c, c.* Anterior valves. *d, d.* Cut edge of arterial wall. *e, e.* Muscular substance of ventricle.

Fig. 33. Heart and great vessels of Chick at the end of the 13th day of incubation. The muscular substance of the front of the ventricles has been removed. The septum and apex are left.

a. Aorta. *p.* Pulmonary artery. *l. v.* Left ventricle. *r. v.* Right ventricle. *m. v.* Mitral valve.

Fig. 34. shows the distal portion of the truncus arteriosus and its principal branches at the 140th hour of incubation. The truncus arteriosus has been cut across just above the rudimentary semilunar valves. The shaded portions are the clots in the vessels. The right side of the preparation is here seen.

a. Truncus arteriosus. *b.* Aorta. *c.* Pulmonary artery. *d.* Origin of branch to right lung. *e.* Descending aorta. *f.* Ductus arteriosus. *g.* Portion of third branchial artery between carotid and fourth branchial artery (descending aorta) nearly obliterated. 3, 4, 5. Third, fourth, and fifth branchial arteries (right side).

Fig. 35. Front view of same preparation. Part of its left side is also seen. Owing to the absence of colour in the clots in the fourth vascular arch and left branch of the pulmonary artery, the latter vessel is not traceable to its termination, and the fourth arch seems to be obliterated.

i. Left carotid. *l.* Arch of aorta. Other letters the same as in fig. 34.

Fig. 36. Left side of same preparation. Outline of left branch of pulmonary artery only just traceable to its termination in descending aorta. Fourth vascular arch apparently obliterated.

Letters the same as in figs. 34 & 35.

Fig. 37. Diagram of magnified truncus arteriosus about 106th day of incubation. The forked septum between the fourth and fifth pairs of branchial arteries is seen just extending into the truncus arteriosus. The dotted lines indicate the directions taken by the right and left legs of the fork in their growth down the vessel.

a. Rudimentary anterior valves. *b.* Commencement of ridge on posterior surface of vessel. *c.* Forked septum growing into vessel between fourth and fifth pair of branchial arteries. *d.* Thickening of truncus arteriosus at ventricular opening. 3, 4, 5. Third, fourth, and fifth pair of branchial arteries respectively.

Fig. 38. Diagram of upper portion of septum of ventricles and base of the truncus arteriosus, to illustrate the way in which the lower border of the forked

septum between the aorta and pulmonary artery becomes united with the lower half of the right ventricular border of the aperture in the septum of the ventricles, and thus completes the separation of the aortic from the pulmonary infundibulum. The semilunar valves are omitted for the sake of clearness. The dotted line shows the part which afterwards becomes closed up.

a. Cut surface of truncus arteriosus. *b.* Cut edge of ventricle. *c.* Interventricular septum. *d.* Left ventricle. *e.* Right ventricle. *f.* Aorta. *g.* Pulmonary artery. *h.* Orifice in septum becoming the opening of the aorta into the left ventricle. *i.* Opening of right ventricle into pulmonary artery. *j.* Dotted line showing position of hourglass-shaped aperture. *k.* Forked lower border of arterial septum.

Fig. 39. Imaginary sections of the truncus arteriosus made just above the semilunar valves at different periods of development, showing the gradual changes in the position of the roots of the aorta and pulmonary artery during the process of the division of the truncus arteriosus, and their position at its termination.

1. At commencement of process of division of the truncus arteriosus. 5. At termination of same. 2, 3, 4. At intermediate stages. *a.* Aorta. *b.* Pulmonary artery.

XI. *On the Diurnal and Annual Inequalities of Terrestrial Magnetism, as deduced from Observations made at the Royal Observatory, Greenwich, from* 1858 *to* 1863; *being a continuation of a communication on the Diurnal Inequalities from* 1841 *to* 1857, *printed in the Philosophical Transactions,* 1863. *With a Note on the Luno-diurnal and other Lunar Inequalities, as deduced from observations extending from* 1848 to 1863. *By* GEORGE BIDDELL AIRY, *Astronomer Royal.*

Received July 27,—Read December 10, 1868.

THE paper which I have the honour now to submit to the Royal Society is similar, in its general character, to that which was printed by the Society in the Philosophical Transactions for 1863, as containing in a contracted form the results of very extensive observations which have been printed, and of detailed calculations founded on them which are prepared for printing, in the legitimate organ of publication of the observations made in the Royal Observatory. For the principal part of the work, the results are here exhibited to the eye in the shape of diagrams.

The instruments employed are precisely the same which were used in the second part of the former investigation, from 1848 to 1857, mounted in the same place, and treated in the same manner; and the observations are reduced by application of the same formulæ. The only difference in the form of exhibition is, that Greenwich Mean Time is here exclusively adopted instead of Göttingen Mean Time, which was used in the former paper. It will be remembered that the longitude of Göttingen is $0^h\ 39^m\ 46^s{\cdot}5$ East of Greenwich. The nominal time, therefore, of the occurrence of a phenomenon is less in the results now presented than in those of the former paper; or, the position on the curves of the figures 1, 2, 3, &c. for hours of time is more advanced than in the former paper, by 40^m nearly.

TREATMENT OF THE PHOTOGRAPHIC CURVES.

The first operation, in the treatment of the photographic records of the magnetometers, was to withdraw the sheets for those days on which the disturbances were so great as to make it difficult to draw a curve-line representing the mean path of the photographic spot. The actual list of days will be given in the detailed Greenwich publication; but it may be interesting here to state the number of days thus omitted, including the back series from 1841 to 1857.

1841 . . 9	1845 . . 5	1849 . . 2	1853 . . 18	1857 . . 10	1861 . . 3
1842 . . 10	1846 . . 17	1850 . . 6	1854 . . 13	1858 . . 6	1862 . . 5
1843 . . 7	1847 . . 20	1851 . . 13	1855 . . 4	1859 . . 15	1863 . . 2
1844 . . 5	1848 . . 20	1852 . . 17	1856 . . 0	1860 . . 14	

The estimate of the amount of irregularity which rendered it necessary to withdraw a sheet has been made throughout by the same person (Mr. Glaisher); and, through the years collected in the two last of the columns above, the records are of the same kind (photographic). I think therefore that, without going through a formal process claiming numerical accuracy for comparison of the tendency to magnetic disturbance in different years, these numbers may be presented as giving very important information. If they point to any cycle at all, it is one of 6 or $6\frac{1}{2}$ years.

These sheets being withdrawn, a pencil-curve was drawn by eye through the slightly disturbed photographic records; and the ordinates, by means of which the diagrams now presented to the Society were constructed, were measured to the pencil-curve. The number used for forming the ordinates of the diagrams was the excess of the curve-ordinate for each special hour above the mean for the twenty-four hours; and these are not here exhibited for every day, but their monthly means are taken, which monthly means themselves are the basis of further operations of grouping; in one operation the same nominal months of different years are combined and their means taken, and in another operation all the months for each year are combined and their means taken. In forming from these means the ordinates of the diagrams, in the two first plates, exhibiting the inequalities in the horizontal plane, it will be remembered that the vertical ordinate represents the change of northerly horizontal magnetic force (supposed to act on the northerly or marked end of the magnet), the measure upwards corresponding to increase of force; and that the horizontal ordinate represents the change of westerly declination, exhibited as a westerly disturbing force on the same scale as the northerly force, by considering a deviation of 1′ to be produced by a westerly force equal to $\frac{1}{3433}$ of the whole northerly force.

ANNUAL INEQUALITY.

Following generally the order of the former paper, I will here advert to the question of annual inequality. The mean westerly declination for the six months called January in the six years was taken; in like manner, the mean for the six months called February; and so for all the months. These means are not comparable for the purpose of ascertaining annual inequality, because they are subject to the influence of secular inequality. The mean annual change produced by secular inequality was found to be −9′·2. The proportional part of this, corresponding to the interval of each month from January, was applied with sign changed to the monthly results. After smoothing the numbers by taking a second mean of adjacent numbers, the following values of Westerly Declination were found, applying, as regards secular values, to the month January in the mean of the six years:—

January	21° 12′·9
February	12·9
March	13·4
April	12·5
May	11·8

June	21° 12·2
July	13·0
August	13·3
September	13·1
October	13·1
November	13·6
December	13·5

From these are obtained the following Excesses of Corrected Westerly Declination for each month above the mean of all the months:—

Annual Inequality of Western Declination.

	In Minutes of Arc.	In parts of Horizontal Force.
January . . .	−0·04	0·0000
February . .	−0·04	0·0000
March . . .	+0·46	+0·0001
April	−0·44	−0·0001
May	−1·14	−0·0003
June	−0·74	−0·0002
July	+0·06	0·0000
August . . .	+0·36	+0·0001
September . .	+0·16	0·0000
October . . .	+0·16	0·0000
November . .	+0·66	+0·0002
December . .	+0·46	+0·0001

The monthly values of the Northerly Horizontal Force were treated in the same way, the correction applied being based on the supposition that the annual increase, in terms of the whole Horizontal Force, is about 0·0017. This number is derived from the following values of Absolute Measure of Horizontal Force; of which it is to be remarked that the numbers for 1858, 1859, 1860 are obtained from observations with the old instrument, corrected for the constant difference between the old and new instrument, those for 1861 from both instruments, and those for 1862 and 1863 exclusively with the new instrument. The values are:—

1858	3·789
1859	3·786
1860	3·864
1861	3·812
1862	3·819
1863	3·824

from which the annual change was found to be +0·0063, or in parts of the whole horizontal force +0·0017.

Applying the proportional parts of this annual change for the several months with the sign changed, the following numbers (all requiring the addition of 0·8830 nearly) are obtained as expressing the monthly values of northerly horizontal force:—

January	0·1172
February	0·1175
March	0·1185
April	0·1196
May	0·1195
June	0·1189
July	0·1172
August	0·1171
September	0·1172
October	0·1167
November	0·1167
December	0·1167

From these are obtained the following Excesses of Corrected Horizontal Force for each month above the mean of all the months:—

Annual Inequality of Northern Horizontal Force.

January	−0·0005
February	−0·0002
March	+0·0008
April	+0·0019
May	+0·0018
June	+0·0012
July	−0·0005
August	−0·0006
September	−0·0005
October	−0·0010
November	−0·0010
December	−0·0010

On comparing these two series of monthly numbers, for Declination and Horizontal Force, with those in the two periods treated in the preceding paper, it does not appear, I think, that there is such agreement as justifies us in concluding that there is any real yearly inequality distinct from the secular inequality. There will, however, always be some doubt on conclusions applying to horizontal force in different months, on account of the uncertainty of correction for temperature; but no such doubt attaches to the declination.

DIURNAL INEQUALITIES IN THE HORIZONTAL PLANE.

I will now call attention to the curves which represent the laws of diurnal inequalities of force in the horizontal plane. And, first, the curves (Plate XXXIII.) which represent the inequalities in different months (the same nominal months in all the years being combined, and their means being taken), for the period 1858–1863, agree very closely with those for the period 188–1857. But, secondly, the curves (Plate XXXIV.), which represent the diurnal inequalities in different years, have undergone in the course of the period 1858–1863 a most striking change. In the years 1858 and 1861 they are similar to those of the winter months; in the years 1859, 1860, 1862, 1863 they resemble those of the summer months. And, on comparing these curves with those in the former paper, the following order of changes will be seen. From 1841 to 1847 the magnitude and summer character of the annual curves had slightly increased; but from 1848 to 1857 they rapidly diminished, giving the smallest and most winter-like curves in 1856 and 1857. The curves in the present paper have risen to the summer form steadily (with the exception of a little irregularity in 1861), and have at length sensibly attained to their original character. Thus

1858	nearly resembles		1856
1859	„	„	1851
1860	„	„	1850
1861	„	„	1851
1862	„	„	1847
1863	„	„	1841.

In other words, the magnetic action of the sun, which had during several years greatly diminished, has now increased till it has attained sensibly the same energy as before.

DIURNAL INEQUALITIES OF VERTICAL FORCE.

I now advert to the changes in the diurnal inequality of Vertical Force. In these, perhaps, the uncertainty connected with temperature-correction is even greater than for the Horizontal Force; but as the same instrument and the same numerical correction are adopted throughout, little uncertainty can attach to the comparisons of different years. On comparing the month-curves (Plate XXXV.) of the period 1858–1863 with those of the period 1848–1857, I think it appears that the curves of autumn are not now quite so bold as in the former period; there is not, however, any remarkable difference. The times, however, of nodal passage (or points where the actual Vertical Force = mean Vertical Force for the day) are not quite the same, as will appear from the following Table, in which the hours &c. are all referred to Greenwich Time.

Hours of nodal passages.

	Period 1848–1857.		Period 1858–1863.	
	h m	h m	h m	h m
January	4 0	16 0	4 25	17 25
February	4 0	15 20	4 50	17 30
March	4 0	15 25	4 55	16 40
April	3 20	14 30	5 0	16 40
May	1 25	13 45	3 0	14 50
June	1 20	13 20	4 40	16 0
July	1 30	13 45	2 50	15 30
August	2 10	13 35	2 30	14 40
September	2 45	14 15	3 20	16 0
October	3 35	15 50	4 25	16 25
November	4 10	15 45	4 50	17 15
December	4 40	15 30	5 15	18 0

The differences are probably connected with those which I have next to exhibit.

On comparing the annual curves for Vertical Force, it appears that to the year 1855 they gradually increased in boldness, and that from that year to 1862 (Plate XXXVI.) they have gradually diminished. The times of passage of node in the annual curves merit examination. In the former paper I pointed out a remarkable change which appeared to have gone on from 1842 to 1857. I will now bring together the nodal hours, all referred to Greenwich Time, for the entire period.

	Hours of nodal passage.	
	h m	h m
1842	8 40	20 30
1843	9 50	23 40
1844	10 40	24 5
1845	9 35	24 0
1846	11 20	23 40
1847	8 40	19 20
1848	No observations.	
1849	6 20	14 30
1850	4 10	14 40
1851	3 10	13 35
1852	3 25	14 15
1853	3 15	15 20
1854	3 30	15 15
1855	2 50	14 20
1856	3 20	14 55
1857	2 30	13 50
1858	2 10	13 35
1859	4 0	15 45
1860	4 50	17 25
1861	4 20	16 20
1862	3 45	15 35
1863	5 25	17 35

It appears here that there is a rude relation between the hours of nodal passage of

the Vertical Force and the magnitudes of the curves in the horizontal plane; the early passage of node synchronizing (but with some irregularity) with the small curves in the horizontal plane.

There is no appearance of any kind of cycle in these results.

LUNO-MENSTRUAL INEQUALITIES IN THE HORIZONTAL PLANE.

The times of the upper and lower transits of the moon at Greenwich were computed, in Göttingen Time from 1848 to 1857, and in Greenwich Time from 1858 to 1863; and these times were laid down on the time-scales of the photographs, and the intervals were divided each into twelve lunar hours. For each of these hours a new measurement of ordinates was made to the pencil-curve above mentioned. Remarking that the lunar phenomena occur, in rapid succession, at all hours of day and night, and consequently with all orders of change of temperature, it did not appear necessary to apply any correction for temperature to the readings of the Horizontal-Force Photographs.

The means being taken for every lunar day, and the first days of all the lunations being grouped together from 1848 to 1863, the second days being grouped together, &c., the means were taken for each lunation-day. The results were corrected for secular inequality, in the same manner as those mentioned in preceding sections (the maximum corrections being, at the beginning of the lunation, about $+\frac{1}{3}\frac{1}{5}$ of the annual correction, and at the end about $-\frac{1}{3}\frac{1}{5}$ of the annual correction). The deduced lunation-inequalities were the following, the unit for Declination being the minute of arc, and that for Horizontal Force being the whole Horizontal Force:—

Day of Lunation.	Inequality in Declination.	Inequality in Horizontal Force.
1	+0·04	−0·00017
2	0·00	− 21
3	+0·09	− 15
4	+0·21	− 6
5	+0·25	− 1
6	+0·24	0
7	+0·19	+ 5
8	+0·12	+ 9
9	+0·09	+ 9
10	+0·05	+ 6
11	−0·01	+ 5
12	−0·08	+ 2
13	−0·08	− 2
14	−0·07	− 10
15	−0·07	− 17
16	−0·08	− 17
17	−0·10	− 8
18	−0·11	+ 6
19	−0·05	+ 16
20	0·00	+ 19
21	−0·05	+ 19

Days of Lunation.	Inequality in Declination.	Inequality in Horizontal Force.
22	−0·13	+0·00016
23	−0·18	+ 11
24	−0·18	+ 8
25	−0·15	+ 6
26	−0·12	0
27	−0·04	− 4
28	+0·04	− 3
29	+0·11	− 6

LUNO-DIURNAL INEQUALITIES IN THE HORIZONTAL PLANE.

The Luno-diurnal inequalities were found in the same way as the Solar Diurnal inequalities; and the results are the following:—

The annual means from 1848 to 1857 are printed in the Greenwich Magnetical and Meteorological Observations, 1859, pages ccxxv and ccxxxi; and it is unnecessary to repeat them here. Those from 1858 to 1863 are contained in the following Tables:—

Lunar Hour.	Luno-diurnal Inequality in Western Declination.							
	1858.	1859.	1860.	1861.	1862.	1863.	Mean, 1858–1863.	Equivalent in terms of Horizontal Force.
0	+0·1	+0·3	+0·5	+0·1	+0·1	+0·4	+0·25	+0·000073
1	0·0	+0·3	+0·5	+0·1	+0·1	+0·4	+0·23	+ 67
2	+0·1	+0·4	+0·4	+0·1	+0·3	+0·3	+0·27	+ 79
3	0·0	+0·4	+0·2	+0·3	+0·2	+0·1	+0·20	+ 58
4	−0·2	+0·4	+0·2	+0·1	+0·2	0·0	+0·12	+ 35
5	−0·2	+0·2	0·0	+0·1	−0·1	−0·1	−0·02	− 6
6	−0·2	+0·2	−0·3	0·0	−0·1	−0·3	−0·12	− 35
7	−0·1	0·0	−0·4	−0·3	−0·2	−0·5	−0·25	− 73
8	−0·1	−0·1	−0·4	0·0	−0·3	−0·5	−0·23	− 67
9	+0·1	−0·1	−0·1	+0·4	−0·3	−0·4	−0·07	− 20
10	+0·2	0·0	−0·1	+0·3	0·0	−0·2	+0·03	+ 9
11	+0·1	+0·1	−0·1	+0·4	+0·2	−0·2	+0·08	+ 23
12	+0·1	+0·3	+0·3	+0·3	+0·4	+0·2	+0·27	+ 79
13	+0·2	+0·2	+0·3	+0·2	+0·3	+0·1	+0·22	+ 64
14	−0·1	+0·1	+0·3	+0·2	+0·2	+0·1	+0·13	+ 38
15	0·0	+0·1	+0·1	0·0	+0·2	+0·1	+0·08	+ 23
16	0·0	−0·2	−0·2	−0·3	+0·1	+0·3	−0·05	− 15
17	−0·2	−0·3	−0·3	−0·3	0·0	+0·1	−0·17	− 49
18	−0·1	−0·5	−0·5	−0·3	−0·3	0·0	−0·28	− 81
19	0·0	−0·6	−0·3	−0·5	−0·3	−0·2	−0·32	− 93
20	+0·1	−0·7	−0·3	−0·4	−0·5	−0·2	−0·33	− 96
21	+0·2	−0·5	−0·2	−0·4	−0·3	−0·2	−0·23	− 67
22	+0·1	−0·4	+0·2	−0·3	−0·1	+0·2	−0·05	− 15
23	+0·1	0·0	+0·2	−0·1	+0·2	+0·3	+0·12	+ 35

Lunar Hour.	Luno-Diurnal Inequality in Horizontal Force.						
	1858.	1859.	1860.	1861.	1862.	1863.	Mean, 1858–1863.
0	−0·00004	+0·00009	+0·00002	+0·00012	+0·00003	+0·00004	+0·000043
1	− 1	+ 10	+ 4	+ 6	+ 5	− 2	+ 38
2	0	+ 10	+ 13	+ 1	+ 3	+ 2	+ 48
3	+ 4	+ 6	+ 14	− 1	+ 2	+ 4	+ 48
4	+ 4	+ 6	+ 12	− 3	+ 2	− 3	+ 30
5	+ 2	+ 3	+ 4	− 3	− 5	− 2	− 2
6	− 6	+ 2	0	− 7	− 17	− 5	− 55
7	− 5	− 2	+ 1	− 13	− 11	− 4	− 57
8	− 8	− 4	+ 1	− 11	− 13	− 2	− 62
9	− 2	− 9	+ 6	− 4	− 11	+ 1	− 32
10	+ 3	− 8	0	− 5	− 16	+ 7	− 32
11	+ 8	− 12	+ 7	− 3	− 1	+ 8	+ 12
12	+ 14	− 6	+ 9	− 5	+ 2	+ 8	+ 37
13	+ 8	− 5	+ 6	− 1	+ 12	+ 11	+ 52
14	+ 7	− 5	+ 4	+ 5	+ 14	+ 11	+ 60
15	+ 4	− 2	0	+ 6	+ 14	+ 7	+ 48
16	+ 3	+ 3	− 3	− 1	+ 13	− 1	+ 23
17	− 1	+ 4	− 5	+ 3	+ 6	− 2	+ 8
18	− 2	+ 1	− 13	+ 3	+ 2	− 6	− 25
19	− 6	− 1	− 12	+ 5	− 5	− 4	− 38
20	− 7	− 5	− 13	+ 6	− 1	− 8	− 47
21	− 7	− 4	− 18	− 1	− 8	− 13	− 85
22	− 7	+ 3	− 10	+ 7	− 4	− 11	− 37
23	− 4	+ 5	+ 2	+ 9	+ 2	0	+ 23

The accordance, in general character, of the numbers in the last columns, derived from the observations 1858 to 1863, with those derived from the observations 1848 to 1857 (Greenwich Magnetical and Meteorological Observations, 1859) is so striking, as to give great confidence in the mean of a few years' observation, and to justify the breaking-up of the entire period 1848 to 1863 (of which the observations and reductions necessarily form one connected and uniform series) in any way that physical considerations may suggest. Now, remarking the singular difference for different years which has presented itself in the discussion of the solar inequalities, it appeared to me very desirable to examine whether there is any discoverable difference in the lunar inequalities for the same years. For this purpose, I divided the years into the following groups defined by the magnitude of the curves in the solar-diurnal inequalities:—

Large curves . . . 1848 to 1852;
Small curves . . 1853 to 1857;
Small curves . . . 1858 and 1861;
Large curves . . . 1859, 1860, 1862, 1863.

The means for these periods were taken; the first and fourth were united, forming a double mean for large curves, and the second and third were united, forming a double mean for small curves; and in each system the numbers for the groups of three adjacent hours, 0^h to 2^h, 3^h to 5^h, &c., were added together. Thus were obtained the following:—

Double three-hourly sums of Means of Luno-Diurnal Inequalities.

Extent of Group in Lunar Hours.	Western Declination.		Horizontal Force.	
	Years of large solar curves.	Years of small solar curves.	Years of large solar curves.	Years of small solar curves.
h h	′	′		
0 to 2	+1·33	+0·90	+0·000284	+0·000182
3 to 5	+0·10	−0·34	+ 176	+ 91
6 to 8	−1·31	−0·86	− 313	− 372
9 to 11	−0·17	+0·72	− 86	− 19
12 to 14	+1·35	+0·94	+ 387	+ 354
15 to 17	+0·27	−0·33	+ 147	+ 156
18 to 20	−1·14	−0·89	− 387	− 201
21 to 23	−0·43	−0·11	− 215	− 169

The inequalities are so evidently semidiurnal* that we may at once proceed to treat them on that assumption. Then each of the groups corresponds to a quadrant of luno-semidiurnal tide, and the first quadrant must be understood to begin at $23\frac{1}{2}^h$ lunar time, or, in arc, 15° before the moon is on the meridian. And when, from the numbers above, we shall have ascertained the argument of the luno-semidiurnal tide as measured from that lunar epoch $23\frac{1}{2}^h$, we must represent the argument as measured from lunar noon, or the argument in lunar time, by adding 15° to the argument reckoned from $23\frac{1}{2}^h$.

In each of the semidiurnal tides, the coefficient of the sine of double lunar angle from $23\frac{1}{2}^h$ may be found by taking 1^{st} number $+2^d$ number -3^d number -4^{th} number, and dividing by $8\times\{\sin 15°+\sin 45°+\sin 75°\}=15{\cdot}4544$. And the coefficient of the cosine may be found by summing 1^{st} number $+4^{th}$ number -2^d number -3^d number, and using the same divisor. The sum of a multiple of sine and a multiple of cosine may then be converted into a single sine; and the correction +15° may be applied, to render the argument measurable from lunar noon. Thus we obtain, as expressions for the Lunar Inequality,

In Western Declination from North,

Years of large solar curves
- First Semidiurnal Wave, 0′·246 × sine (double moon's hour-angle + 39° + 15°)
- Second Semidiurnal Wave, 0′·237 × sine (double moon's hour-angle + 31° + 15°)
- Mean, 0′·242 × sine (double moon's hour-angle + 50°)

Years of small solar curves
- First Semidiurnal Wave, 0′·188 × sine (double moon's hour-angle + 14° + 15°)
- Second Semidiurnal Wave, 0′·169 × sine (double moon's hour-angle + 38° + 15°)
- Mean, 0′·179 × sine (double moon's hour-angle + 41°)

* The numbers in each of the four columns of figures above may be resolved into a semidiurnal and a diurnal series, as follows:—

+1·34	−0·01	+0·92	−0·02	+335	−51	+268	−86
+0·18	−0·08	−0·34	0·00	+161	+15	+124	−32
−1·22	−0·08	−0·88	+0·02	−350	+37	−286	−86
−0·30	+0·13	+0·30	+0·42	−150	+65	− 94	+75
+1·34	+0·01	+0·92	+0·02	+335	+51	+268	+86
+0·18	+0·08	−0·34	0·00	+161	−15	+124	+32
−1·22	+0·08	−0·88	−0·02	−350	−37	−286	+86
−0·30	−0·13	+0·30	−0·42	−150	−65	− 94	−75

In Horizontal Force.

Years of large solar curves	First Semidiurnal Wave,	$0{\cdot}0000596 \times \sin$ (double moon's hour-angle $+21° + 15°$)
	Second Semidiurnal Wave,	$0{\cdot}0000781 \times \sin$ (double moon's hour-angle $+20° + 15°$)
	Mean,	$0{\cdot}0000688 \times \sin$ (double moon's hour-angle $+36°$)
Years of small solar curves	First Semidiurnal Wave,	$0{\cdot}0000517 \times \sin$ (double moon's hour-angle $+34° + 15°$)
	Second Semidiurnal Wave,	$0{\cdot}0000588 \times \sin$ (double moon's hour-angle $+15° + 15°$)
	Mean,	$0{\cdot}0000552 \times \sin$ (double moon's hour-angle $+41°$)

There does not appear to be any sufficient reason for concluding that one of the semi diurnal waves certainly differs from the other, or that the constant angle in the argument is certainly different in any of the several cases. But there appears to be no doubt that the coefficient for years of large solar curves is greater than that for years of small solar curves. The proportion $\frac{\text{lunar semidiurnal inequality in years of large solar curves}}{\text{lunar semidiurnal inequality in years of small solar curves}}$ is,

$$\text{for declination,} \quad \frac{0{\cdot}242}{0{\cdot}179} = 1{\cdot}35;$$

$$\text{for horizontal force,} \quad \frac{695}{549} = 1{\cdot}25.$$

It would seem possible to suggest two conjectural reasons for this remarkable association in the time-law of changes of solar effect and lunar effect. One is, that the moon's magnetic action is really produced by the sun's magnetic action; and a failure in the sun's magnetic power will make itself sensible both in its direct effect on our magnets and in its indirect effect through the intermediation of the moon's excited magnetism. The other is, that, assuming both actions (solar and lunar) to act on our magnets indirectly by exciting magnetic powers in the earth, which alone or principally are felt by the magnets, the earth itself may go through different stages of magnetic excitability, increasing or diminishing its competency to receive both the solar and the lunar action.

The arguments of lunar inequality, in western declination from north, and in horizontal force to magnetic north, are sensibly the same; so that we may consider the two disturbances to be synchronous, or that they are the effect of one disturbance in a definite straight line. The mean coefficient in western declination $= 0'{\cdot}210$, which expressed in terms of horizontal force $= 0{\cdot}0000611$. The mean coefficient in force to magnetic north, similarly expressed, $= 0{\cdot}0000621$. The direction of the composite disturbing force is therefore in the direction magnetic N.W. and S.E. very nearly; or, in astronomical bearing, making an angle 65° west of the north meridian. This may be described roughly as in the line from the Red Sea to the south of Hudson's Bay.

The laws of the lunar action and the solar action are widely different. The lunar action is semidiurnal; the solar action is mainly diurnal. The lunar action is in the N.W. direction; the solar action is mainly in the S.W. direction. (See the curve for the mean of years 1858–1863, in Plate XXXIV., accompanying this paper; and the curve for the mean of years 1848–1857, Greenwich Magnetical and Meteorological Observations, 1859, page clxxxv.)

LUNO-MENSTRUAL AND LUNO-DIURNAL INEQUAITIES OF VERTICAL FORCE.

The luno-menstrual inequality has been investigated; but the resulting numbers are small and uncertain, and it does not appear necessary to insert them here.

The luno-diurnal inequality for the period 1858–1863, expressed in terms of the horizontal force, is found to be as follows:—

Lunar Hour.	Inequality of Vertical Force, in terms of Horizontal Force.	Lunar Hour.	Inequality of Vertical Force, in terms of Horizontal Force.	Lunar Hour.	Inequality of Vertical Force, in terms of Horizontal Force.
0	+0·000005	8	−0·000120	16	+0·00008
1	+ 18	9	− 77	17	− 18
2	+ 64	10	− 33	18	− 64
3	+ 26	11	− 51	19	+ 20
4	+ 43	12	+ 20	20	− 38
5	+ 33	13	+ 38	21	− 26
6	− 84	14	+ 95	22	+ 97
7	− 110	15	+ 69	23	+ 102

On comparing these numbers with the corresponding numbers for the period 1849–1857 (Greenwich Magnetical and Meteorological Observations, 1859, page ccxxxvi), there appears to be sufficient resemblance between them to induce us to combine them. The general mean is as follows:—

Luno-diurnal Inequality of Vertical Force, for the period 1849–1863.

Lunar Hour.	Inequality of Vertical Force, in terms of Horizontal Force.	Lunar Hour.	Inequality of Vertical Force, in terms of Horizontal Force.	Lunar Hour.	Inequality of Vertical Force, in terms of Horizontal Force.
0	+0·000011	8	−0·000078	16	+0·000049
1	+ 10	9	− 76	17	+ 61
2	+ 16	10	− 33	18	+ 31
3	− 20	11	− 51	19	− 8
4	− 12	12	− 23	20	− 18
5	− 53	13	+ 10	21	− 6
6	− 74	14	+ 60	22	+ 47
7	− 11	15	+ 79	23	+ 104

These numbers may be rudely resolved into a Luno-diurnal term and a Luno-semidiurnal term, in the following form:—

Luno-diurnal Inequality of Vertical Force, for the period 1849–1863.

Lunar Hour.	Luno-diurnal term.	Luno-semidiurnal term.	Lunar Hour.	Luno-diurnal term.	Luno-semidiurnal term.
0	+0·000017	−0·000006	12	−0·000017	−0·000006
1	0	+ 10	13	0	+ 10
2	− 22	+ 38	14	+ 22	+ 38
3	− 50	+ 30	15	+ 50	+ 30
4	− 31	+ 19	16	+ 31	+ 19
5	− 57	+ 4	17	+ 57	+ 4
6	− 53	− 21	18	+ 53	− 21
7	− 1	− 10	19	+ 1	− 10
8	− 30	− 48	20	+ 30	− 48
9	− 35	− 41	21	+ 35	− 41
10	− 40	+ 7	22	+ 40	+ 7
11	− 77	+ 26	23	+ 77	+ 26

The numbers do not appear sufficiently regular to justify the examination with reference to the classification of the years by large solar curves and small solar curves.

XII. *Spectroscopic Observations of the Sun.*—No. II. *By* J. Norman Lockyer. *Communicated by Dr.* Sharpey, *Sec. R.S.*

Received November 19,—Read November 19 and 26, 1868.

Preliminary Remarks.

In my first paper under the above title, kindly communicated by Dr. Sharpey to the Royal Society in 1866*, was contained an account of the determination of the nature of Sun-spots by means of the spectroscope. The paper concluded as follows:—

"May not the spectroscope afford us evidence of the existence of the 'red-flames' which total eclipses have revealed to us in the sun's atmosphere, although they escape all other methods of observation at other times? and if so, may we not learn something from this of the recent outburst of the star in Corona?"

Before the paper was written I had diligently swept round the solar disk in search of evidence of the red flames, but without result. This want of success I attributed to the excessive brilliancy of the spectrum of the circumsolar regions in the field of view of the instrument employed. I found in fact (although I did not discontinue my efforts) that both for these observations and for those on sun-spots more dispersion was necessary; in one case to weaken the atmospheric light, in the other to widen the spectrum.

I therefore represented my requirements to the Government-Grant Committee, and was at once supplied with funds to procure a spectroscope of the requisite dispersive power.

The construction of this instrument was commenced in the beginning of 1867 by Mr. Cooke, on a plan which had been arranged between us, but unfortunately it was never finished. Mr. Cooke's health was then already failing, and at last, at the end of the year, he begged me to consider the order cancelled. Under these circumstances, at the beginning of the present year I sought the assistance of Mr. Browning; but the construction was further delayed, partly on account of an illness which necessitated my absence from England. At last the instrument, which reflects great credit on Mr. Browning's skill, arrived on the 16th of October, 1868, not quite complete, but in a condition which enabled me to commence work.

I mention these facts, first to account for my apparent inaction, and secondly in order that the coincidence in time of my results with those obtained by the observers of the recent eclipse may not be misinterpreted†.

* Proceedings of the Royal Society, vol. xv. p. 256.

† October 11, 1869.—It is important that I should be allowed further to emphasize this remark, for M. Faye, who was unaware of the date on which my new Spectroscope was received, has stated (Comptes Rendus, t. lxvii. (1868), p. 840), "L'insuccès des tentatives premières de M. Norman Lockyer (il est aisé de s'en rendre compte

I began my work with the new instrument by continuing my search after the prominences. I found that the circumsolar light was now so greatly reduced that, although the lines were faintly seen on a dimly coloured background, the background itself was apparently dark enough to render a bright line distinctly visible. My first attempts, however, with the new instrument, not yet in adjustment, were as unsuccessful as those made with the smaller one; and it was not till the 20th of October that, after sweeping for about an hour round the limb and arriving at the vertex of the image, near the south pole of the sun, I saw a bright line flash into the field.

My eye was so fatigued at the time that I at first doubted its evidence, although, unconsciously, I exclaimed "at last!" The line, however, remained—an exquisitely coloured line absolutely coincident with the line C of the solar spectrum, and, as I saw it, a prolongation of that line. Leaving the telescope to be driven by the clock, I quitted the observatory to fetch my wife to endorse my observation.

Detail of the Observations.

October 20.—Having settled that the new line was absolutely coincident with C, I commenced to search for more lines. This I found very difficult, as the instrument requires several movements and adjustments for the various parts of the spectrum, and the rate of the driving-clock was not properly adjusted for the sun's motion; the prominence was therefore lost at times; moreover the observations were impeded by clouds.

I commenced the search for lines from C to A. B was first brought into the field with the newly discovered line at C. There were no new lines visible. I then made an excursion to A with no result, and returned to C to assure myself that the prominence was still on the slit.

I then worked from the line at C towards D. A little beyond D, the lines of which are widely separated in my instrument, I detected another single and less vivid line, by estimation 8° or 9° of KIRCHHOFF'S scale more refrangible than the more refrangible of the strongest D lines. I could detect no line corresponding to it in the solar spectrum, but the definition was not good.

b was next tried, the excursion now being made from the new line near D. There was no line at *b*, though the new D line was still visible when I returned to it.

In the same manner, after many interruptions from clouds, I tried F; here I found

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aujourd'hui) me paraît tenir à ce que ce savant, dans l'impossibilité où il était alors de prévoir de quelles raies lumineuses se composerait le spectre des protubérances supposées gazeuses, ne savait sur quelles particularités délicates du spectre si compliqué des régions circumsolaires il devait porter son attention. *Cela est si vrai, que c'est seulement quand il a su, par les observateurs français et anglais de l'éclipse, la nature détaillée du spectre des protubérances,* qu'il a réussi à trouver en Angleterre les traces de ce spectre dans celui des régions voisines du bord du soleil."

I think that the illustrious French astronomer, who has otherwise done me such ample justice, will not object to my pointing out this slight inaccuracy, due entirely to the fact that the first communication of my discovery was incomplete in its statement of the circumstances which attended it. The bright lines as seen in my instrument are so obvious and brilliant that a child could not overlook them.—J. N. L., October 10, 1869.

another line. As at first caught it was very long; and by moving the telescope very slightly backwards and forwards in right ascension, in one direction the line shortened and brightened, and was visible on the solar spectrum for some distance, in another direction it became disconnected with the spectrum altogether. I was hence able to determine roughly the shape and dimensions of the prominence.

It was extremely difficult to fix the exact position of the line at F, although I had had no difficulty or even cause for hesitation about the others. It seemed at times to lie athwart the F line in the faint spectrum, although at first it had appeared more refrangible, especially when it was visible on the solar spectrum itself.

October 22.—Two days afterwards I had another opportunity of observing the prominence spectrum, and of endorsing everything I had observed on the former occasion. With regard to the F line, I traced the bright line parallel to the dark line and outside it (*i. e.* more refrangible) further than I did on the 20th, but once or twice I caught it inside F on the sun.

I found that a narrow slit was best for D and F, as there are bright bands in the solar spectrum at those points. The lines were always visible with the usual width of slit. C was bright enough to allow the slit to be widened.

On this day, as on the 20th, my attention was strongly drawn to certain bright regions in the spectrum.

October 27.—On this date the prominence first observed was no longer on the limb, I had to search therefore for a new one; I found one on the eastern equatorial limb. The method of observation previously adopted was changed, and a mixed spectrum of the sun's limb and prominence was obtained. The two spectra were in fact superposed.

In the case of the line at C, the result was to absolutely eclipse the dark line in the spectrum, and to replace it by a vividly bright band (Plate XXXVII. fig. 5). The behaviour of the F line was still a puzzle to me. In the spectrum of the light proceeding from the exact limb of the sun the bright line was seen more refrangible than F, but in the spectrum of the prominence at some distance above the sun the black line F was eclipsed (Plate XXXVII. fig. 6). This experiment, which I repeated several times, seemed in a measure to explain what I had before observed; and under this date I entered in my note-book: "It appears that away from the sun's surface the substance gives out less refrangible light than it does when apparently at the surface."

November 5.—The next observations were made on this date under superb atmospheric conditions, and after an important alteration had been made in the instrument, enabling me to make the several adjustments with the utmost nicety.

After the adjustments to the sun's limb had been made, I at once saw what I imagined to be the indication of a small prominence, and swept for a development of it, thinking that the portion observed might be one of the loops or lower levels which generally separate the higher peaks. Having swept for some distance on both sides the region on which the telescope was clamped in the first instance, and finding everywhere the same uniformity of height, it at once struck me that I was in presence of something

new, and that possibly what I was seeing might indicate a solar envelope. I rapidly, therefore, tried several other parts of the limb to test the idea. It was soon established. *In every solar latitude both the* C *and* F *bright lines were seen extending above the solar spectrum.* The spectrum near D was so bright that I was compelled to refrain from examining it, but I caught the line near D once.

The thickness of this envelope I found to be sensibly uniform, except in the regions where it was heaped up with prominences.

The spectrum of the envelope cleared up all the difficulties connected with the F line. Perfect definition and adjustment now enabled me to see that the base of the bright line widens out as the solar spectrum is approached, and that whereas the line, away from the sun, corresponds, in the case of an ordinary prominence, in refrangibility and thickness to the FRAUNHOFER line F, close to the sun it widens out, so as to overlap the F line on both its sides to an extent about equal to its thickness, so that it is three times broader, or perhaps more, on and close to the limb.

In the spectrum of a prominence in which violent action was going on the line thickened out in the same manner at some distance above the limb (Plate XXXVII. fig. 4). There was no thickening observed in the C line at the base, or in the case of the phenomenon just referred to.

I obtained on this day the outlines and dimensions of two prominences.

November 6.—The observations made on the preceding day were confirmed, and one of the prominences reobserved. I also suspected a new line a little less refrangible than C.

November 8.—The line suspected on the 6th was seen several times to-day. It came out beautifully with very varying brilliancy and even colour, to judge by appearance and impressions. It is very much thinner than the bright line C, and seems only to flash out where great action is going on, which slightly thickens the C line at its base. It is so near the C line that when both are shining brilliantly the line looks a double one, like D in a spectroscope of moderate dispersive power.

To-day for the first time I got definite evidence of what I had suspected on several occasions with regard to different parts of the spectrum. A bright line was observed in the solar spectrum itself, a little less refrangible than the line C.

On the Spectrum of the Prominences.

The existence of three lines in the spectrum of the prominences and their approximate positions were determined and communicated to the Royal Society on the 20th of October. See Plate XXXVII. figs. 1, 2, 3.

The coincidence of one of the lines with the solar line C was at once determined.

The coincidence of another line with F at a certain distance from the sun's surface was finally determined on the 5th of November, when the fact of the widening out of the lines towards the sun was discovered.

The exact position of the line near D is shown in Plate XXXVII. fig. 2, in which it

is laid down from the mean of three careful micrometrical measurements made under far from good atmospheric conditions on the 15th of November. In KIRCHHOFF's map the new line falls in a region where no line was measured by him. I may also add that, by the kindness of Mr. GASSIOT, I have been enabled to inspect the very elaborate maps of the spectrum constructed at Kew Observatory. The measures above given make the new line fall between two lines of almost inconceivable faintness; in Mr. GASSIOT's map, indeed, there are none but such lines for some distance on either side of the region in which the new one falls.

On the 8th of November the existence of another line was definitely established; its position in the spectrum is slightly less refrangible than FRAUNHOFER's C.

Unlike the other lines, which are seen in all prominences, this line is only visible at times, being rendered so apparently by the presence of certain conditions which are not permanent. Intense action going on in a prominence will sometimes render this line visible; I am not, however, prepared to say that this is always the case. The line when visible is much more variable than the others; at times it is the mere ghost of a line, at others it rivals the C line in brilliancy.

Of the three lines C is generally more brilliant than F; but I should add that it is difficult to determine the relative brilliancy of the lines because they are never seen together in the field of view of my instrument. The relative brilliancy of the line near D I am not sure about, because its situation in the brightest portion of the spectrum not only renders comparison difficult, but renders any conclusion which may be formed little worthy of confidence. My observations so far (November 16th) induce me to ascribe great variability, not only to the absolute, but to the relative, brightness of the lines. One instance is of the utmost importance. On the 5th of November, in sweeping round the sun with the F line in the field of view, I came across a prominence in which action of the intensest kind was evidently going on; the light and colour of the F line were most vivid, the luminosity of the line was greater than that of any part of the solar spectrum then visible in the field (Plate XXXVII. fig. 4). The action was not general, but limited to certain points, as if the prominences were built up of clouds, and the action was intensest at their centres; where the light was most vivid the spectrum widened out to almost the same extent as at the base*. The C line exhibited all the variations of brightness, but they were unaccompanied by any very decided widening of the line. The spectrum was too bright to observe the effect on the line near D.

On the Dimensions and Forms of the Prominences.

The prominence first observed on October 20th on the (true) southern limb of the sun was an exceedingly fine one; its shape, as determined by sweeping the slit over it, was as represented in Plate XXXVII. fig. 7; its apparent height was about 3′. I did not stop to measure it more accurately. This prominence was not so visible on the next day of observation (the 22nd), as the sun's rotation had carried it on to the disk.

* This might have been an effect of irradiation, but I can scarcely think so.

On the 5th of November I obtained the outline of two brilliant prominences, one near the southern, the other near the northern limb of the sun. The extreme (measured) apparent height of one was 35,000 miles, of the other somewhat less; the former I estimated to extend along the sun for about 200,000 miles. The shape of the southern prominence had changed considerably by the next day, the bright peak being quite gone; at the same time the length of the main portion had extended as if the peak had been absorbed into it (Plate XXXVII. figs. 8 & 9).

The prominences therefore change from day to day; at present I have not measured any more rapid change, but these observations are of so delicate a nature that it is easy to imagine rapid changes to be going on in any prominence of decided outline; for an error in the adjustment of the instrument with regard to the meridian or latitude, the least variation in the rate of the driving-clock, or any oscillation of the telescope-tube or the spectroscope, brings the slit on another part of the outline of the prominence, and under these circumstances the length of the line is perpetually varying.

It must be borne in mind that the dimensions of the prominences cannot be determined absolutely, as we do not know whether they are actually on the sun's limb at the time of measurement. Measurement can only fix a minimum.

On the Continuous Solar Envelopes.

The continuity of this envelope, which I propose to name the CHROMOSPHERE, a name suggested by Dr. SHARPEY, was spectroscopically established on the 5th of November, and an account of the observations was transmitted to the Royal Society on the same day.

By careful estimations made on the 6th of November (which are estimations only, for I had not yet mounted a wire-micrometer on the spectroscope-telescope), its general thickness was determined to be about 5000 miles; the level of its upper surface was not absolutely uniform in all latitudes, but it was very nearly so. I could detect no difference in the general level as between the equatorial and solar regions of the sun.

It would appear that the light by which its existence is revealed proceeds from the same substance or substances of which the prominences are composed; and I hold the prominences to be merely the heaping together of the new envelope in some localities.

Under proper instrumental conditions the spectrum of this envelope can always be seen whenever the sun is shining. The spectrum consists of a line coincident with FRAUNHOFER'S C, another more refrangible than D, and another coincident in the main with F. I say coincident in the main, because when the spectrum of the envelope is viewed so that it appears to rest on the solar spectrum, the line at F takes the form of an arrow-head (Plate XXXVII. fig. 3), the shaft of the arrow being represented by the black line F in the solar spectrum itself. This results from a thickening out of the line as it approaches the sun. At the outer surface of the envelope the refrangibility of the light is the same as that of the line F; but, as the sun's surface is approached, the line widens gradually into a band overlapping F by more than its own thickness on either side, and more on the more refrangible side than on the other. The black line F

also loses its thickness as it approaches the boundary of the spectrum. The amount of widening at the base is subject to variation.

In the spectrum of the chromosphere this widening at the base is not generally observed either in the line at C or in that near D.

None of the lines stop sharply; they all fade out as the limit of the envelope is approached.

On certain Bright Regions in the Solar Spectrum.

From the commencement of my observations with the new instrument my attention has been drawn to certain bright regions in the ordinary spectrum; but it was not till the 8th of November, 1868, that I succeeded in observing a definite bright band extending for a certain distance on the sun near the limb.

I should state that I have observed this behaviour in the F band on either side of FRAUNHOFER's dark line F, and in the C line, when the prominence, as I have imagined, has extended from the limb over the earth's side of the sun.

The position of the bright band observed on the 8th of November is near C, but slightly less refrangible, not far from the place in the scale occupied by the last discovered red line, the position of which as yet has not been micrometrically determined.

Other regions to which my attention has been particularly drawn from the first, although up to the present time I have obtained no results, lie, one between the *b* lines, another between *b* and F, another less refrangible than B, one near D, and another near G.

It is quite possible that these bright regions, the light of which is variable, may be due to faculæ; this conclusion is strengthened by the fact that diligent sweeping within the limb has not revealed the bright lines of the chromosphere spectrum. If this be so, the faculæ are not the prominences, although they may be possibly connected with them.

On the nature of the Chromosphere and Prominences.

It has already been concluded by M. JANSSEN, from the coincidence of two of the bright lines with C and F, that the prominences are composed of hydrogen.

So far as our present knowledge goes, however, this does not dispose of the other two bright lines, the positions of which have been determined by myself: I allude to the lines near D and near C.

At the present moment I am engaged on a series of experiments on gaseous spectra, which I hope will afford additional information on these points; in the interim, on the assumption that the chromosphere and prominences are wholly, or in part, composed of hydrogen, several considerations which appear to me of great importance may be touched upon.

These considerations are based upon the experiments of MM. PLÜCKER and HITTORF* on the one hand, and of Professor FRANKLAND on the other†. In MM. PLÜCKER and HITTORF's paper, entitled "On the Spectra of Ignited Gases and Vapours, with especial regard to the different Spectra of the same elementary gaseous substance," these investi-

* Philosophical Transactions, vol. clv. (1865) Part I. pp. 1–29. † Proc. Roy. Ins. vol. v. p. 419.

gators point out the effect of temperature on the different spectra, the temperature of the discharge of RUHMKORFF's induction-coil being increased by increasing the power of the inducing current, or, preferably, by diminishing the duration of the induced one, by means of the Leyden jar.

Among the important results obtained in the case of nitrogen were the following. With increase of temperature the light passes through the following colours:—

Golden,
Bluish violet,
White,

and at an extreme temperature the lines expand, approaching thus to a continuous spectrum.

We now come to hydrogen. When a GEISSLER's tube filled with extremely rarefied hydrogen is used, the spectrum produced by a spark of low temperature consists of three lines of the same width as the slit—one red, another bluish green, another violet*; these are respectively termed Hα, Hβ, and Hγ by MM. PLÜCKER and HITTORF. The places of these lines in the solar spectrum are at C, at F, and one at some distance from G towards F.

I quote the following results of increase of temperature from the memoir under notice.

"Hydrogen shows in the most striking way the expansion of its spectral lines, and their gradual transformation into a continuous spectrum. When the direct discharge of RUHMKORFF's large induction-coil is sent even through the old spectrum-tubes enclosing hydrogen, the formerly obtained spectrum is essentially altered. By increasing the power of the coil, the violet line Hγ first expands; while it continues to expand, the expansion of the bluish-green line Hβ becomes visible. Let the aperture of the slit be regulated so that the double sodium-line will separate into two single lines nearly touching one another. Then, the angular breadth of Hβ becoming two or three minutes, the breadth of Hγ is about double. The expansion takes place as well towards the less as towards the more refracted part of the spectrum. Hα remains almost unchanged after Hγ has passed into an undetermined large violet band, and Hβ extended its decreasing light on its two sides. On employing the Leyden jar, and giving to the gas in our new tubes a tension of about 60 millims., the spectrum is already transformed into a continuous one, with a red line at one of its extremities. At a tension of 360 millims. the continuous spectrum is highly increased in intensity, while the red line Hα, expanded into a band, scarcely rises from it. If the electric spark passes through hydrogen at the ordinary tension, the ignited gas on its way always gives the spectrum of the three expanded lines.

"Even in the old spectral tubes enclosing highly rarefied hydrogen, the ground, from which the three characteristic lines rise, did not appear always of the same darkness; in some instances new bright lines appeared, *especially in the neighbourhood of the sodium-*

* I am aware that ÅNGSTRÖM has discovered another line, but it is not necessary here to consider it.

line. In resuming the subject, we pointed out the existence of a new hydrogen-spectrum, corresponding to a lower temperature, but having no resemblance at all to the spectra of the first order of nitrogen, sulphur, &c. In this spectrum, of a peculiar character, if fully developed, we observe a great number of well-defined bright lines, almost too numerous to count and represent by an engraving, but brilliant enough to be examined with a magnifying-power of 72, after the light has passed through four prisms.

"On sending the direct discharge of RUHMKORFF'S coil through a tube of glass from one-fourth to one-eighth of an inch in diameter, provided with electrodes of platinum or of aluminium, enclosing hydrogen at a tension of 5 to 10 millims., a luminous thread of light of a bluish-white colour was seen passing along the axis of the tube, without touching the glass. When analyzed by the prism, it gave a faint spectrum of the above-mentioned numerous bright lines, especially within the red and the yellow. Among these lines neither Hα nor Hγ were seen; Hβ only appeared, but less bright than many other lines. By interposing the Leyden jar and gradually increasing its charge, all the lines became brighter, Hβ surpassing all other lines in brilliancy; Hα appeared beautifully, Hγ fainter. Hence we conclude that the numerous bright lines belong neither to the vaporized metal of the electrodes, nor to the decomposed interior surface of the glass, but solely to the hydrogen, constituting a new spectrum of it. This spectrum may be seen simultaneously with the three characteristic lines Hα, Hβ, Hγ; but at an increased temperature, when these lines begin to expand, it entirely disappears."

MM. PLÜCKER and HITTORF also determined the action of the discharge of a small induction-coil through a tube enclosing hydrogen which was gradually rarefied to the highest tenuity to be reached by a GEISSLER'S exhauster; they found that, as the exhaustion proceeded, the red colour of the gas gradually faded into an "undetermined violet," and that Hα disappeared, while Hβ, though fainter, remained well defined.

With hydrogen gas in GEISSLER'S tubes, then, the following facts are established:—

I. With a certain degree of rarefaction and temperature, we obtain three characteristic lines, Hα, Hβ, and Hγ.

II. By increasing the temperature, we expand Hγ first towards both ends of the spectrum, then Hβ, Hα remaining almost unchanged after Hγ has passed into an undetermined large violet band.

III. By increasing the tenuity, Hα disappears first, Hβ remaining well defined, and moreover the colour of the ignited gas changes to the eye.

IV. Under certain conditions, which are not stated in the memoir, new lines appear in the spectrum, especially in the neighbourhood of the sodium-line.

Assuming that hydrogen gas is present in the chromosphere and prominences, we have the following facts to place side by side with those just stated:—

I. In place of the three lines we have but Hα and Hβ.

II. Hβ is in process of expansion, the expansion increasing as the sun is approached, and Hγ is so far expanded that it no longer exists as a line; most careful observations

repeated several times have failed to detect it. Were it a broad band having the same total amount of light, it would be invisible in the spectroscope; it has probably therefore reached this stage.

III. The prominences have been observed of various colours (this fact is not here stated merely with reference to the observation recorded in III. *ante*).

IV. *There is a line in the yellow*, most probably proceeding from the substance which gives off the light at C and F, as the length of this line, as far as the later observations with the more correctly adjusted instrument go, is the same as that of those in C and F.

I am aware that the conditions as to density cannot for one moment be held to be the same in the two cases; but as at present (so far as I know) we have no similar experiments ranging over greatly varying densities, I have thought it desirable to bring these striking facts forward at once. We are justified in thinking that the density of the chromosphere, always assuming that it is composed wholly or in part of hydrogen, cannot be very great; if it were, the spectrum would most probably be continuous; for Professor FRANKLAND has shown, in the lecture before alluded to, that hydrogen burning in oxygen under a pressure of ten atmospheres gives out a spectrum, bright, and perfectly continuous from red to violet.

It is possible that experiments in which both density and temperature are varied may enable us to match accurately the spectrum of the chromosphere, and thereby determine both the temperature and pressure at the surface of the sun.

The bright lines which have been observed in several stars, especially in the remarkable one in Corona, the outburst of which was spectroscopically watched by Mr. HUGGINS in 1866, shows us that under certain conditions of constitution a chromosphere may be a part of the regular star-economy, and be liable to changes more or less great, and effected with more or less rapidity. The continuous observation of the sun's chromosphere may therefore lead to a knowledge of many important facts.

It is remarkable that in the outburst in the star in Corona, two of the lines of the chromosphere were coincident with C and F, and that two other lines were visible. If, therefore, either or both of these lines be due to hydrogen, we must regard that star, leaving the density of the envelope out of consideration, as being of lower temperature than our sun.

The position of the prominences in all parts of the sun's limb has generally been held to be sufficient evidence that they are not connected with spots. They may, however, still result from some common cause. It may be noted that the eclipse which was photographed by Mr. DE LA RUE in Spain happened near the time of maximum sun-spots, and that nearly the whole limb of the sun was covered with prominences. My observations since 1866 have been carried on at the minimum sun-spot period, and the prominences observed during the eclipse this year were few in number, and covered but a small part of the sun's limb.

Received November 26, 1868.

Additional Note on the Chromosphere.

Since my last communication to the Royal Society I have received, through the kindness of Admiral MANNERS, the following extract of a note from Father SECCHI, in which, although the existence of the new continuous envelope is not announced, important corroboration of its existence is contained. Father SECCHI says:—

"Rome, 15 Nov. 1868.

"I have been able to verify the observations of Mr. LOCKYER on the sun, but I find that, even where the lines do not become brilliant, their blackness vanishes by a partial inversion. I have found also some luminous lines which become exceedingly brilliant near the edge of the sun. One is near the ray D, and the other in contact almost with the line B on the side of C.

"*I find that all around the full limb of the sun the inversion takes place.* If the slit is perpendicular to the edge the inversion is a very short part, so , of 10″ or 15″, but if the slit is parallel to the edge then the inversion is complete. It is a very beautiful fact. It is perhaps one that will modify our ideas on the origin of these lines."

I have italicized the most important part of the letter.

Since the 20th of the present month, in consequence of a conversation on that day with Mr. DE LA RUE, I have gone over my observations of the sun's limb with great care, and have also reexamined Mr. DE LA RUE'S photographs with a view to ascertain the evidence which they give of the continuity of the envelope. The result strongly confirms my former views. It is true that the photographs do not show a continuous chromosphere of anything like uniform thickness; but that arises from the fact that the only part of the sun's limb where the envelope was visible at all during totality happened to be covered by irregular prominences, which were probably very abundant at the time. In fact, owing to the relative sizes of the sun and moon during the eclipse of 1860, and the direction of the moon's motion, the top and bottom of the sun's limb, as shown in plate 15 of Mr. DE LA RUE'S memoir, were considerably within the moon's limb during the totality; only small portions of the sun's limb on the preceding and the following sides were uncovered, and there the chromosphere is seen to be almost continuous, though, as I have said, very irregular. The absolute want of continuity in the photographs, which, as far as I recollect, is not observable in Mr. DE LA RUE'S hand drawing, is not a final argument against its real spectroscopic continuity. On the 24th of November 1868, when I had an opportunity of observing the sun, I employed the whole time in examining into the absolute continuity of the chromosphere; I was enabled, until interrupted by clouds, to examine continuously and carefully about 300 degrees of the sun's contour, and during the time I did not for one moment lose sight of the chromosphere spectrum. It is very gratifying to find that photographs taken many

years ago by so excellent an observer as Mr. De La Rue should be found to bear independent testimony to the accuracy of the conclusion which I have arrived at, by an entirely different line of research.

Received April 9, 1869.

Historical Notice of the growth of our knowledge of the Chromosphere.

When I was first enabled, by means of the spectroscope, to determine that the prominences are merely heapings up of an envelope which gave everywhere the spectrum of hydrogen (at pressures which Dr. Frankland and myself have since approximately determined) and is continuous round the sun, or at all events continuous in the same sense that the photosphere is continuous, I was not aware that I had been anticipated in any part of the discovery.

I have since found, however, that the continuity of the envelope, apart from its nature and place in the solar economy, has been suspected for many years, although it had never been demonstrated, as it easily might have been, by eclipse observations at properly chosen stations, and although it has been very variously interpreted.

I think it desirable, now that the spectroscope has determined the existence and nature of such an envelope, that, in justice to myself and to those who have gone over the same ground before me, a brief historical notice of the progress of our knowledge on this point should be given.

It is easy now to divide the phenomena observed in the many eclipses between 1706 and 1842 into two classes:

I. Observations of the prominences properly so called;

II. Observations of the chromosphere;

and all the observations of both these classes, accumulated during the eclipses which happened up to and including the year 1842, have been discussed by two eminent astronomical authorities,—I refer to Arago and Professor Grant, to both of whom, long even before the eclipse of 1851, it was perfectly obvious that the prominences were solar and not lunar phenomena.

Arago* considered the prominences to be merely clouds floating in the sun's atmosphere—an atmosphere rendered evident to us by the Corona, and to these clouds he ascribed the spots without a nucleus; to the Corona also he attributed the darkening of the limb. He says:—

"Il faut admettre une enveloppe extérieure qui diminue (éteint) moins la lumière qui vient du centre que les rayons qui viennent sur le long trajet du bord à l'œil. Cette enveloppe extérieure forme la couronne blanchâtre dans les éclipses totales du soleil."

It is not easy to reconcile *all* Arago's statements as to the nature of this atmosphere or envelope; but I shall return shortly to a later enunciation of them by himself, merely

* Annuaire du Bureau des Longitudes, 1846. MSS. quoted by Humboldt, Cosmos, Sabine's Translation, vol. iv. Notes, p. cii.

remarking here that there is nothing said about the "clouds" forming a continuous envelope round the sun.

Professor GRANT*, who went over the same ground as ARAGO, and had ARAGO's results before him, was led to consider before the eclipse of 1851 that "The observations of solar eclipses would seem to indicate that above the luminiferous envelope there exists a STRATUM of nebulous matter which is visible only by means of reflected light. Various interesting questions present themselves for solution in connexion with the admitted existence of such a STRATUM. In the first place, does this THIRD ENVELOPE exercise an influence in the production of any of the other phenomena which have been disclosed by observations on the physical constitution of the sun?"

Then, referring to Sir J. HERSCHEL's hypothesis of convection currents throwing up the then imagined non-luminous envelope below the photosphere, through the photosphere, he adds:—

"This view of the subject, while it carries with it considerable probability, obviates the necessity of introducing into the theory of the physical constitution of the sun the idea of a third envelope independent of the two others."

We next come to the eclipse of 1851, which was observed by Professor SWAN, among others, who contributed to the Royal Society of Edinburgh three valuable memoirs on the eclipse with special reference to the red prominences. Assuming that they existed in the solar atmosphere, and agreeing with the prescient remarks of Sir J. HERSCHEL, that they must be "cloudy masses of the most excessive tenuity," he remarks†:—

"Obviously the simplest view that can be taken of this phenomenon, is to regard the red fringe and the red prominences as of the same nature; and all the observations will then confirm the idea that the matter composing those objects *is distributed all round the sun*. It therefore seems probable that when we are furnished with observations of a tangential phase of the eclipse from stations on the north side of the moon's shadow, it will be found that a sierra appeared towards the sun's north point, of which the detached prominences seen in that region, by observers situated near the middle of the moon's shadow, were only the highest peaks. . . . . Since, then, it has been shown to be highly probable that the matter composing the red prominences is distributed with little interruption all round the sun, we may conceive the luminous strata of the solar atmosphere to be surmounted by an envelope of clouds of which the higher portions are visible beyond the moon's limb, at the central phase of a total eclipse, and which then constitute the red prominences. If [he continues, throwing out the suggestion previously made by GRANT] it be thought that the hypothesis of two envelopes of cloud, one above and another below the luminous strata of the sun's atmosphere, introduces too great complication, we may avoid the objection by supposing that the envelope which occasions the penumbræ around the spots penetrates the luminous stratum, and exists, although in greatly different degrees of density, both above and below it.

* History of Physical Astronomy, p. 400.

† Transactions of the Royal Society of Edinburgh, vol. xx. part iii. pp. 462, 463, 464.

"If, then, we conceive that a stratum of cloudy matter surrounds the sun, of which the red prominences are the higher portions, the serrated appearance of the long range of prominences, seen by Mr. DAWES and Mr. HIND, sufficiently indicates that its general surface is exceedingly uneven, presenting the appearance of being covered with numerous eminences or ridges. But these irregularities are small when compared with the large hook-shaped prominence, and its companion the detached cloud, which were seen by most of the observers of the eclipse. . . . . .
Now, as the spots have been supposed to arise from upward currents causing apertures in the sun's luminous atmosphere, I conceive the higher red prominences, or those which remain visible at the middle of the total phase of a central eclipse, may in like manner be formed by the same, or similar, currents in the sun's atmosphere, breaking through the envelope of cloud that surrounds him, bending back the edges of the apertures they have formed, and sometimes carrying up detached masses of cloud, such as that which was seen at the late eclipse. We may, however, suppose the envelope of cloud to be sometimes simply raised, without being broken through; and in that state it may form the conical prominences which were observed at the late eclipse.

"Since the prominences reflect, they must also absorb light; and thus the hypothesis which has been proposed regarding them *assumes the presence of an envelope of cloud surrounding the sun's luminous atmosphere, capable of absorbing part of his light and subject to occasional interruptions of its continuity.*"

Mr. SWAN thus formally sums up his most important addition to our knowledge of the solar envelopes, agreeing with ARAGO that the Corona is also a solar appendage. We have now four envelopes.

I. The dark cloud below the photosphere. [The cloudy stratum of HERSCHEL.]
II. The photosphere itself.
III. The envelope of cloud so often referred to.
IV. The sun's atmosphere surrounding all, in which the other envelopes may be supposed to float.

LITTROW, who observed the same eclipse, after describing some of the observations, goes on to say:—

"Tout cela me fortifie dans l'opinion conçue déjà par mon observation seule, que ce bord rouge forme une couche environnant toute la photosphère du soleil, et gonflée çà et là en protubérances"*.

ARAGO in his 'Astronomie'† defends his first view, and criticises Mr. SWAN's hypothesis. He remarks:—

"Tout s'explique dans l'hypothèse de nuages flottants dans l'atmosphère diaphane qui entoure le soleil.

"J'avais cherché de rendre compte des protubérances lumineuses en les assimilant à des nuages flottants dans l'atmosphère diaphane dont je supposais la photosphère

* Astr. Nachrichten, t. xxxiv. p. 31, and Comptes Rendus, Feb. 22, 1869.
† Ed. 1856, t. iii. p. 623.

entourée. M. SWAN ayant sans doute remarqué dans ma notice cette phrase 'L'éclipse de 1842 nous a mis sur la trace d'un troisième enveloppe située au-dessus de la photosphère et formée de nuages obscurs ou faiblement lumineux,' accumule à la fin de son mémoire citations sur citations, pour prouver que nonobstant ce que cette phrase paraît de renfermer de positif, je n'ai pas eu la pensée qu'il existât au-dessus de la photosphère une couche continue de nuages. JE RECONNAIS LOYALEMENT QUE L'IDÉE DE LA COUCHE CONTINUE APPARTIENT EN PROPRE À M. SWAN."

The next criticism we find brings in the Corona in a way we shall see it again brought forward subsequently.

"La limite extérieure *de la première couronne* lumineuse indiquerait, dans l'hypothèse de M. SWAN, la région qu'occupe la couche continue de nuages dont il croit avoir besoin pour expliquer tous les phenomènes des éclipses totales. Il faudrait donc supposer que, lorsque la couronne est unique, cette couche de nuages s'est abaissée jusqu'à être presque en contact avec la photosphère solaire. C'est alors qu'apparaîtraient les longs arcs courbés, colorés et fortement dentelés, qui ont été signalés par les observateurs comme étant visibles quelques instants après le commencement de l'éclipse totale, et quelques instants avant la fin. Mais admettant pour un moment que ces grands mouvements oscillatoires en hauteur de la couche nuageuse existent, pourquoi cette couche se présenterait-elle comme une ligne circulaire sans couleur lorsqu'elle serait à une grande hauteur, et descendrait-elle irisée et très-irrégulière dans son contour lorsqu'elle se rapprocherait du soleil. Suivant M. SWAN, les protubérances sont des portions de son atmosphère continue, soulevées du-dessus du niveau général par le courant ascendant. Mais comment n'a-t-il pas remarqué qu'en 1842 ces protubérances existaient tout notablement au-dessous de la ligne circulaire qui dessinait les limites de la couronne la plus brillante sur la couronne extérieure?

"Je persiste donc à soutenir simplement que la troisième atmosphère solaire que M. SWAN veut bien admettre avec moi est gazeuse et qu'il y flottent seulement des nuages."

This strong criticism would seem to have had great weight, and I believe that much of the strange modern overlooking of the continuous envelope is to be ascribed to it, to say nothing of the invariable (so far as I know) descriptions of the solar atmosphere to be found in our text-books, on which recent ideas and theories have been based.

We next come to the eclipse of 1851, which was observed in the Brazils. The Brazilian Commission, of which M. LIAIS was a member, sent in a report to the Paris Academy of Sciences, which was reported on by M. FAYE, who commenced his report by thus defining the Corona and prominences*.

"La couronne lumineuse ne serait autre chose que l'indice sensible d'une troisième enveloppe du soleil, d'une atmosphère extérieure. . . . *Les protubérances seraient les nuages de cette troisième atmosphère.*"

I have not been able to obtain the Commissioner's report; but M. LIAIS, in a separate work†, states distinctly (I.) that there is a continuous envelope overlying the photosphere,

* Comptes Rendus, t. xlviii. p. 163.

† L'espace céleste et la nature tropicale.

(II.) that it is not the Corona, (III.) that it is the locus of the general absorption of the photospheric light, and (IV.) that its height is about 3″·3—an immense step in advance, as we now know, on the ideas of Arago. At the same time M. Liais was convinced that the Corona was in reality a solar appendage.

The famous Spanish eclipse of 1860 is next on the list. Mr. De La Rue's admirable photographs have made us all familiar with the solar appendages then visible. Specially to be noticed in them are the points where the limbs of the sun and moon were nearly in contact both at the commencement and at the end of the totality.

In discussing the results of this eclipse, both M. Le Verrier and Father Secchi found themselves compelled to use the word "envelope" to explain all the phenomena observed.

M. Le Verrier*, after a preliminary discussion of the results of this eclipse, remarked, "Faut il croire que la surface entière de l'astre en (nuages rouges) est parsemée jusqu'à une faible hauteur comme elle est semée de facules, et que les nuages roses en sont des émanations commes les taches qui apparaissent sur la disque de l'astre."

And then, after a more complete discussion, he endorses the idea of the complete continuity of the envelope, and makes it at the same time not only the only solar atmosphere, but the origin of spots!!

After referring to Herschel's hypothesis, and remarking "à cette constitution si complexe on eût dû ajouter une troisième enveloppe formée de l'ensemble des nuages roses," he goes on, "or je crains que la plupart de ces enveloppes ne sont de pures fictions; que le soleil ne soit simplement un corps lumineux, en raison de sa haute température, et recouvert par une couche continue de la matière rose dont on connaît aujourd'hui l'existence. L'astre, ainsi privé d'un corps central, liquide ou solide, recouvert d'un atmosphère rentre dans la loi commune de la constitution des corps célestes. . . . . Il paraît clair qu'ils (les protubérances roses) émanent accidentellement d'une couche de matière qui recouvre toute la surface du soleil jusqu'à une hauteur de 8 à 10 secondes. . . ."

M. Le Verrier then proceeds to show, as we have seen M. Swan and M. Liais do many years before, that the darkening of the limb is due to this envelope, and then adds, "D'un autre côté il resulte de l'observation des nuages solaires que la matière de l'atmosphère s'accumule quelquefois en quantités plus considérables sur certains points, et comme la lumière de la partie correspondante du soleil peut se trouver plus ou moins éteinte, on arrive à une explication naturelle de l'existence des taches!"

We see, then, that the French results of the eclipse of 1860 entirely endorsed Mr. Swan's assertion of the continuity of the cloudy envelope, although the hypothesis put forward to explain the envelope was completely discordant with observation then and is much more so now.

Father Secchi's idea will be rendered evident by the following extract from his report contained in the memoirs of the Observatory of the Collegio Romano†:—

"La seconda conseguenza, non meno importante è, che questa materia riveste tutta la

* Report quoted in Comptes Rendus, February 8, 1869, page 316.

† New Series, 1860–62, vol. ii. No. 5, p. 43.

superficie solare come un generale inviluppo transparente. Infatti il loro numero prodigioso, e il loro estendersi per archi continuati di molti gradi, ci mostra che è irragionevole supporle, particolarità locali ed eccezionali sulla superficie solare, come sono le macchie, nè possono dirsi eruzioni vulcaniche di pochi punti: al contrario il vederle spuntare congiunte in lunghe catene tanto al principio che al fine della totalità, ci persuade che negli altri punti della circonferenza si rendono visibili solo le cime maggiori e più elevati, restando le minori e più basse coperte dal corpo lunare."

Although these modern results would seem to have settled the question as far as it was possible to settle it by ordinary observations on the central line of totality, I cannot find that the idea of the continuity of the envelope was generally accepted in England or in France.

In 1861 we find M. Faye* calling upon observers of the eclipse that was to happen on the last day of that year to examine "si cette auréole (la partie de cette couronne la plus voisine du soleil) présente ou non le renversement du spectre solaire, c'est à dire si les raies obscures de Fraunhofer seront remplacées dans ce spectre par des raies brillantes." There is not one word about the "couche rose" of M. Le Verrier!

After this in England we find General Sabine, Dr. Balfour Stewart, and Professor Challis independently arriving at the conclusion that the red flames are *solar auroræ*—a theory which I think plainly indicates that the idea that they formed part of a continuous envelope was not in their minds. Mr. Balfour Stewart, in a Lecture at the Royal Institution†, remarked:—

"In support of this hypothesis it may be remarked that, during the late total eclipse in Spain, Mr. De La Rue, by means of the Kew photoheliograph, proved that these red flames belong to the sun, and that they extended in one case to the distance of 70,000 miles beyond his photosphere. But, considering the gravity of the sun, we are naturally unwilling to suppose that there can be any considerable amount of atmosphere at such a distance from his surface; and we are therefore induced to seek for an explanation of these red flames amongst those phenomena which require the smallest possible amount of atmosphere for their manifestation. Now the experiments of Mr. Gassiot, and the observed height of the terrestrial aurora alike convince us that this meteor will answer our requirements best. And besides this, the curved appearance of these red flames, and their high actinic power in virtue of which one of them, not visible to the eye, was photographed by Mr. De La Rue, are bonds of union between these and terrestrial auroræ."

Although Mr. De La Rue in his Memoir refers to the prominences being scattered widely over the sun's disk, it was not till December 1867 that the real meaning of the photographs in this particular was grasped. Mr. Stoney, in a paper communicated to the Royal Astronomical Society in that month, in which he refers to his hypothesis that the sun has an enormous atmosphere, which in eclipses projects far beyond the disk of

* Comptes Rendus, vol. lii. p. 679.

† Proceedings of the Royal Institution, vol. iv. p. 60.

the moon, in which atmosphere the vapours which give rise to the selective absorption of the photospheric light are situated at various heights according to their vapour-density, writes as follows:—

"Directly outside the photosphere there lies a stratum of the sun's atmosphere which is still hotter than the photosphere, and on the outer boundary of this hot region there appears to be a shell of excessively faint cloud, part of which is seen in Mr. De La Rue's photographs of the eclipse of 1860. It probably extends the whole way round the sun. It is therefore very desirable that this faint shell, which seems to lie at a distance of 8″ or 10″ from the edge of the sun's disk, should be observed, both from a central station and from stations close to the northern and southern limits of totality, so as to ascertain whether we have reason to presume it is continuous round the disk"*.

Broadly, then, up to the spectroscopic determination of its continuity and real nature, the story of the chromosphere is as follows:—

1842. Arago refers it to clouds at the base of the Corona, and regards these clouds as the origin of non-nucleated spots, the dimming of the limb being due to the Corona.

1842. Grant acknowledges the continuity of the envelope, ascribes its brilliancy to reflected light, and thinks it may be due to the "cloudy stratum" being driven by convection-currents through the photosphere.

1851. Swan terms it a new envelope of the sun, states that it shines by reflected light, and ascribes to it the dimming of the limb.

1851. Littrow describes it as a continuous thin envelope.

1858. Liais describes it as a continuous envelope, and gives its thickness as 3″·3.

1860. Le Verrier describes it as the unique atmosphere of a solid sun and the origin of all spots.

1860. Secchi acknowledges it as a third envelope.

1867. Stoney considers it as a stratum of cloud at a distance of 8″ or 10″ from the photosphere.

Description of the Plates.

PLATE XXXVII.

Figs. 1, 2, and 3 show the position of the lines observed on October 20, 1868, and their usual form, *i. e.* the line F is broad at the base and gradually tapers upwards, while C, and the line near D (with no corresponding absorption-line ordinarily visible) do not, as a rule, present this peculiarity with the instrument employed.

Fig. 4 shows the appearance of the F line observed on the 5th of November 1868.

Figs. 5 and 6 show the appearances of the C and F lines observed with a tangential slit;

* Monthly Notices of the Royal Astronomical Society, vol. xxviii. p. 19.

in the case of F the bright line observed was sometimes on one side of the absorption-line, and sometimes put it out altogether (October 27).

Figs. 7, 8, and 9 are outlines of the prominences observed on the 20th of October, 5th and 6th of November 1868.

Added October 10, 1869.

PLATE XXXVIII.

The spectroscope of large dispersive power, by means of which the work I have described in the former part of this paper has been done, is attached to a clock-work driven refractor of $6\frac{1}{2}$ inches aperture and $98\frac{1}{2}$ inches focal length, the definition of which is very fine, and worthy of the reputation of its makers, Messrs. Cooke and Sons, York.

Fig. 1 will give a general idea of the spectroscope and of its attachment to the large equatoreal. In this figure are shown the eye-end of the telescope with finder and clamping and adjusting rods, and an internal tube which I have found it convenient to insert into the ordinary one to allow of the adjustment of the spectroscope to the direct solar image, and the variously enlarged images, without disturbing the spectroscope itself on its supports.

This internal tube carries the ordinary eyepiece-end on which the prismatic steel rods which support the train of prisms, collimator, and telescope are fixed and provided with means of rotation.

Fig. 1 also shows the arrangement of the instrument when the direct image is viewed. The eyepiece is taken out, and the image is allowed to fall on the slit-screen outside the telescope. A short collimator collects the light which then passes through the seven prisms (of 45°) of dense flint glass, of which the specific gravity, refractive index, and dispersive power, as determined by Mr. Browning, are as follows:—

Specific gravity	3·91
Refractive index	1·665
Dispersive power	0·0752

Ordinarily the seven prisms are alone used, but when more dispersion is wanted I have found it very convenient to fix an extra prism of 60°, as shown in the figure; this makes the instrument for some part of the spectrum a direct vision one; and I have further increased the dispersion by partly filling the small telescope with direct vision prisms.

The adjustment of the spectroscope to the telescope allows of the slit being brought either tangentially or radially on any part of the sun's limb.

Fig. 2 shows a slit I have designed for comparing various portions of the solar surface with each other. The slit, for purposes of description, may be imagined to be divided into three portions. The central portion admits direct light from some part of the sun;

the upper and lower portions admit light from two other parts of the sun reflected on the slit by means of combinations of prisms.

The slit used in ordinary investigation is similar to that usually employed.

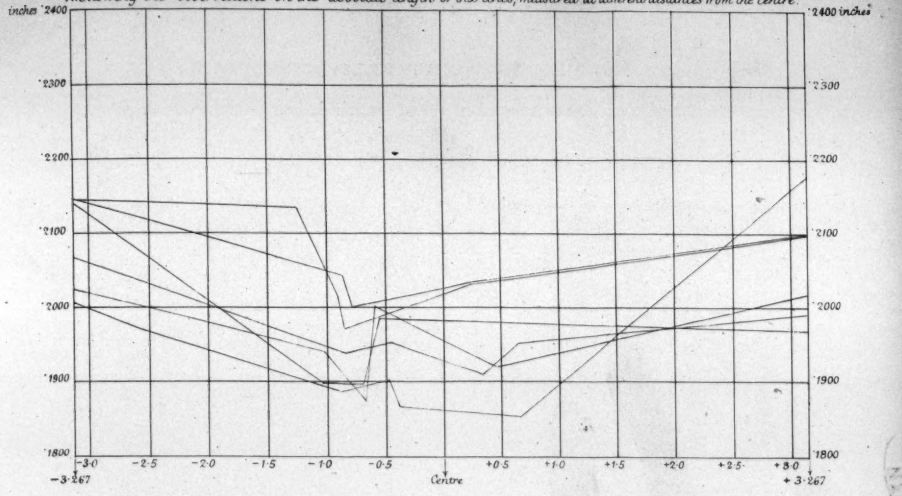
The train of prisms, the collimator, micrometer, and observing telescope are shown in fig. 3 on a larger scale. The prisms are fixed to a plate which it is possible to rotate slightly on its axis, and this, after the more obvious precautions have been taken, constitutes one of the most important adjustments.

The collimator is furnished not only with the usual focusing-screw, but with additional rackwork, which permits of the slit being placed in the image, the colour of which is coincident with the arc under examination. It is only by paying infinite attention to this point that any good results can be obtained; and when this is done, and the atmosphere is pure and calm, the interval between each of the higher cloud-domes on the sun shows an approach to the spot-spectrum, and the spectrum is a mass of horizontal lines. It is rare, however, that the atmosphere is steady enough to show this effect in its greatest perfection.

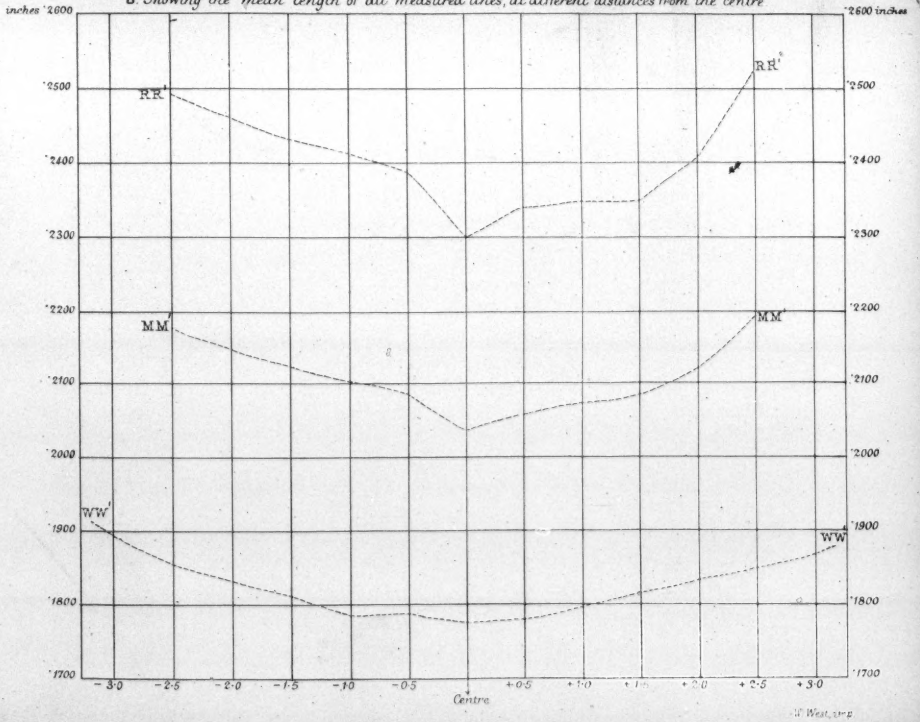
The principal point about the micrometer-arrangement is, that the micrometer-head is read by a little telescope, the eye-end of which is immediately above that of the observing telescope; this saves much time, as the position of the observer is not disturbed when readings are necessary. I have also found it convenient to supplement this arrangement by an ordinary eyepiece-micrometer for differential measures of small range.

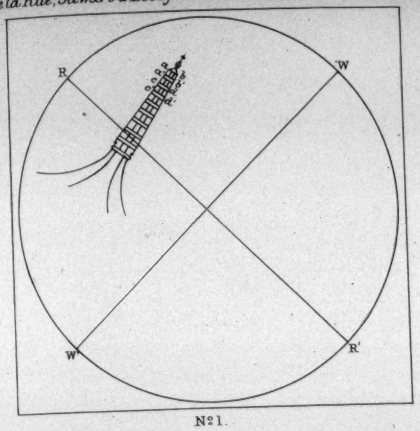
I cannot allow myself to conclude this short description without testifying to the great perfection of the optical finish of the new spectroscope, which enables me to obtain a solar spectrum of the utmost purity. My thanks are due to Mr. BROWNING for coming to my aid as he did, and employing his well-known skill in the construction of an instrument at once so unique, portable, and powerful—a task accomplished at last with results most satisfactory to myself and creditable to him.

A. Showing the observations on the absolute length of six lines, measured at different distances from the centre.



B. Showing the mean length of all measured lines, at different distances from the centre





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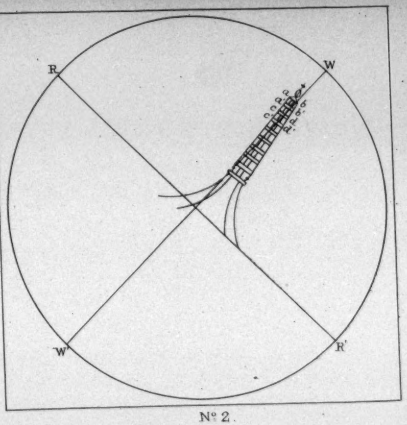

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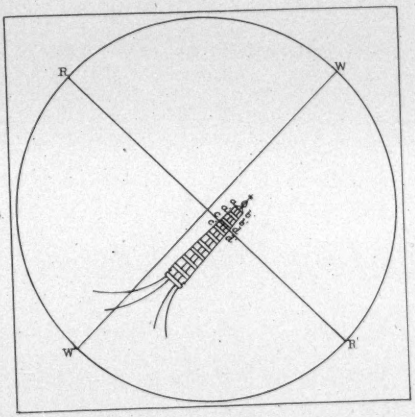

Nº 3.

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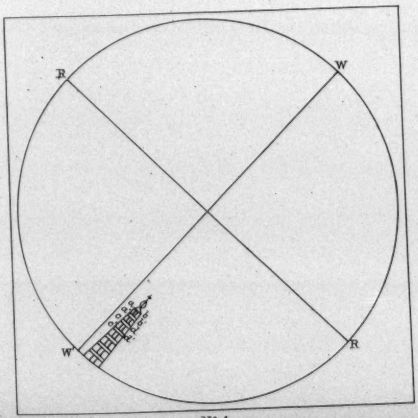

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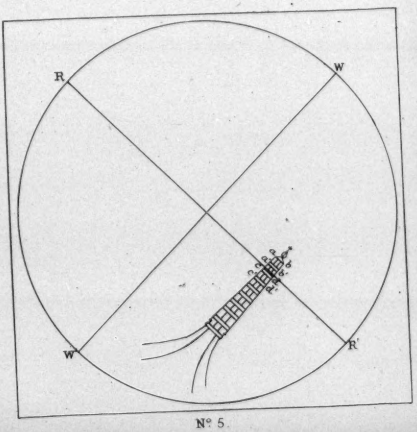
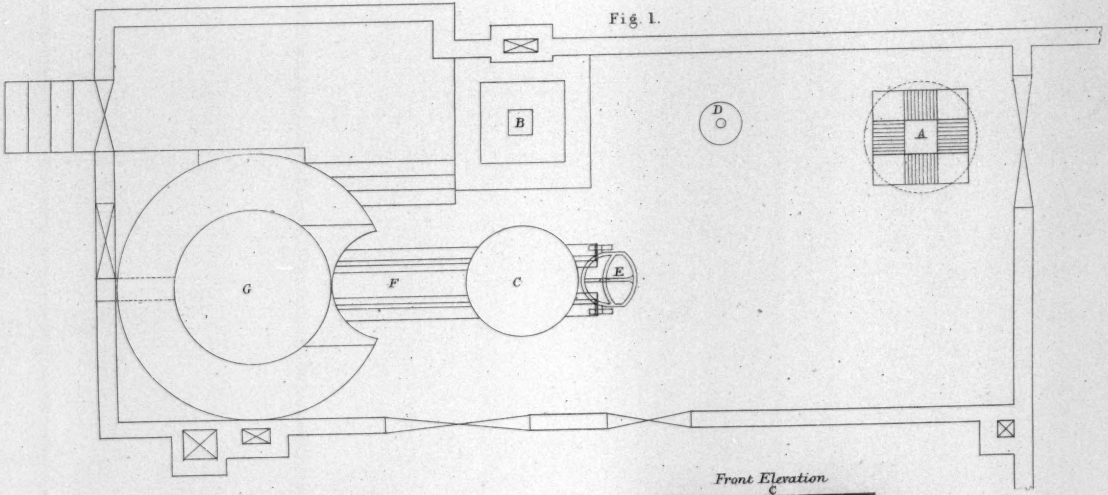


Nº 5.

Fig. 1.



Annealing Oven (1)

Fig. 2

Side Elevation

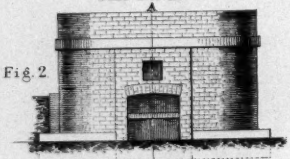


Fig. 3.

Front Elevation

Fig. 4.

Section thr° G.H.

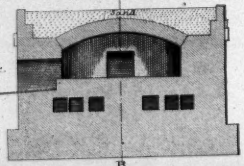
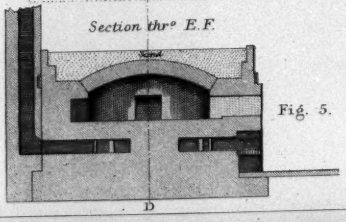


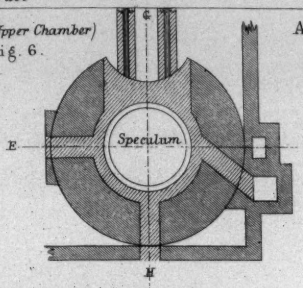
Fig. 5.

Section thr° E.F.



Annealing Oven (2)

Plan (Upper Chamber)
Fig. 6.



Plan – Flues
Fig. 7.

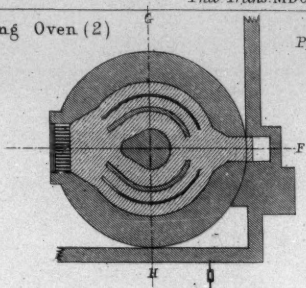


Fig. 8.

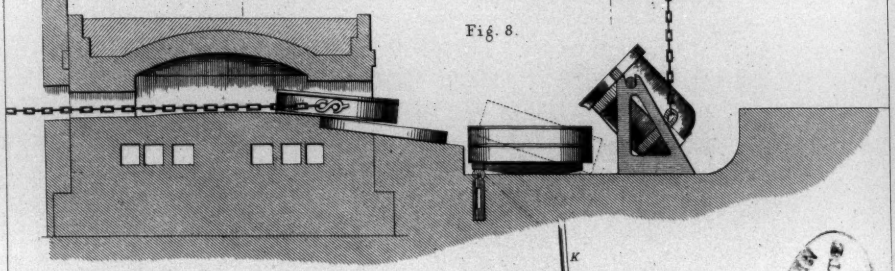
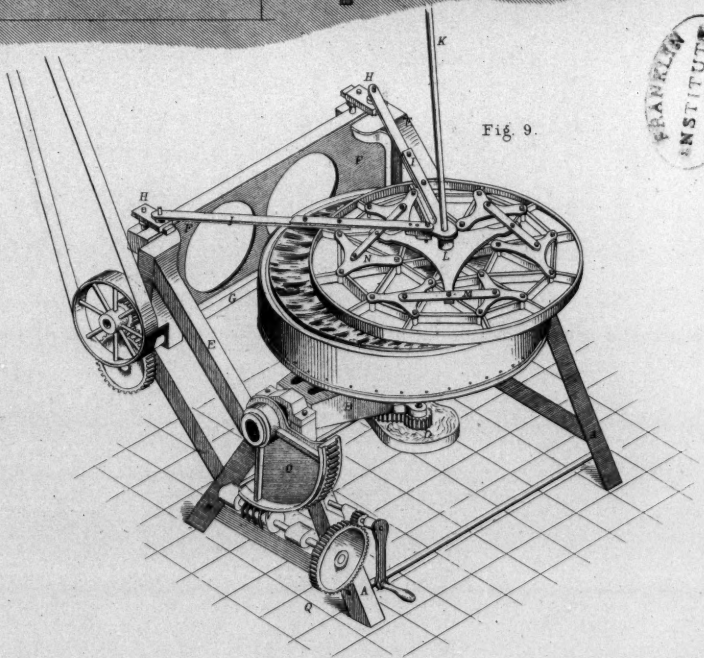


Fig. 9.



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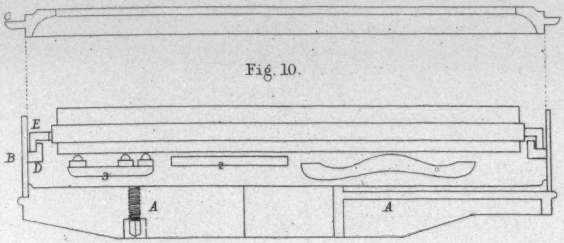

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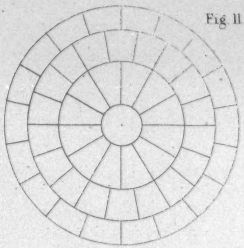

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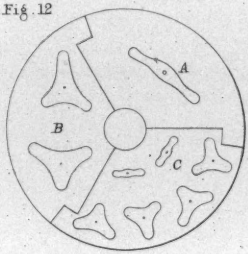

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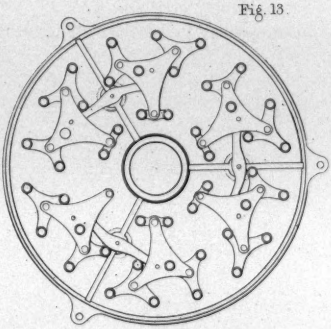

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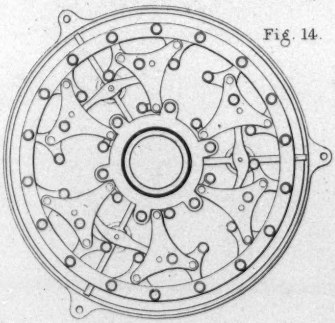

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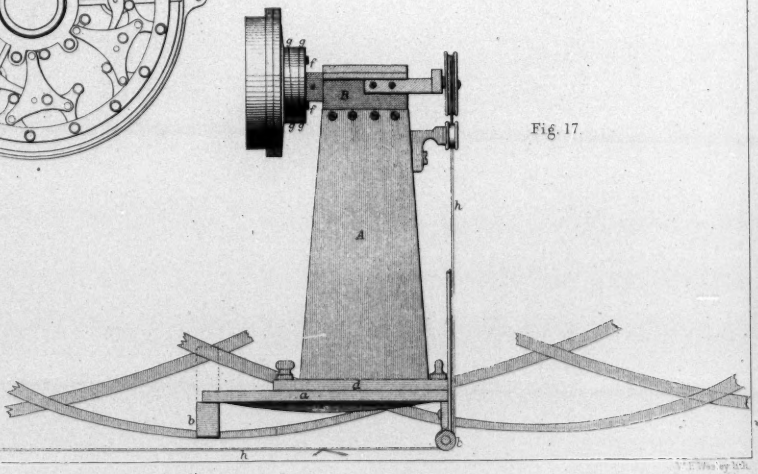

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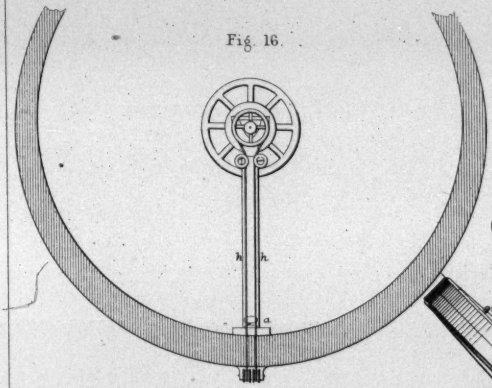

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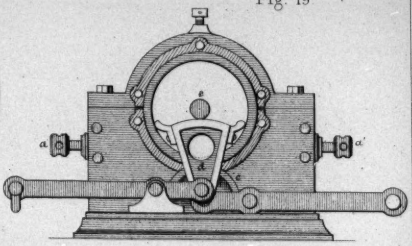

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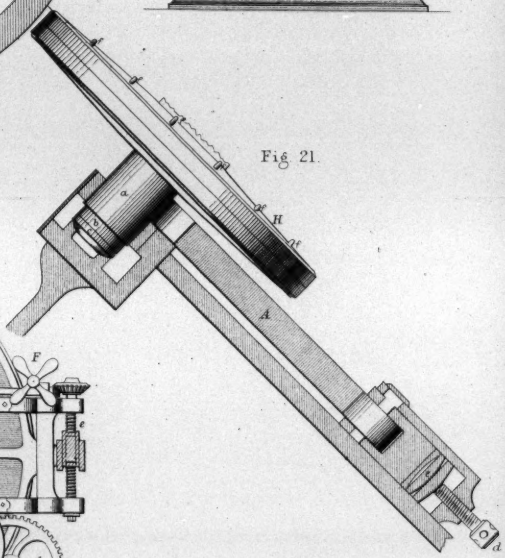

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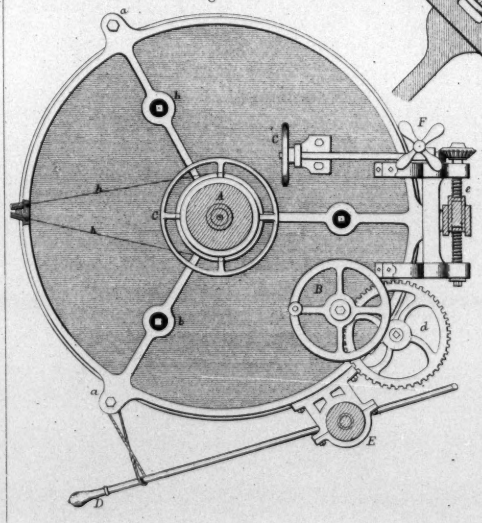

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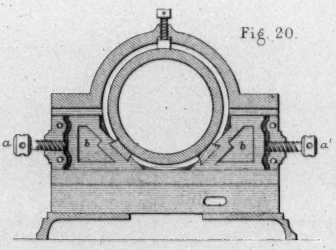
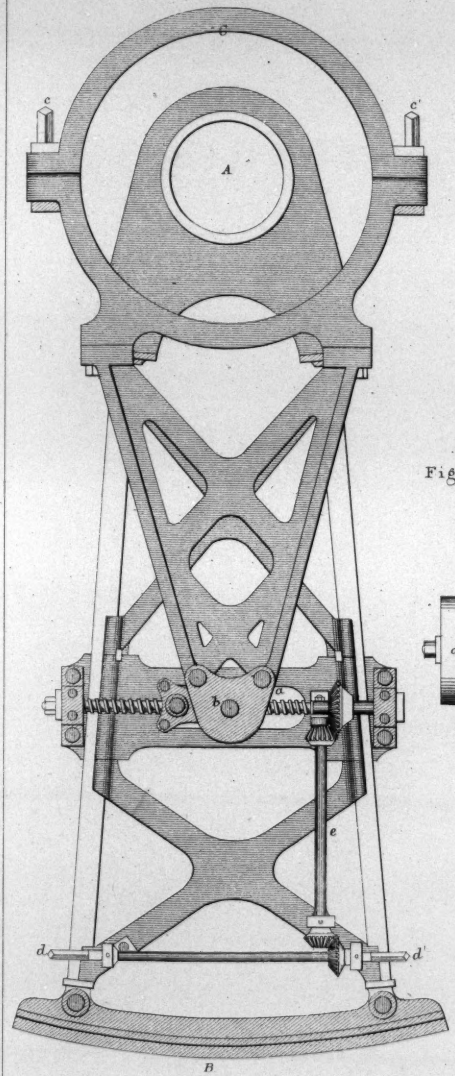


W.H. Wesley Ph.

Fig. 22.



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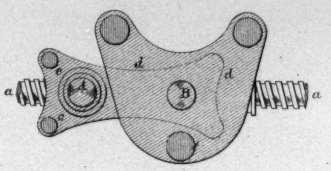

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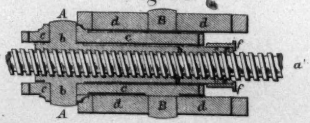

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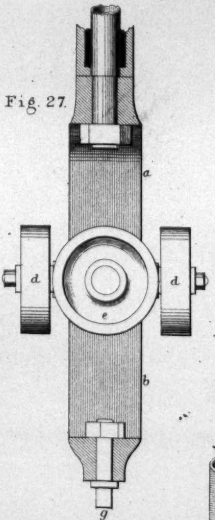

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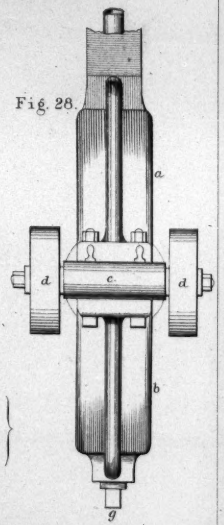

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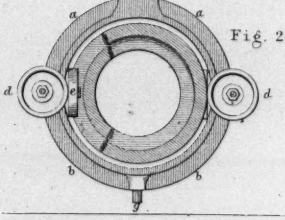

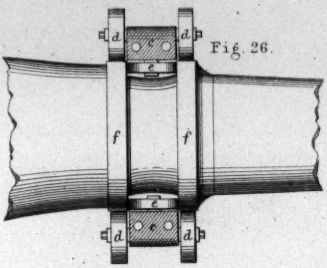

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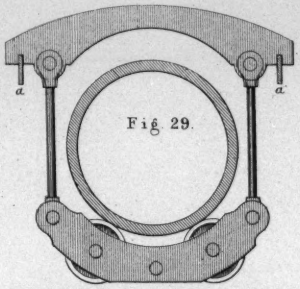

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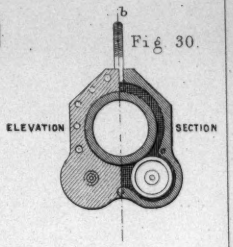

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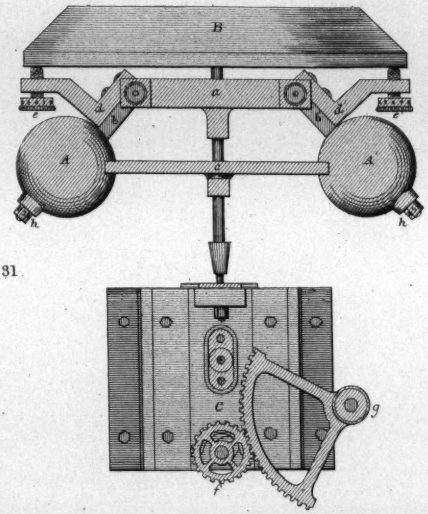

Fig. 31.

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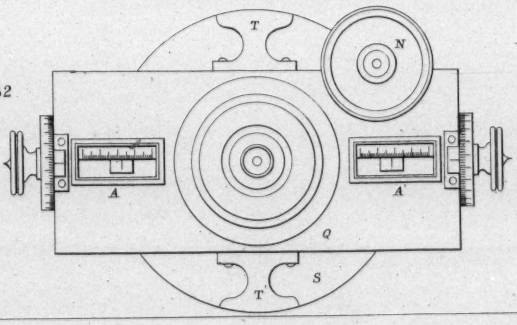
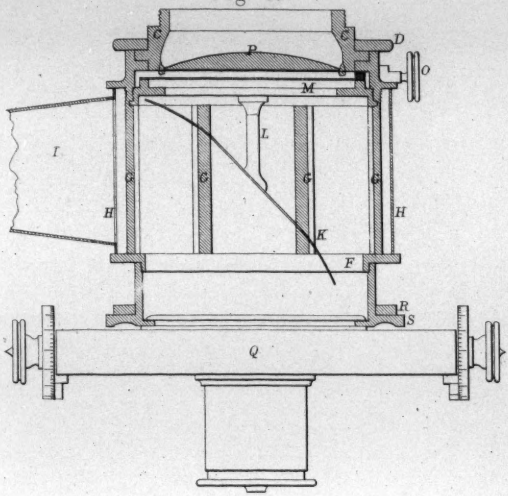


Fig. 32.

Fig. 33.



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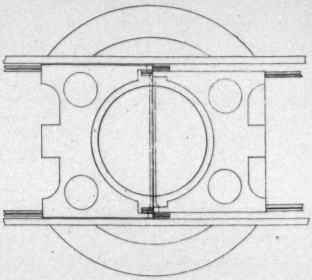

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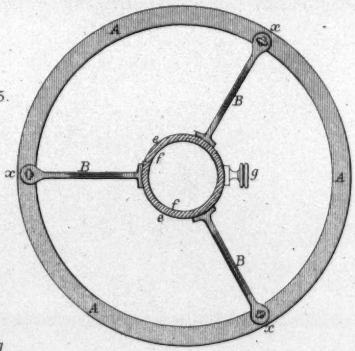

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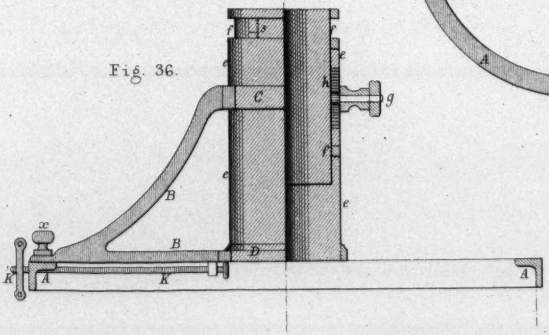

W.H.Wesley lith.

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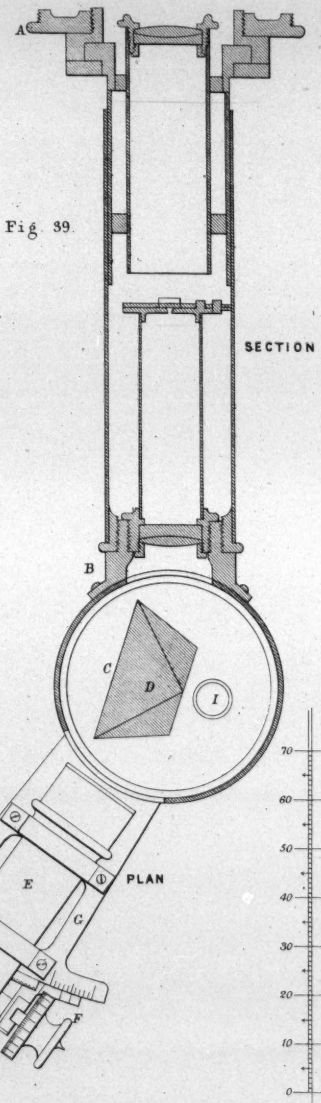

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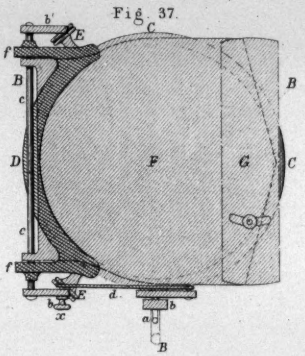

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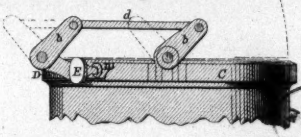

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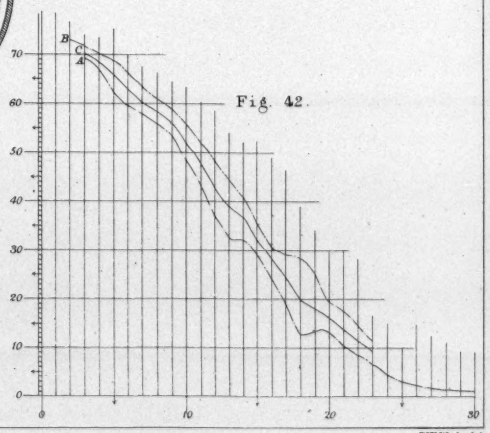

W.H. Wesley lith.

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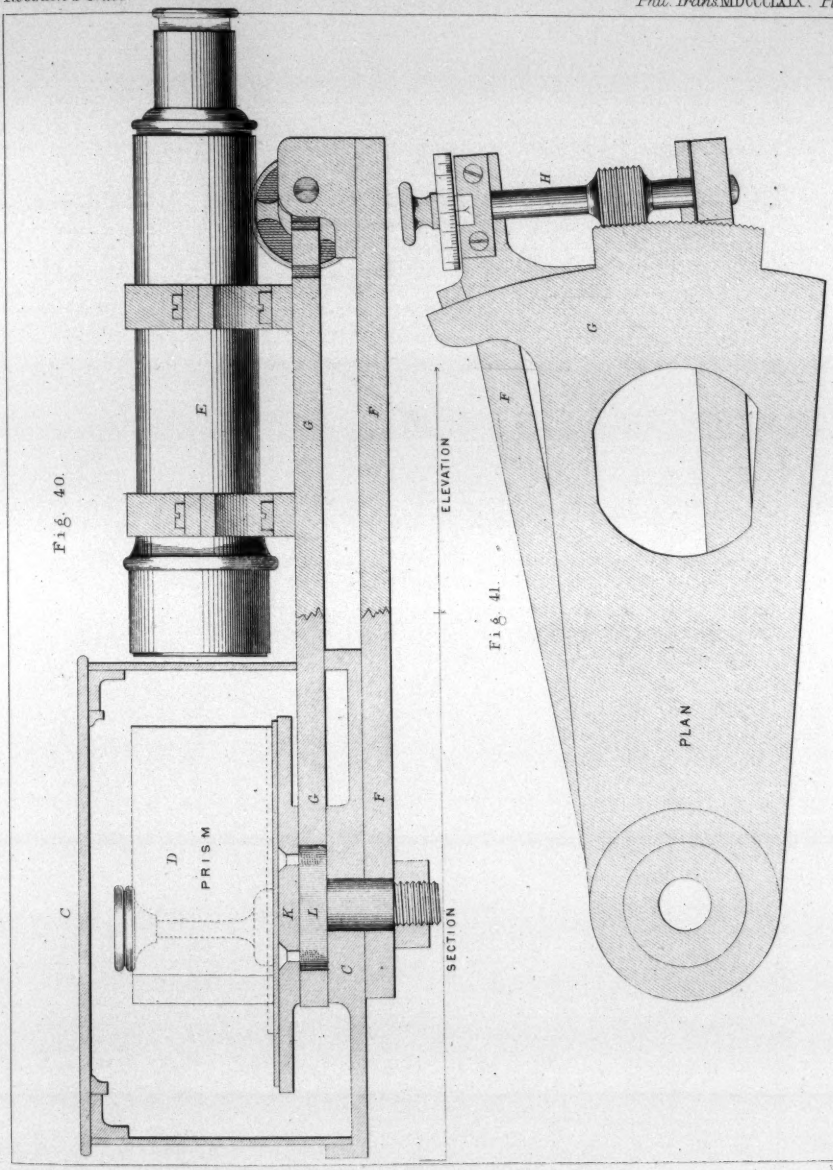

Fig. 40.

Fig. 41.

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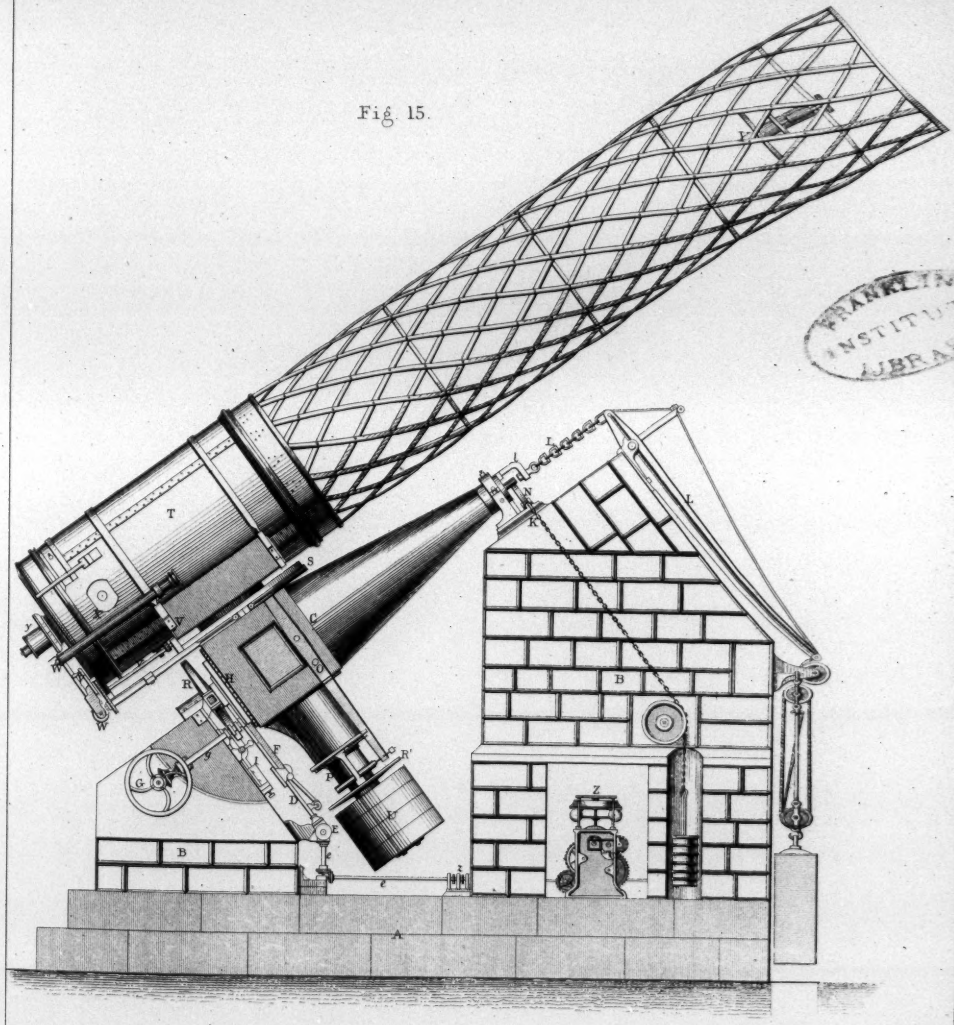

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W. H. Wesley lith.

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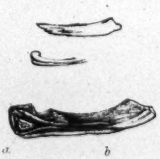

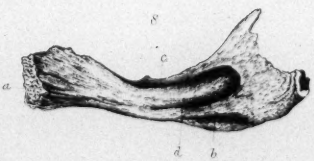

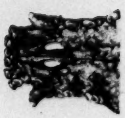

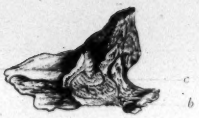

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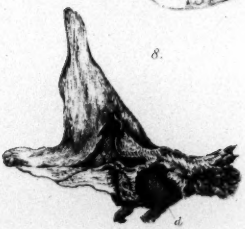
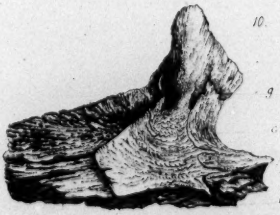
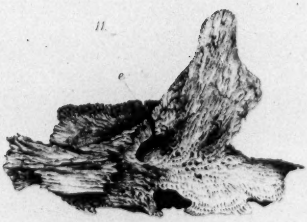
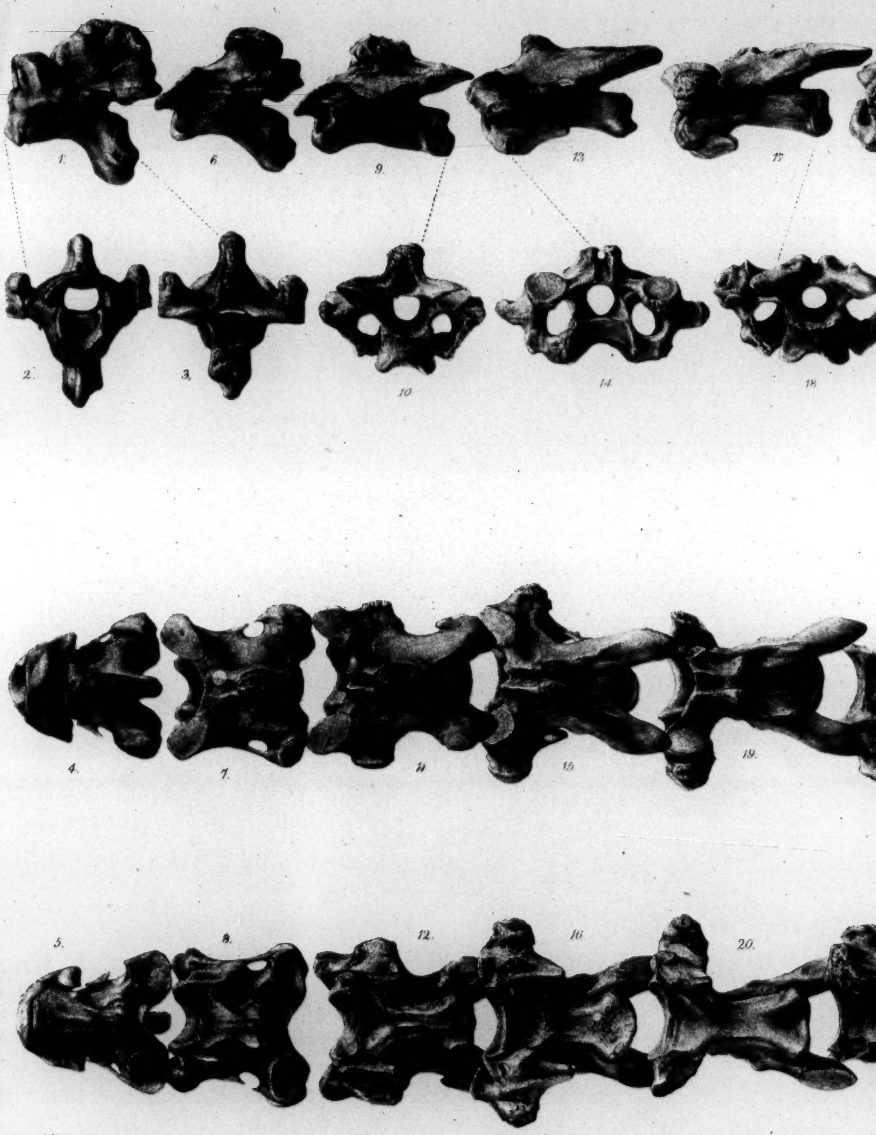


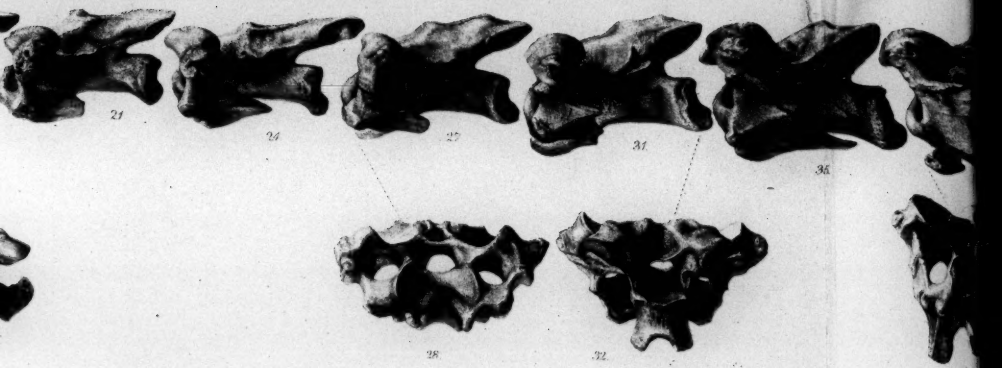




A&E Newton.



G.H. Ford ad nat. lith.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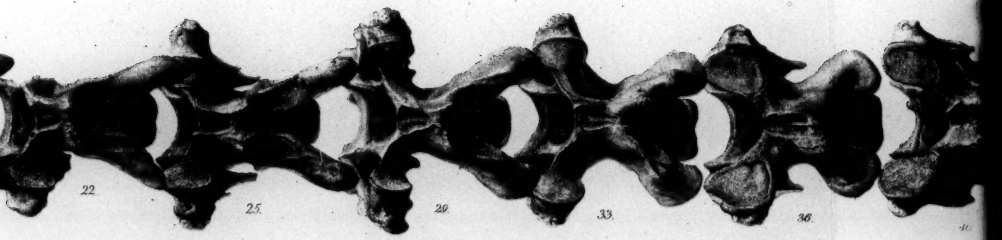
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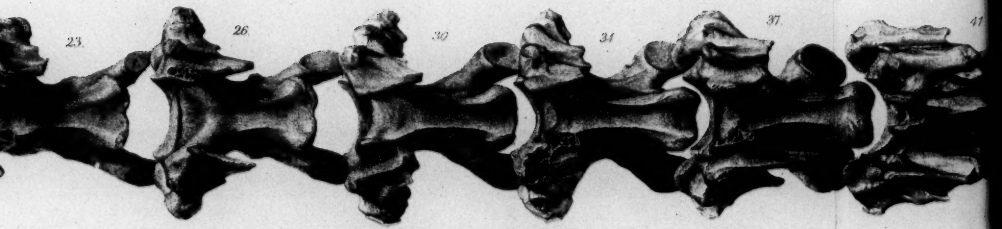
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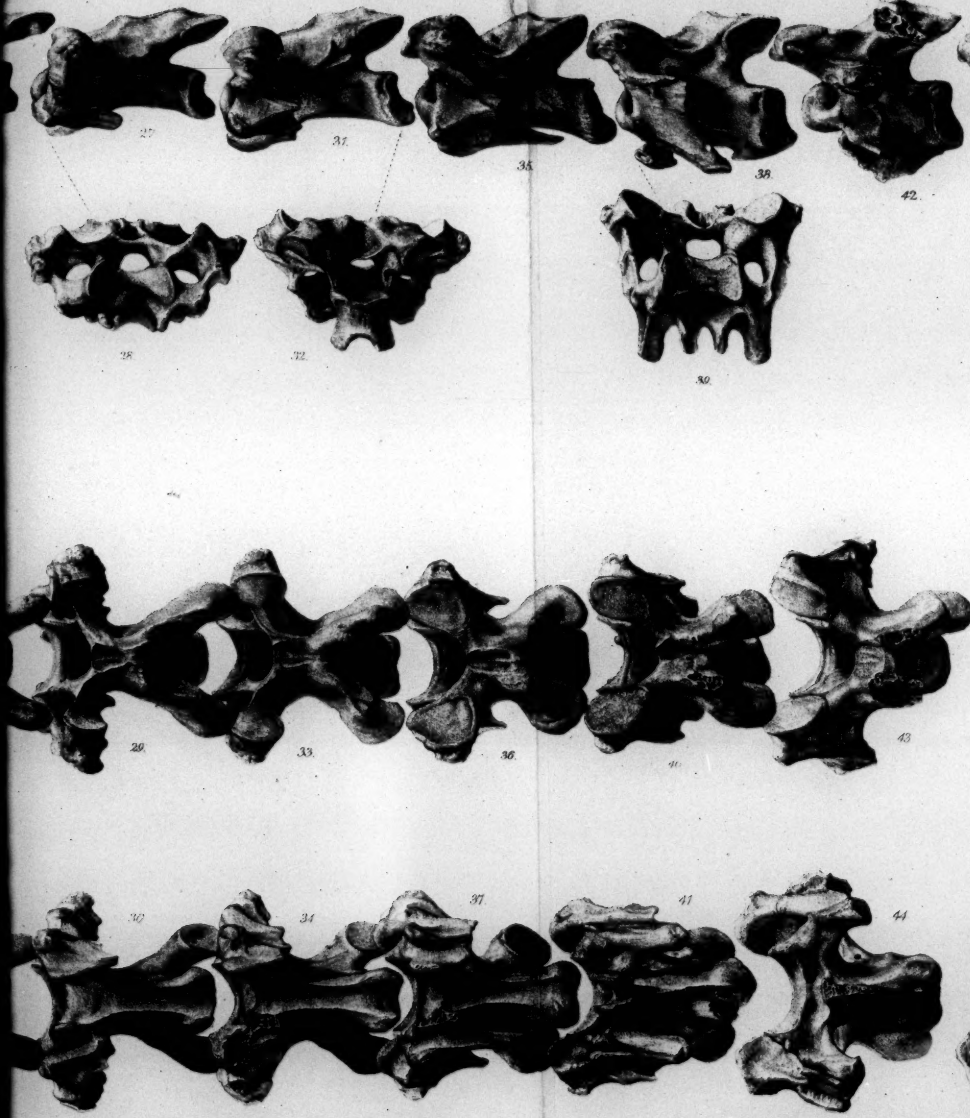
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W. West. imp.

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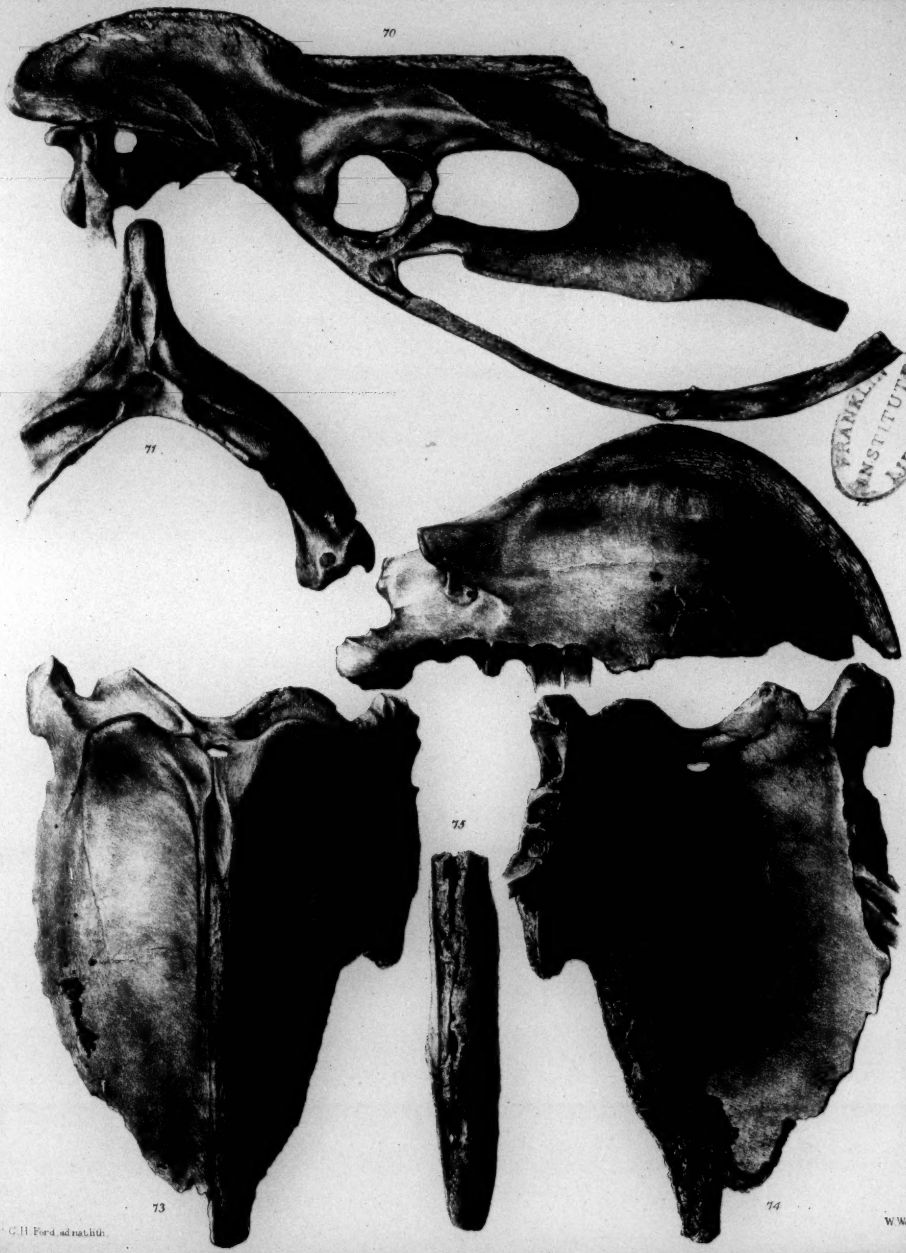

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W. West. imp.

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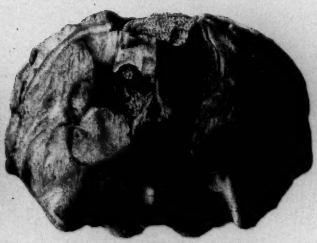

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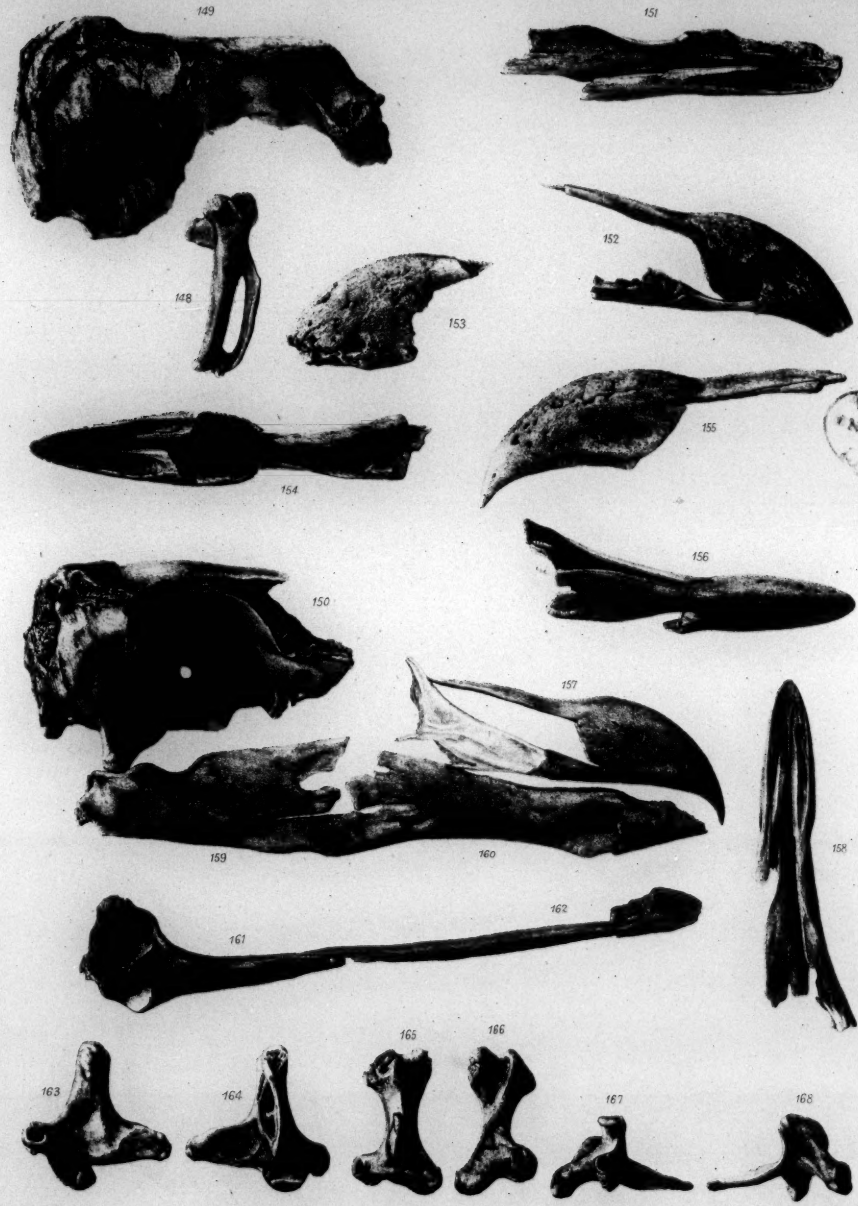

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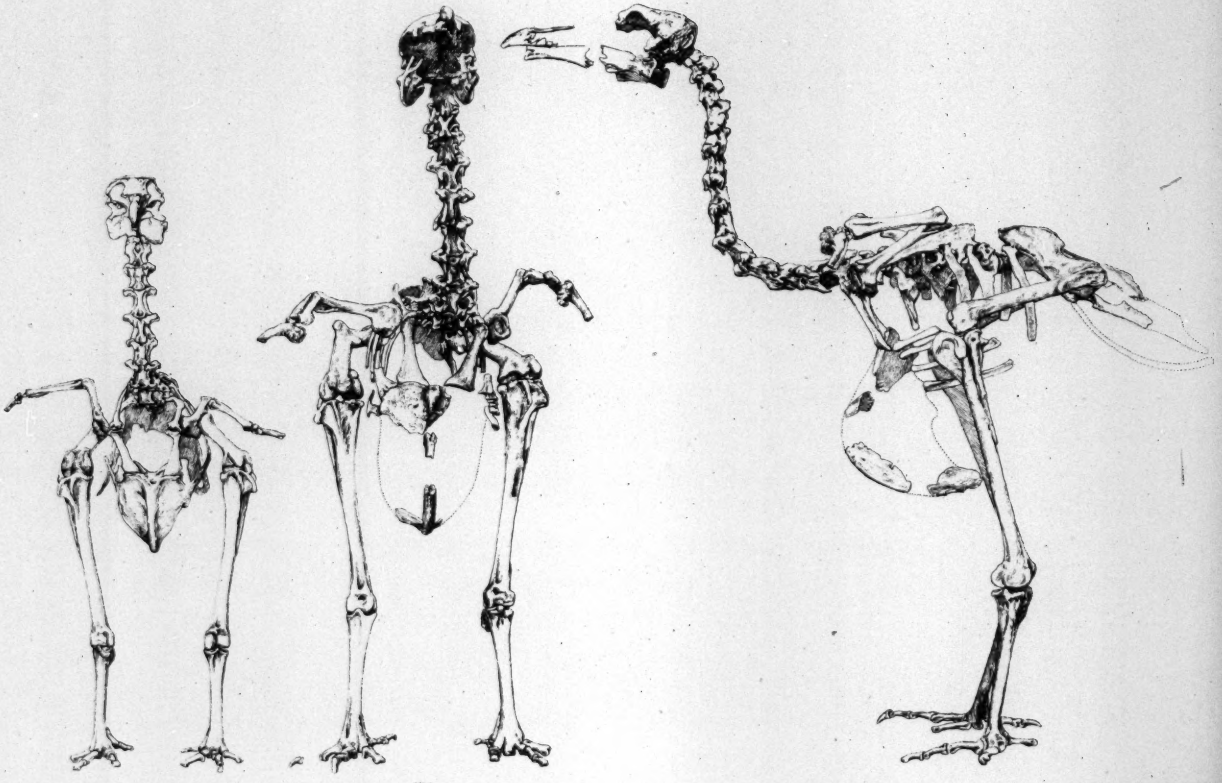

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S. Arling Photog.r
G.H. Ford lith.

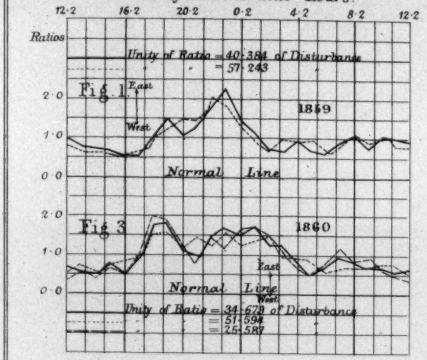
W. West imp.

Diurnal Variations of Easterly and of Westerly Disturbance with different separating values.

Separating Value = 1′.0
= 1′.4
= 1′.7

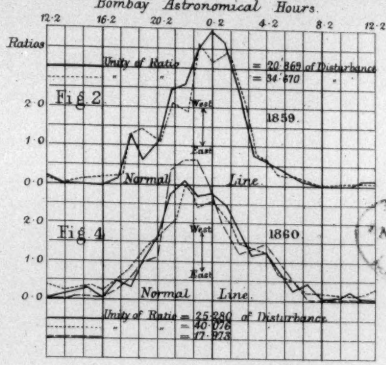
Easterly Disturbances.

Bombay Astronomical Hours.



Westerly Disturbances.

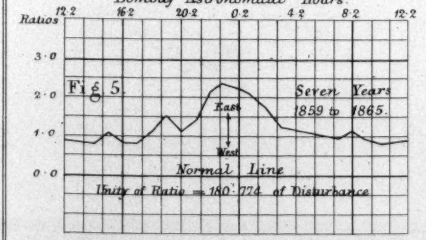
Bombay Astronomical Hours.



Diurnal Variations of Easterly and of Westerly Disturbance.
Separating Value = 1′.4

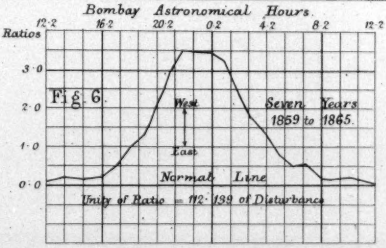
Easterly Disturbances.

Bombay Astronomical Hours.



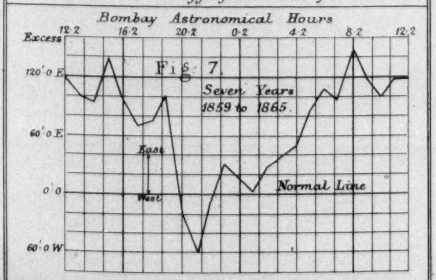
Westerly Disturbances.

Bombay Astronomical Hours.



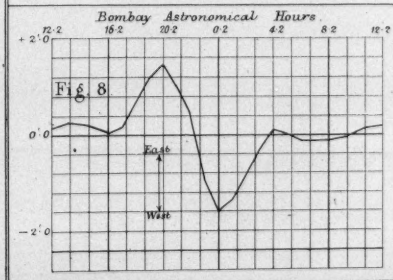
Disturbance Diurnal Variation,
or the excess, at each hour, of the aggregate of Easterly Disturbances over the aggregate of Westerly Disturbances.

Bombay Astronomical Hours



Mean Diurnal Variation of Declination for the seven years 1859 to 1865.

Bombay Astronomical Hours



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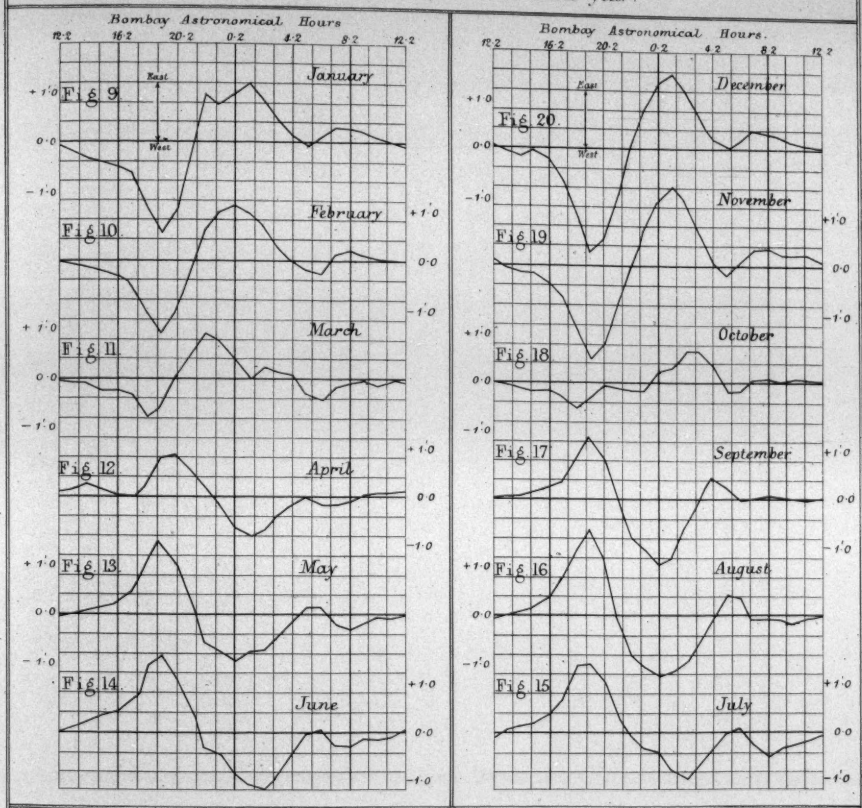
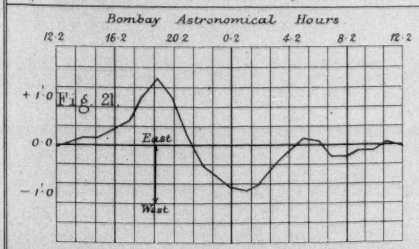
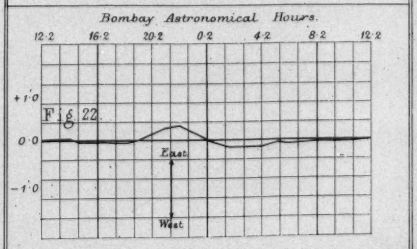
Excess of the Diurnal Variation of Declination in each month over the Mean Diurnal Variation for the whole year.



Excess of the Diurnal Variation of Declination for the half-year April to September over the Mean Diurnal Variation for the whole year.

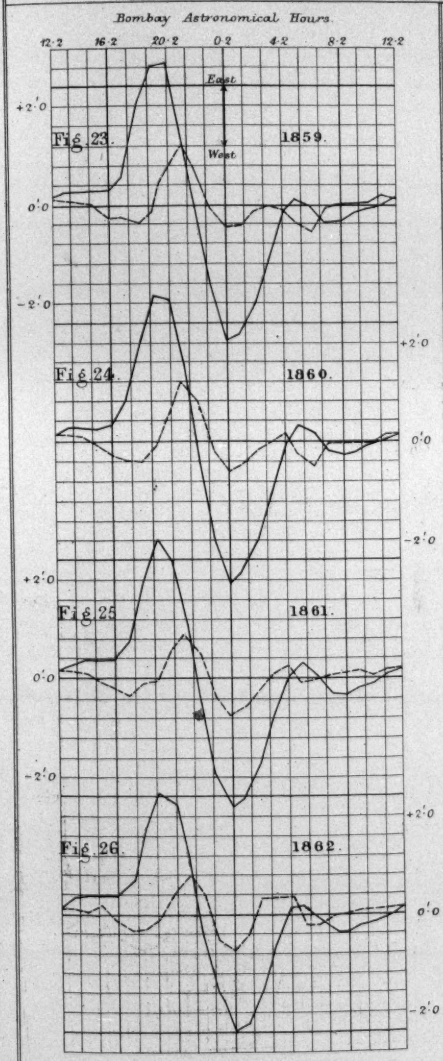


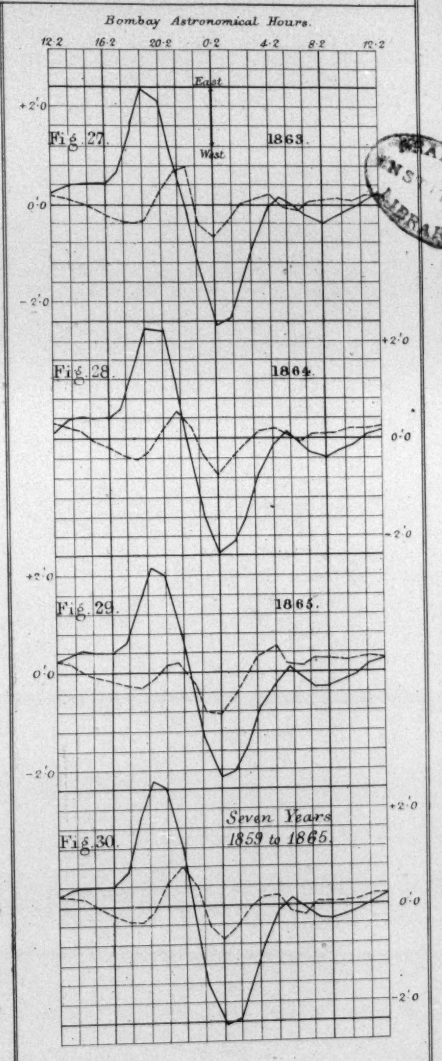
Excess of the Diurnal Variation of Declination for the half-year January to June over the Mean Diurnal Variation for the whole year.



Mean Diurnal Variation of Declination for the half-years April to September and October to March in each of the years 1859 to 1865 and for the whole Period of seven years.

——————— *April to September.*
– – – – – – – – *October to March.*





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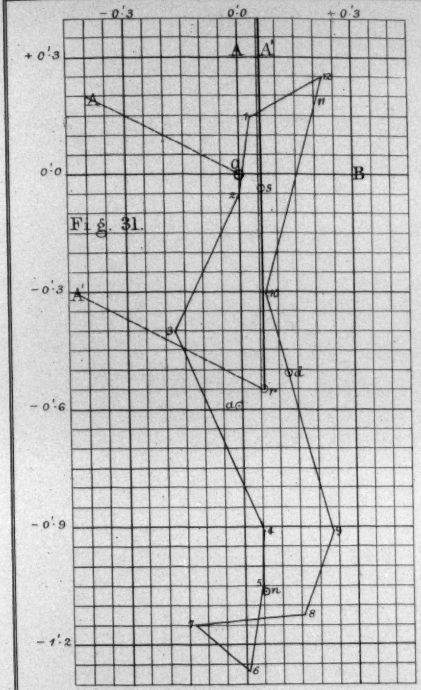
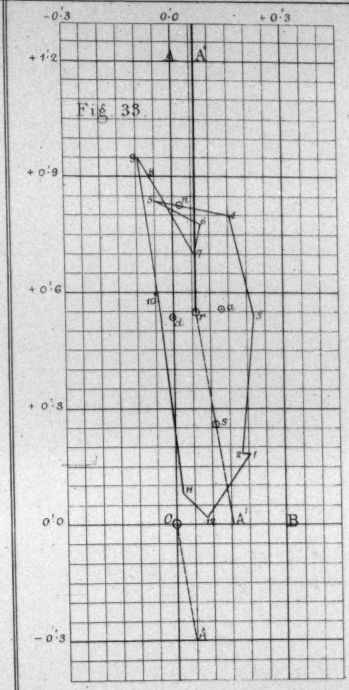
Values of the coefficients of the series (y) which expresses the Mean Diurnal Variation of Declination at Bombay in each month of the year. See paragraph 14. The numbers 1, 2, 3, 4, &c represent the months January, February, March, April &c

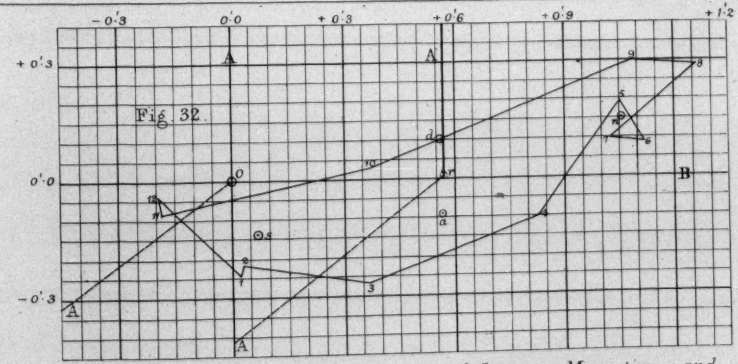
A1 and B1



A2 and B2



A3 and B3



The directions OA and rA correspond to noon of Gottingen Mean time, and the directions OA and rA (broken lines) to noon of Bombay Mean time.

W H Wesley lith

Values of the coefficients of the series (y) which expresses the Mean Diurnal Variation of Declination at Bombay in each of the half-years of 1859 to 1865. See paragraph 14. The numbers 9, 0, 1, 2, 3, 4 and 5 represent the years 1859 to 1865 respectively.
_____ *April to September.* ---------- *October to March.*

A1 and B1

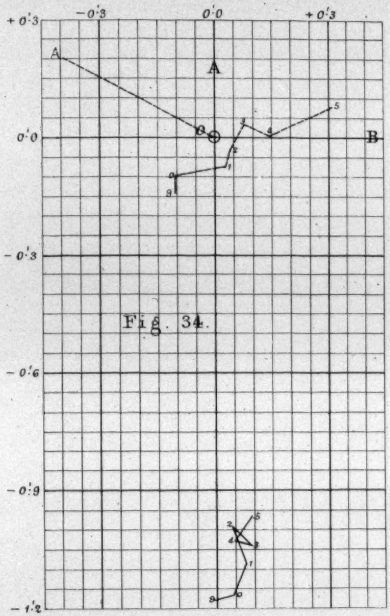


Fig. 34.

A2 and B2

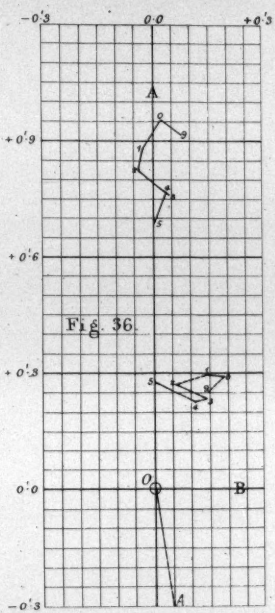


Fig. 36.

A3 and B3

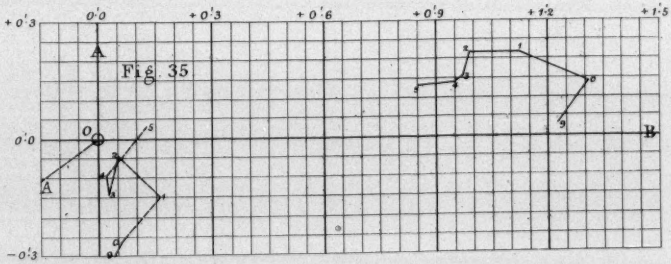
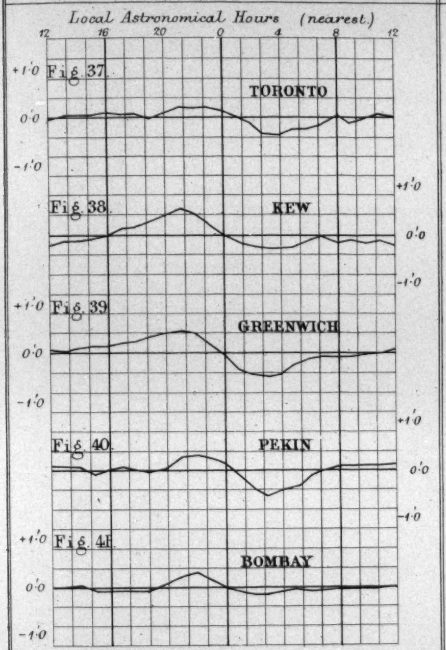


Fig. 35

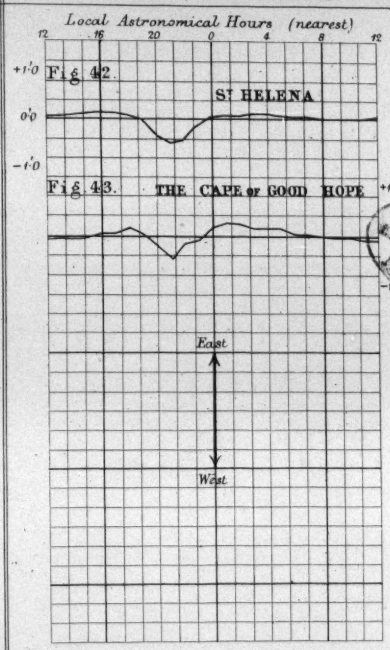
The directions OA correspond to noon of Göttingen Mean time, and the directions OA (broken lines) to noon of Bombay Mean time.

Excess of the Diurnal Variation of Declination for the half-year January to June over the Mean Diurnal Variation for the whole year at different stations.

North Latitude Stations.



South Latitude Stations.



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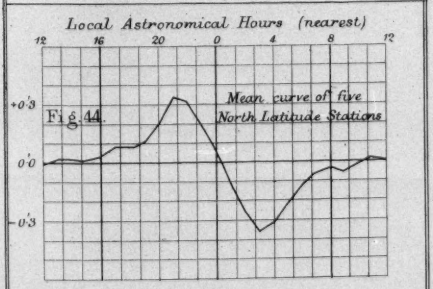
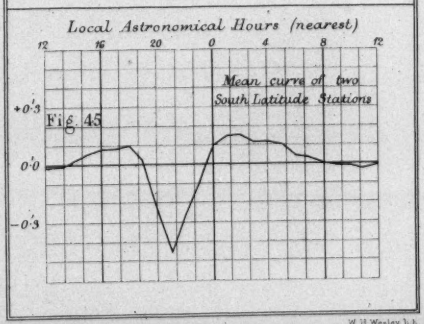
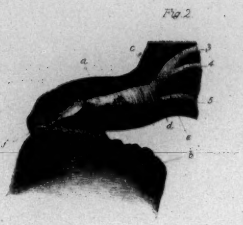
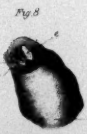
Mean for the five North Latitude Stations, of the Excess Variations represented by Figures 37 to 41.
Scale enlarged four times.

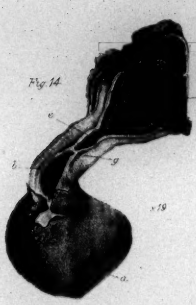


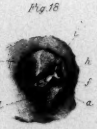
Mean for the two South Latitude Stations, of the Excess Variations represented by Figures 42 and 43.
Scale enlarged four times.

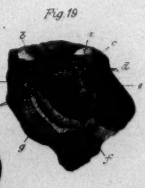












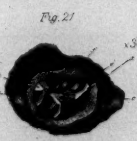


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Fig. 28

Fig. 29

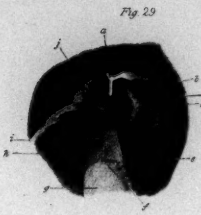


Fig. 30

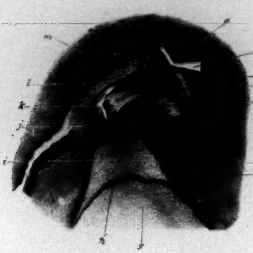


Fig. 31

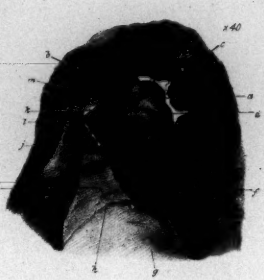


Fig. 32

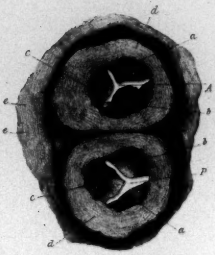


Fig. 33

Fig. 34

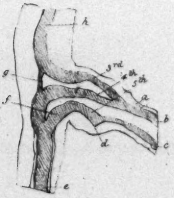


Fig. 35

Fig. 36

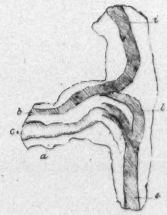


Fig. 37

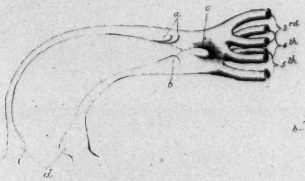


Fig. 38

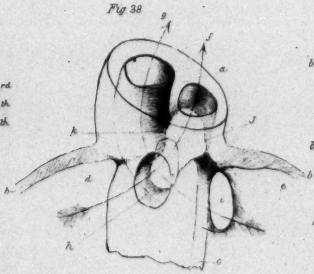
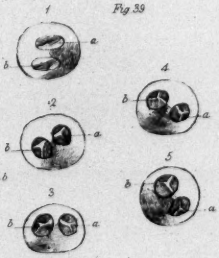


Fig. 39



W.H. Wesley lith.

Curves representing the Diurnal Change in Magnitude and Direction of the Magnetic Forces in the Horizontal Plane acting on the North End of the Needle at the Royal Observatory, Greenwich, for each Month in the Year, on the Mean of the Years 1858 to 1863.

The numerals on the Curves represent Hours of Greenwich Mean Solar Time.

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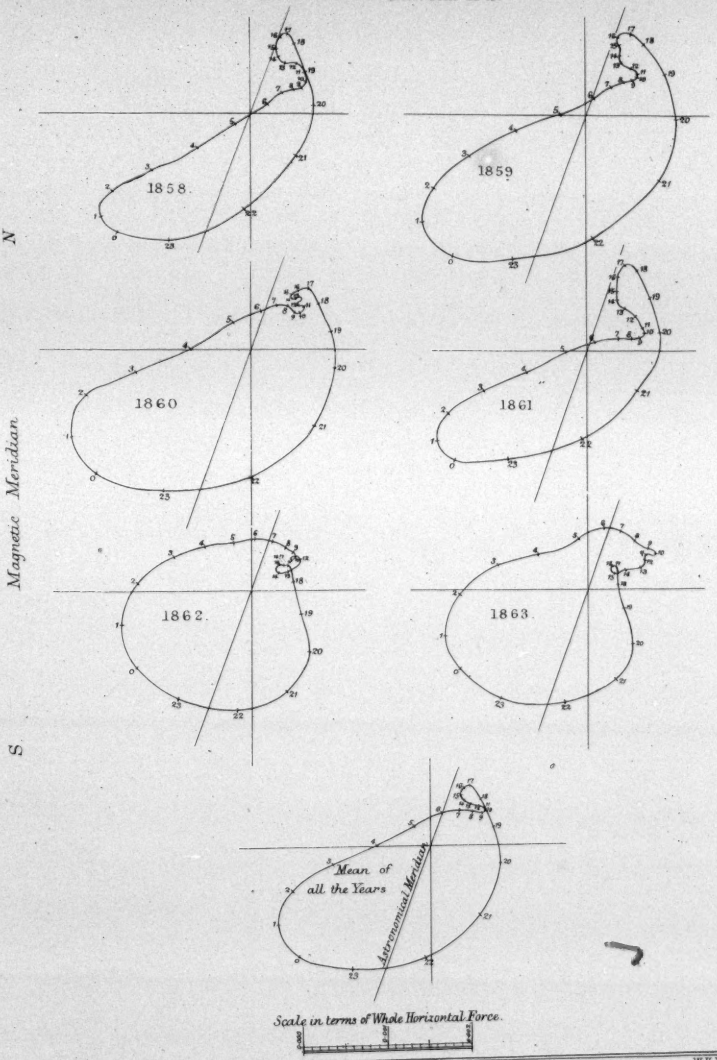
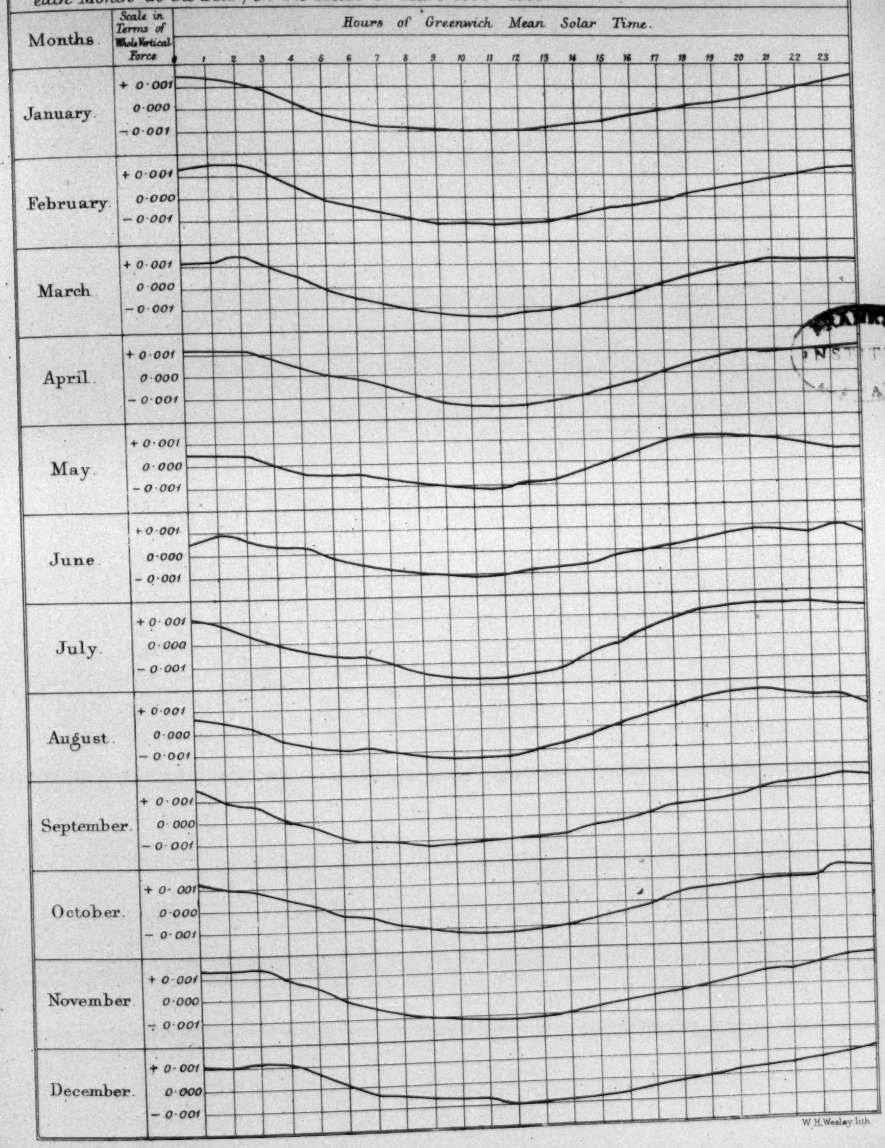
Curves representing the Diurnal Change in Magnitude and Direction of the Magnetic Forces in the Horizontal Plane acting on the North End of the Needle, at the Royal Observatory, Greenwich, for the Years 1858 to 1863, from the Means of the Observations at corresponding Hours through each Year.

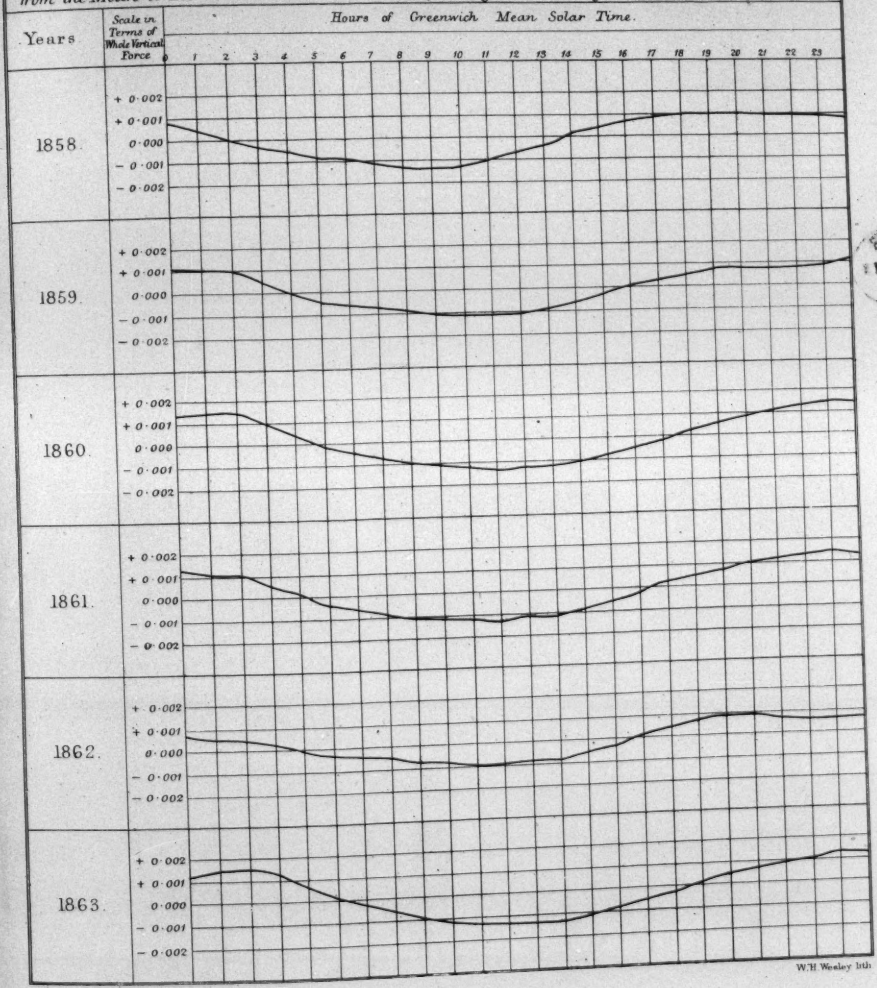
The numerals on the Curves represent Hours of Greenwich Mean Solar Time.



Curves representing the Diurnal Change in Magnitude and Sign of the Magnetic Vertical Force, acting on the North End of the Needle at the Royal Observatory, Greenwich, for each Month in the Year, on the Mean of Years 1858–1863.



Curves representing the Diurnal Change in Magnitude and Sign of the Vertical Magnetic Force acting on the North End of the Needle at the Royal Observatory, Greenwich, for the Years 1858 to 1863, from the Mean of all the Observations at corresponding Hours through each Year.



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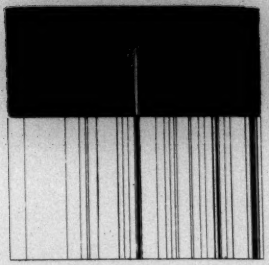

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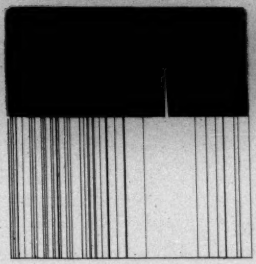

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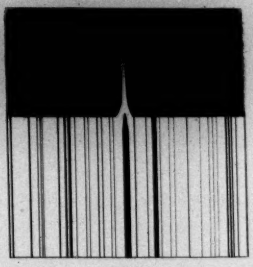

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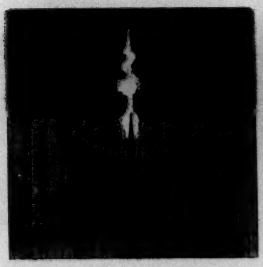

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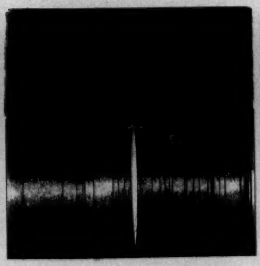

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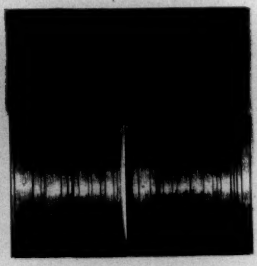

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8.

9.

J. N. Lockyer del. W.H. Wesley lith.

W. West imp.

Fig. 2.

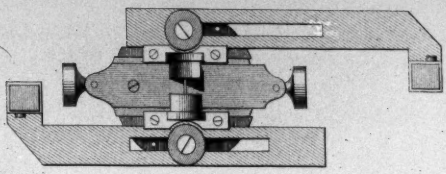

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Fig. 3.

Fig. 1.